February 10, 2022

# Notes on Numerical Simulation for Acoustics

David Herrin
University of Kentucky

Vibro-Acoustics Consortium

# Overview

- Fundamentals
- Example 1 – Engine Cover
- Example 2 – HVAC Duct
- Example 3 – Concentric Perforated Tube Muffler

# The Wave and Helmholtz Equation

$$\nabla^2 p = \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2}$$

The wave equation

$$\nabla^2 p - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0$$

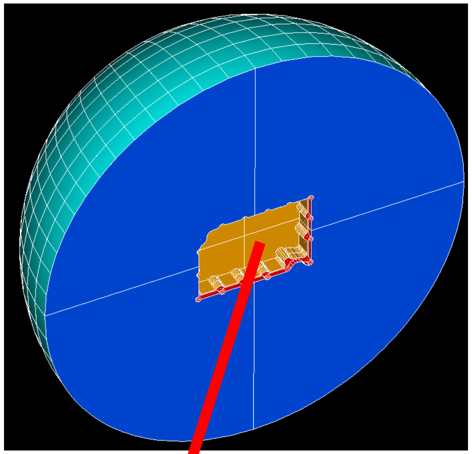
Helmholtz equation (assume constant frequency excitation)

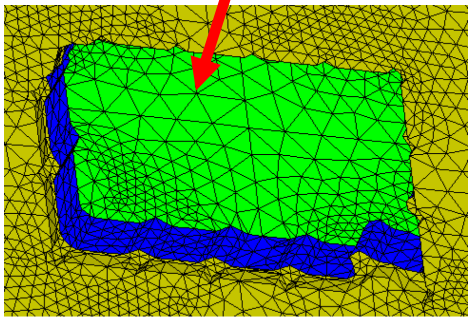
$$\nabla^2 p + k^2 p = 0$$

$$k = \frac{\omega}{c}$$

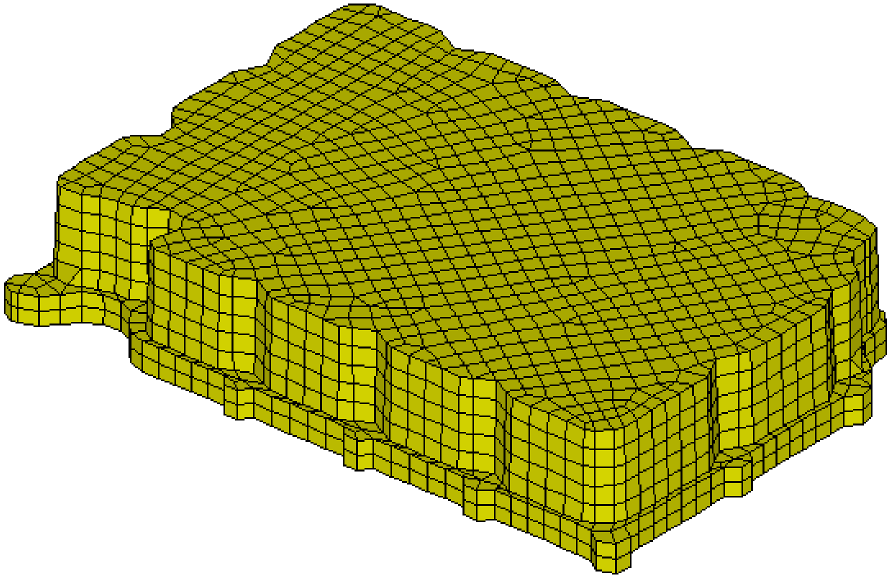
# Finite and Boundary Element Analysis

## FEM

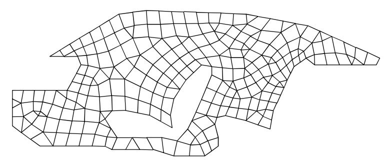
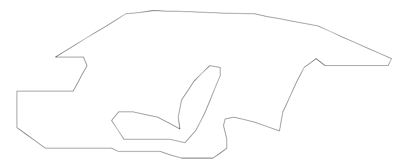




## BEM



# Finite and Boundary Element Analysis

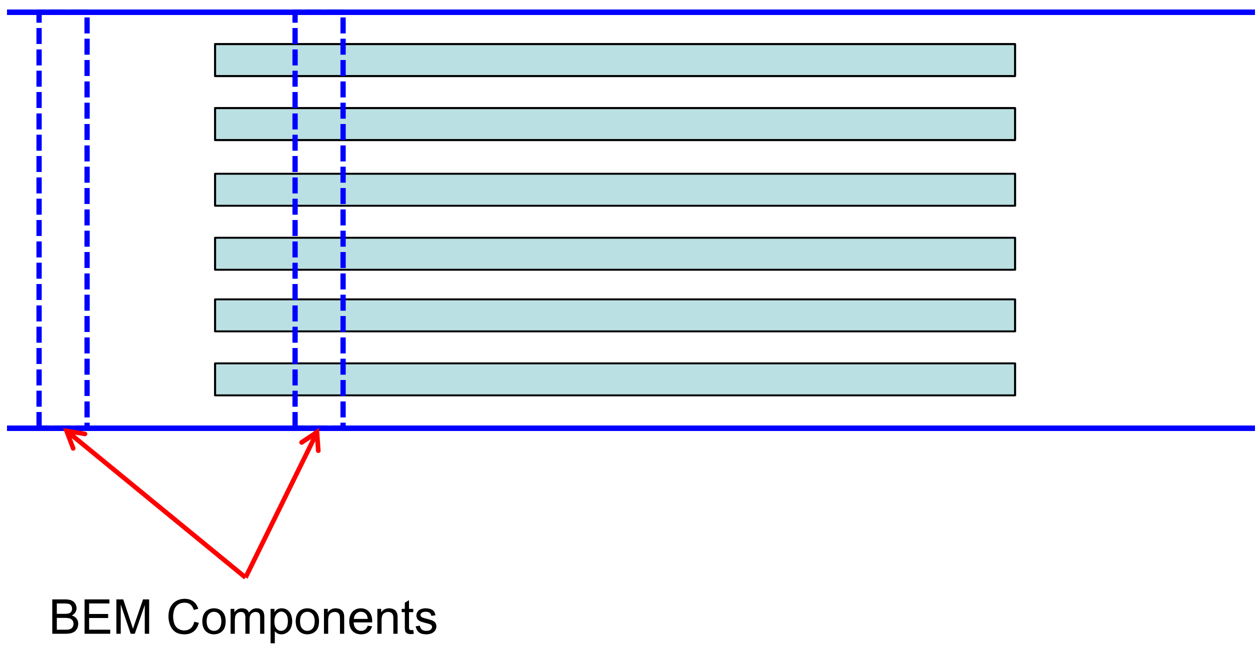
| | FEM | BEM |
|---|---|---|
| | Domain method  | Boundary method  |
| Discretization | volume (3-D) | surface (2-D) |
| Calculation of modes | straightforward | expensive |
| Sound radiation | AML or infinite elements | automatic |
| Different materials | straightforward | multi-domain |
| Solution time (large models) | faster for direct frequency response | slower (fast multi-pole can speed it up) |

# Software Landscape

| Software | FEM | BEM |
|---|---|---|
| Ansol Coustyx | No | Yes |
| ANSYS | Yes | No |
| Comsol Acoustics | Yes | Yes |
| ESI VA-One | Yes | Yes |
| MSC Actran | Yes | No |
| Siemens Simcenter | Yes | Yes |
| Simulia Wave6 | Yes | Yes |

# Muffler Analysis Program

- Tim Wu – University of Kentucky
- BEM based



# Boundary Conditions

- Normal Particle Velocity
  - Default BC is $v_n = 0$
  - Commonly read in from structural FEM
- Sound Pressure
  - Not commonly used
- Impedance ($Z$) or Admittance ($A$)
  - Sound absorption
  - Anechoic termination

$$Z = \frac{p}{v_n} = \frac{1}{A}$$

# Point Monopole Source

- Sound Pressure Amplitude ($A_+$)

$$p = \frac{A_+}{r} e^{-jkr}$$

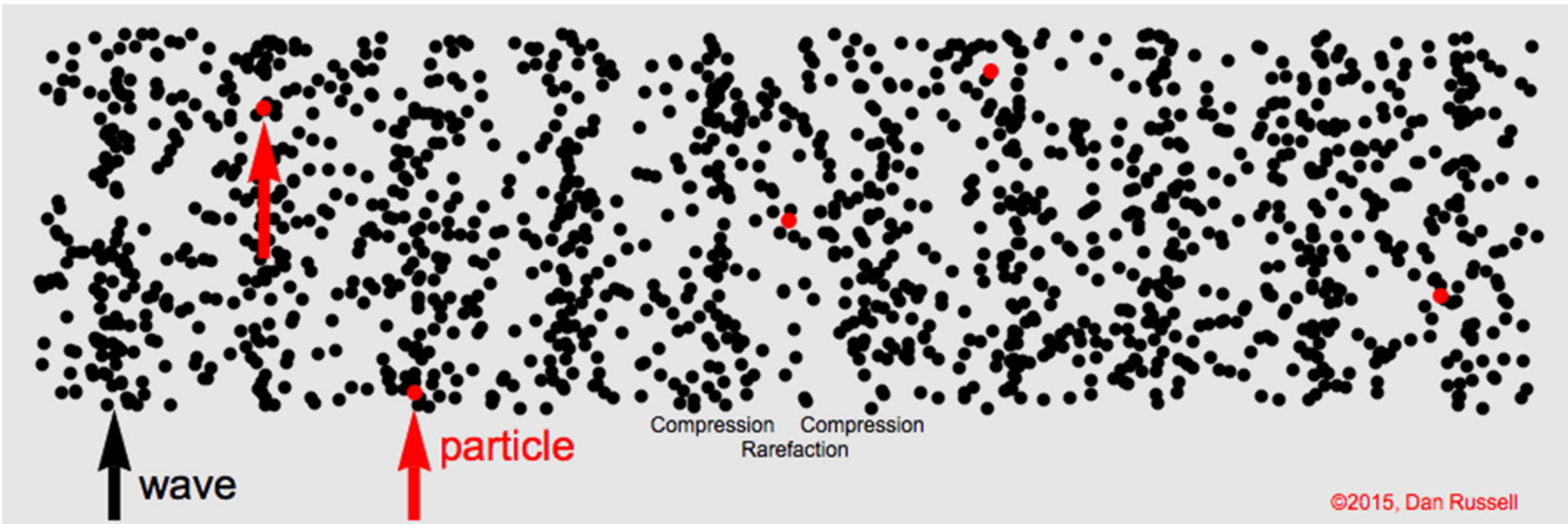
- Volume Velocity ($Q$)

$$Q = \frac{4\pi A_+}{j\rho ck}$$

- Sound Power ($W$)

$$W = \frac{\rho c k^2}{4\pi} Q^2$$

# Acoustic Wavelength




$$\lambda_a = \frac{c}{f}$$

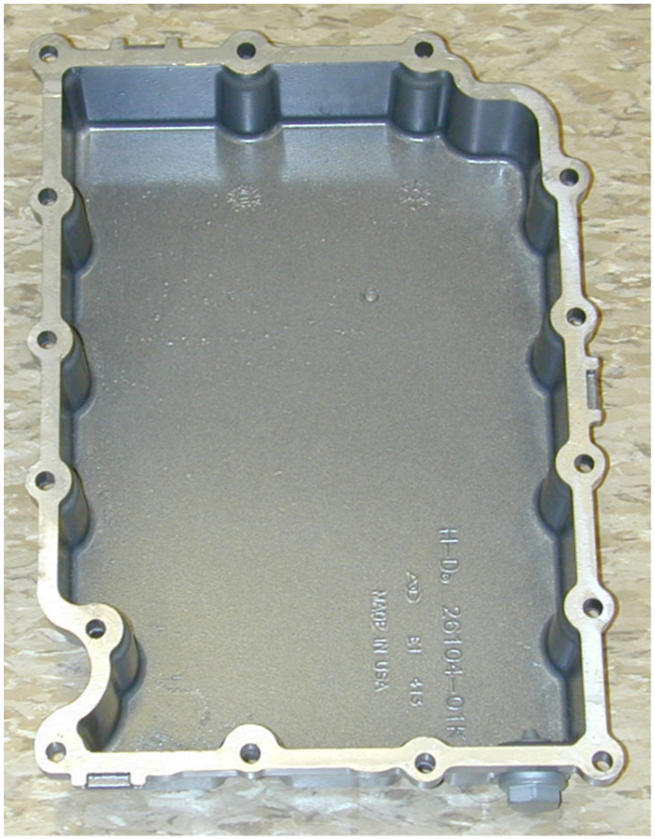
6 to 10 linear elements per acoustic wavelength.

https://www.acs.psu.edu/drussell/demos.html

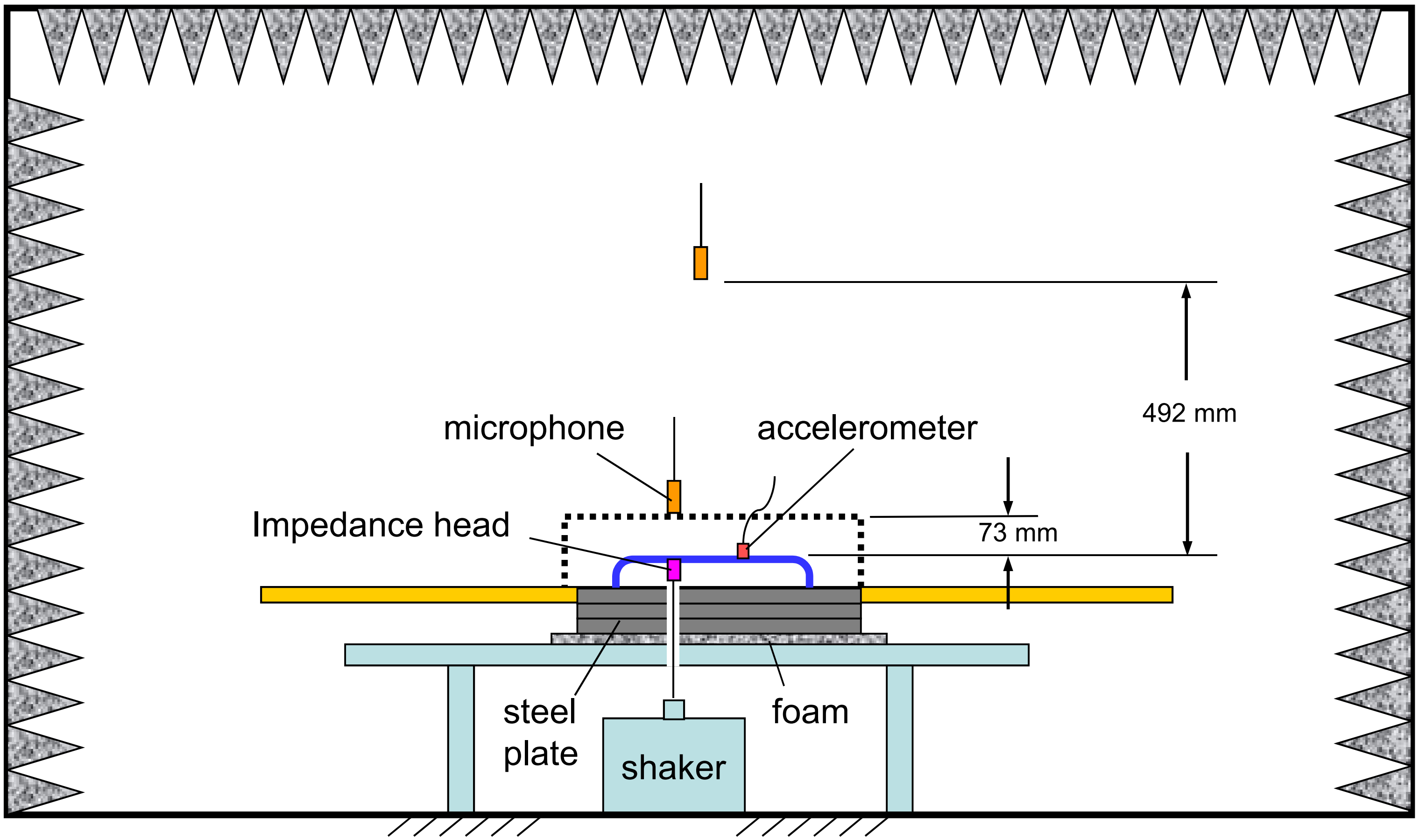
# Overview

- Fundamentals
- Example 1 – Engine Cover
- Example 2 – HVAC Duct
- Example 3 – Concentric Perforated Tube Muffler

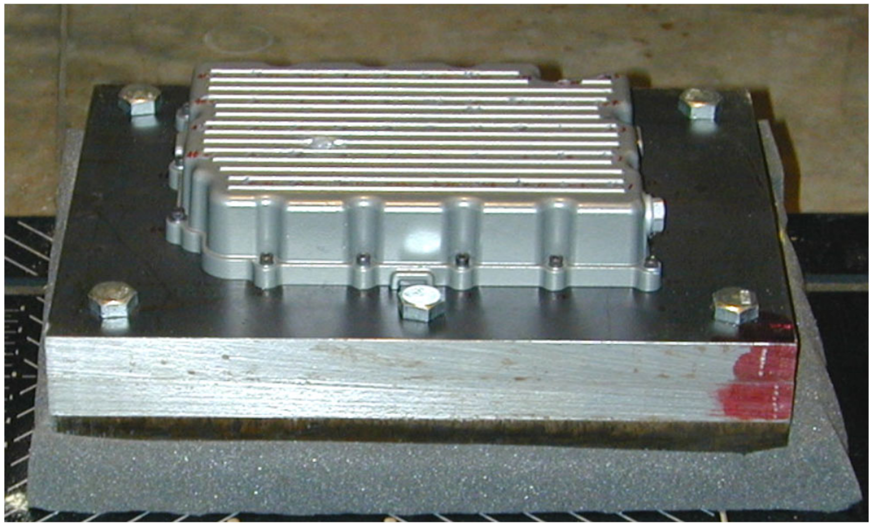
# Aluminum Oil Pan

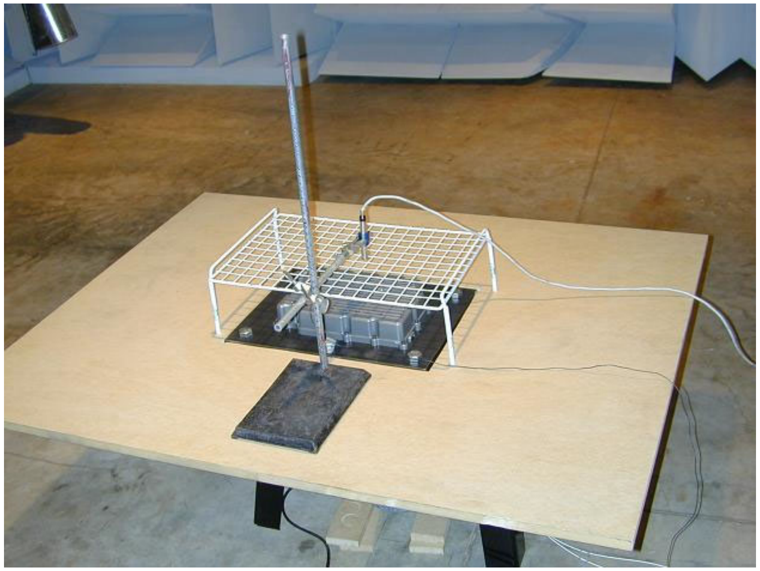


# Experimental Setup



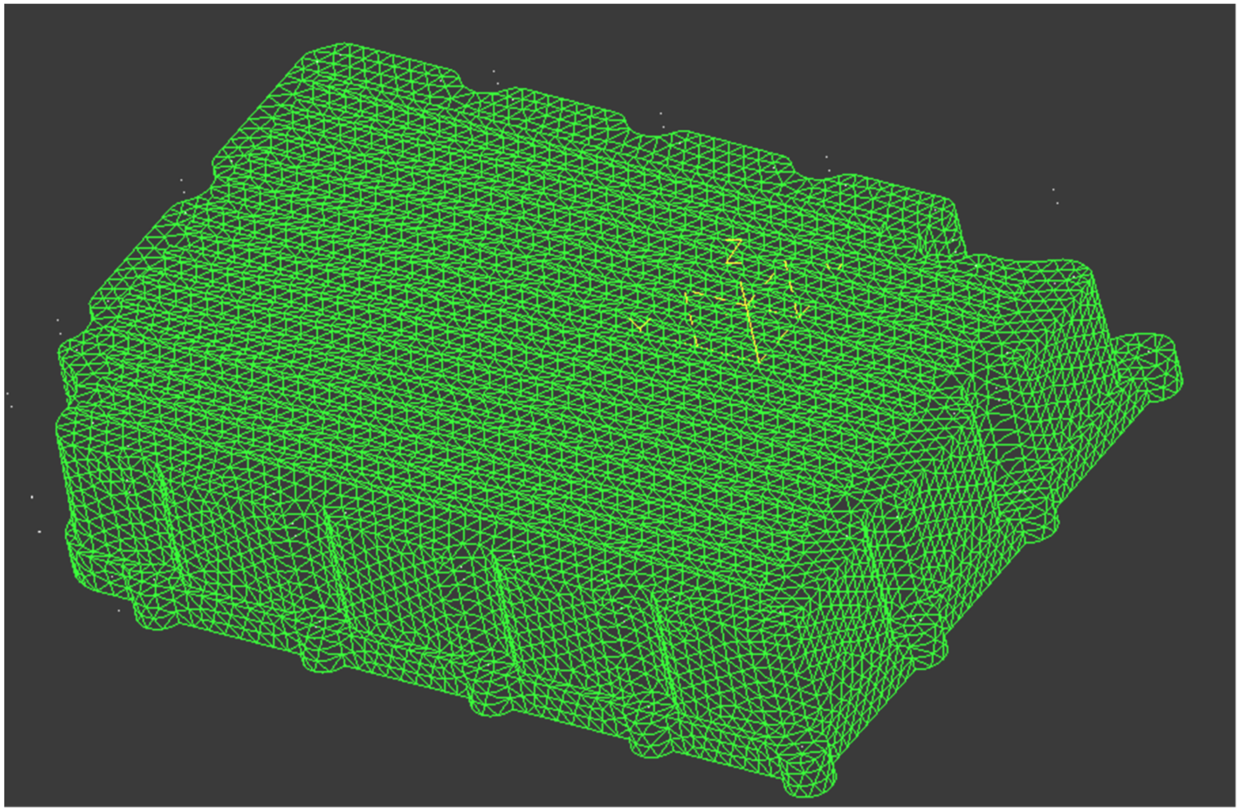
# Experimental Setup





# Structural FEM Model

Solid Parabolic Tetrahedral Elements:
40,944 elements, 83,013 Nodes
Fixed nodes at bolt positions



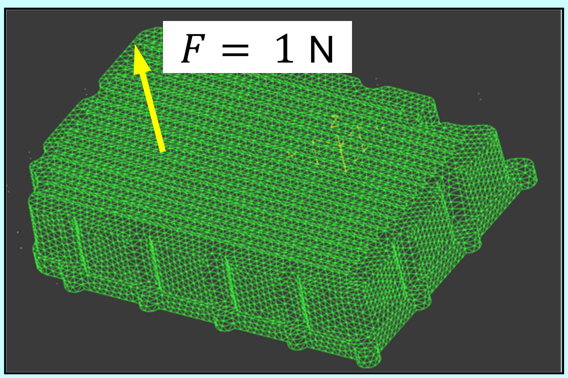
# Verification of FEM Model (15 Fixed Points)

| Mode No. | Measured (Hz) | FEM (Hz) | Error (%) |
|---|---|---|---|
| 1 | 953 | 994 | 4.3 |
| 2 | 1611 | 1672 | 3.8 |
| 3 | 2241 | 2257 | 0.7 |
| 4 | 2646 | 2668 | 0.8 |
| 5 | 2761 | 2735 | -1.0 |
| 6 | 3031 | 3095 | 2.1 |
| 7 | 3953 | 3818 | -3.4 |
| 8 | 4226 | 4015 | -5.0 |
| 9 | 4643 | 4240 | -8.7 |

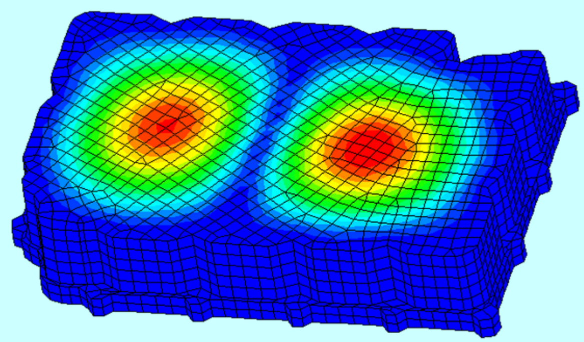
# Forced Response

- Measured damping
- Unit force at stinger

FEA Model




Vibration



# Proportionality Relationships

Vibration ($v$) and Force ($F$)

$$v \propto F$$

Sound Pressure ($p$) and Vibration ($v$)

$$p \propto v$$

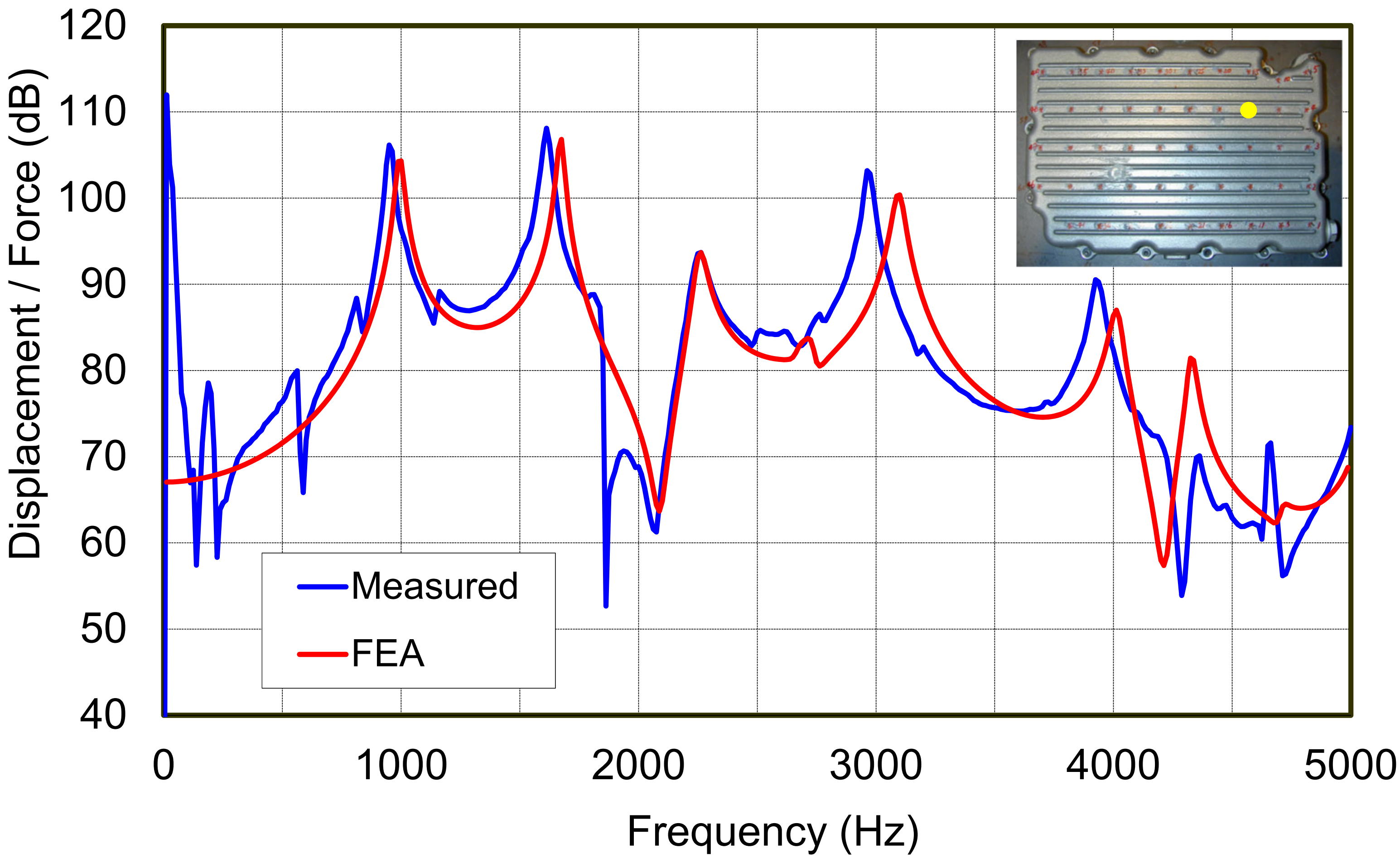
Sound Power ($W$) and Sound Pressure ($p$)

$$W \propto p^2$$

Sound Power ($W$) and Force ($F$)

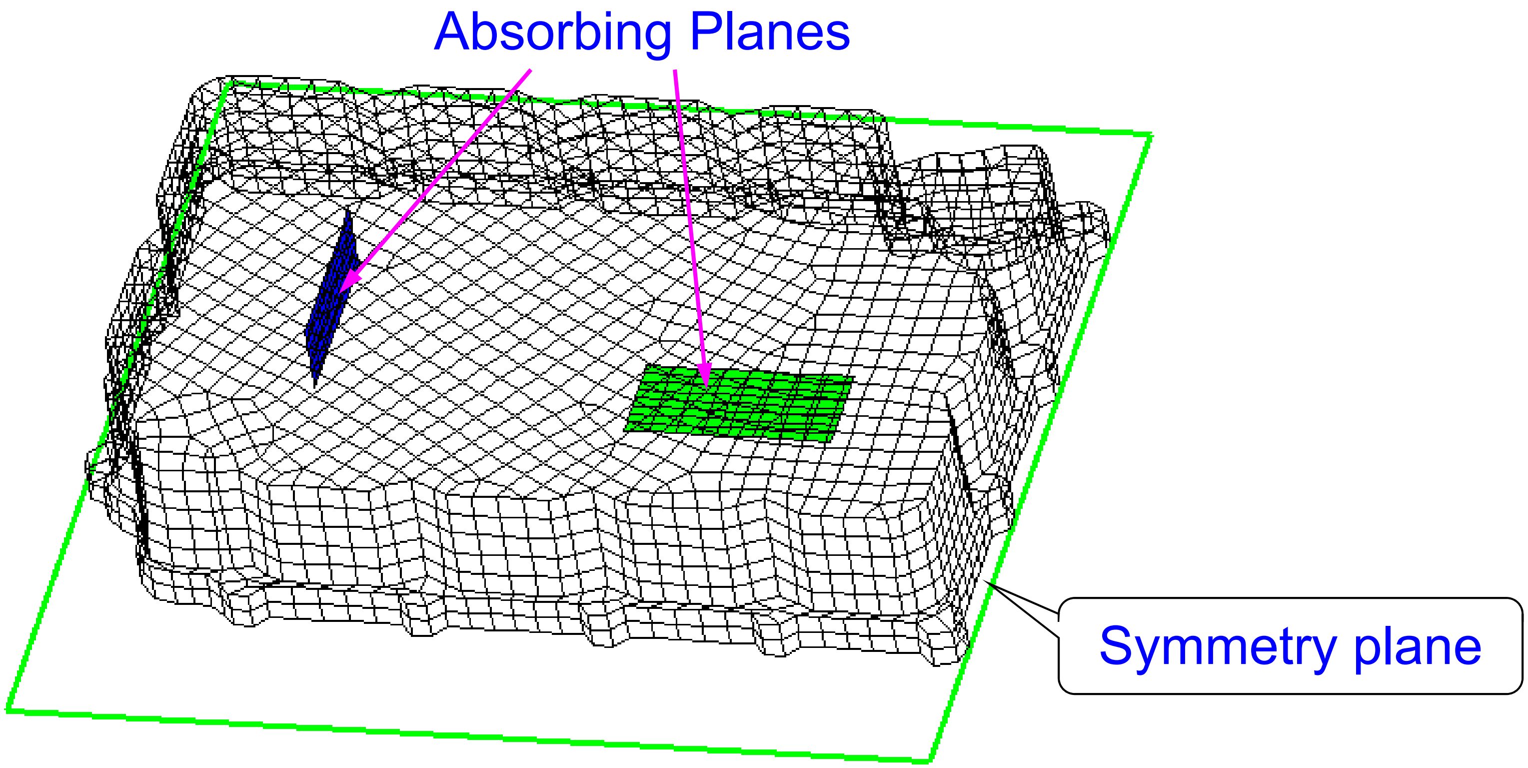
$$W \propto F^2$$

# Vibration Comparison

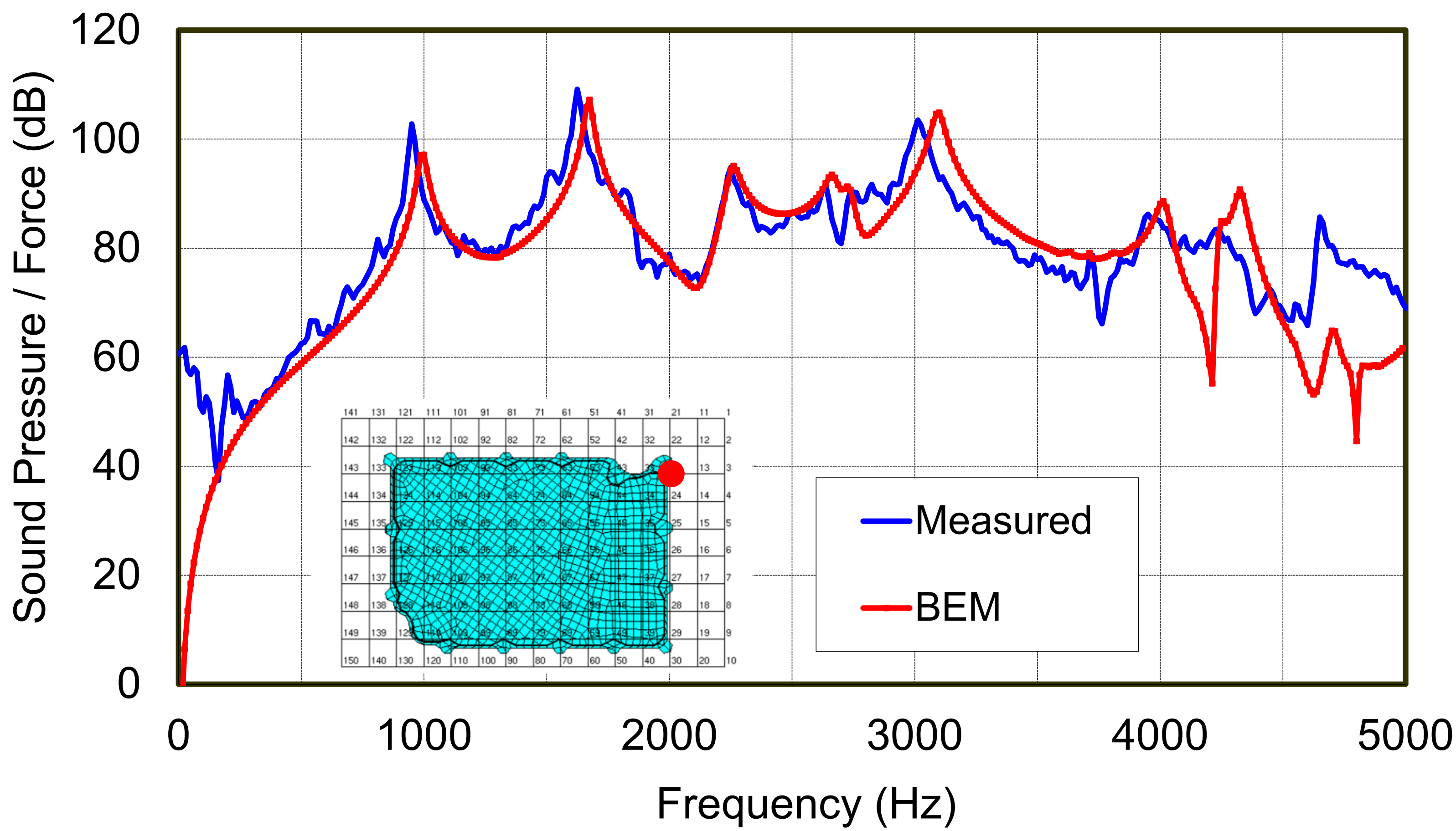


# BEM Model

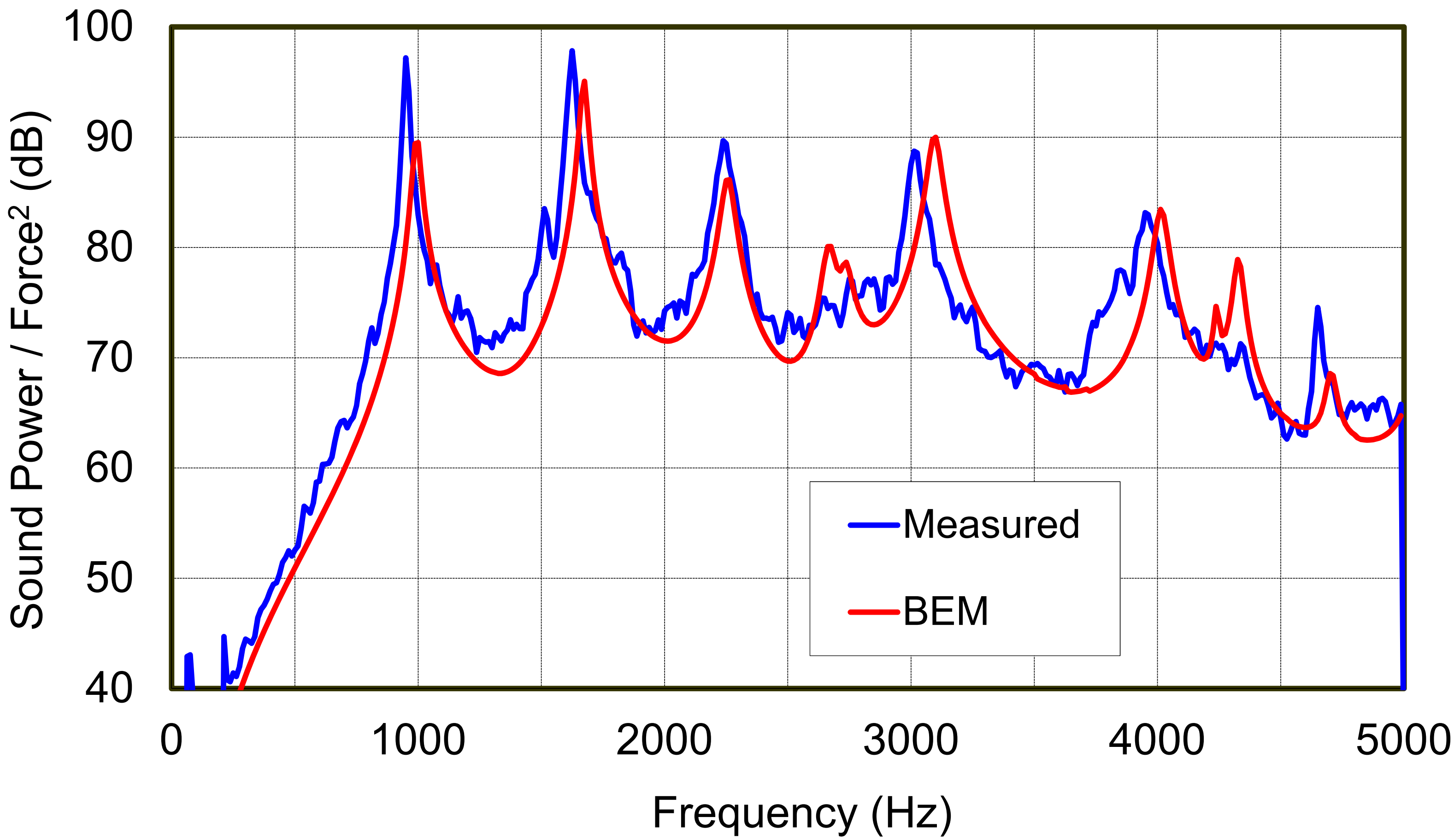
2072 linear elements, 2148 nodes



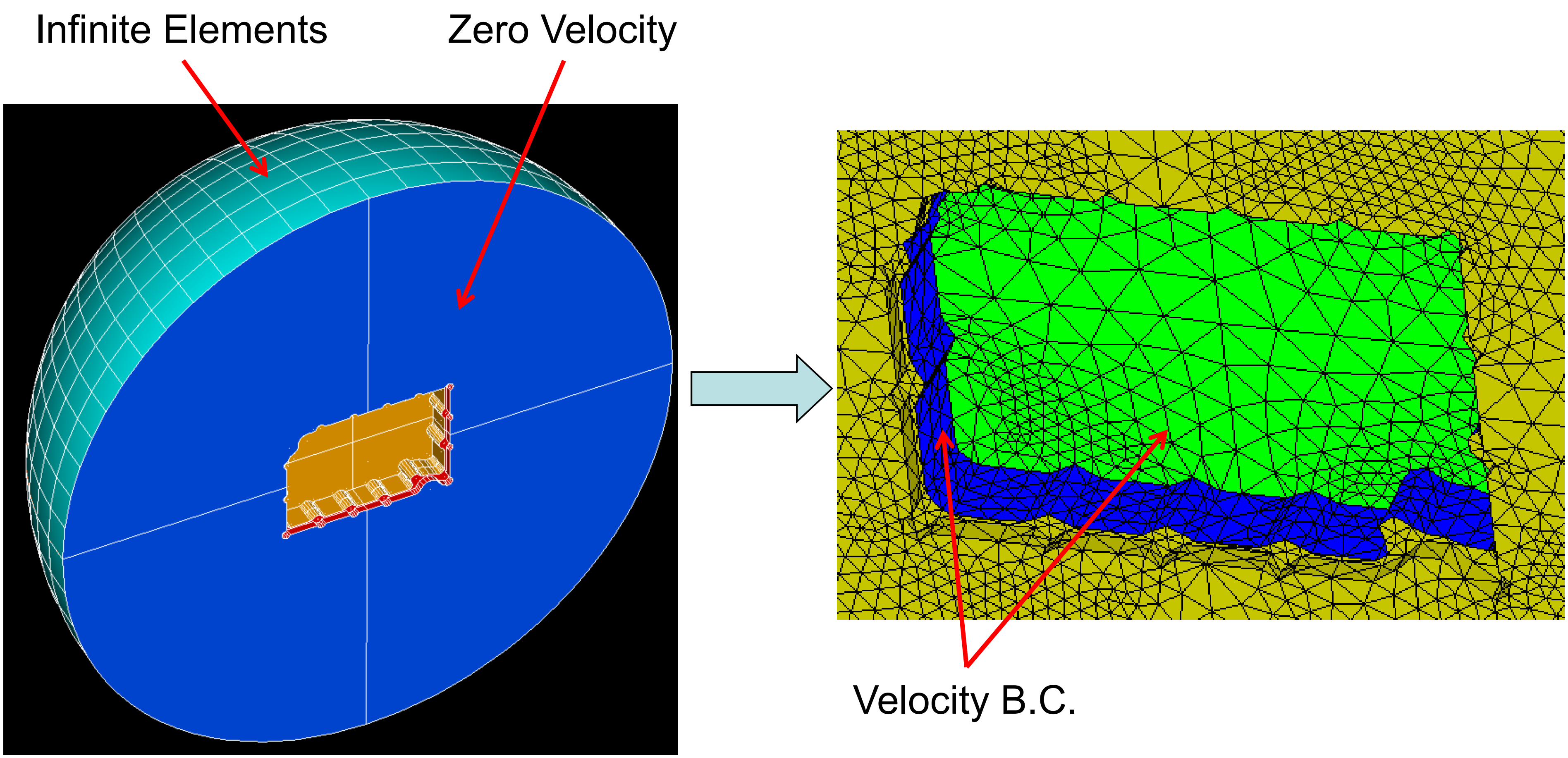
# Sound Pressure Level



# Sound Power Level



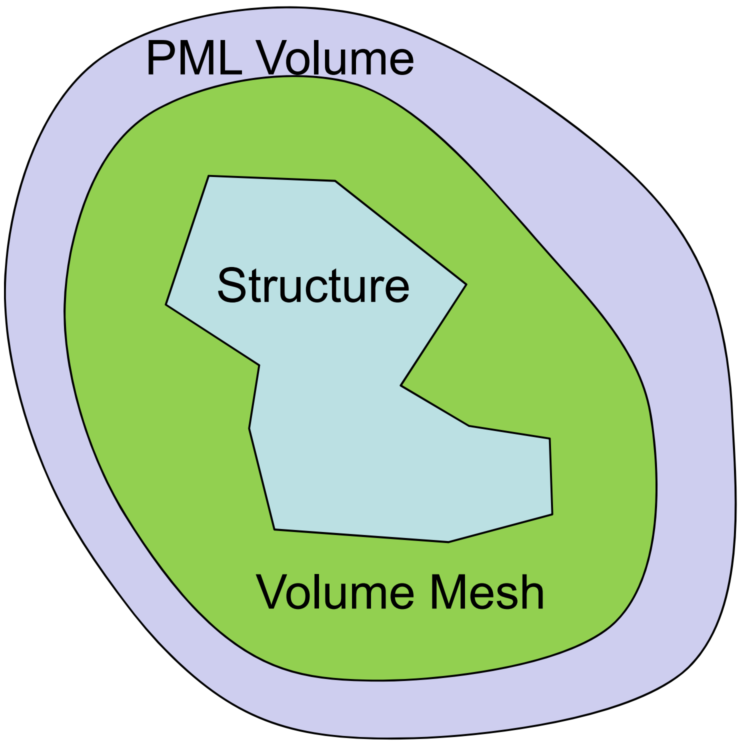
# FEM Model



# Radiation Boundary Conditions

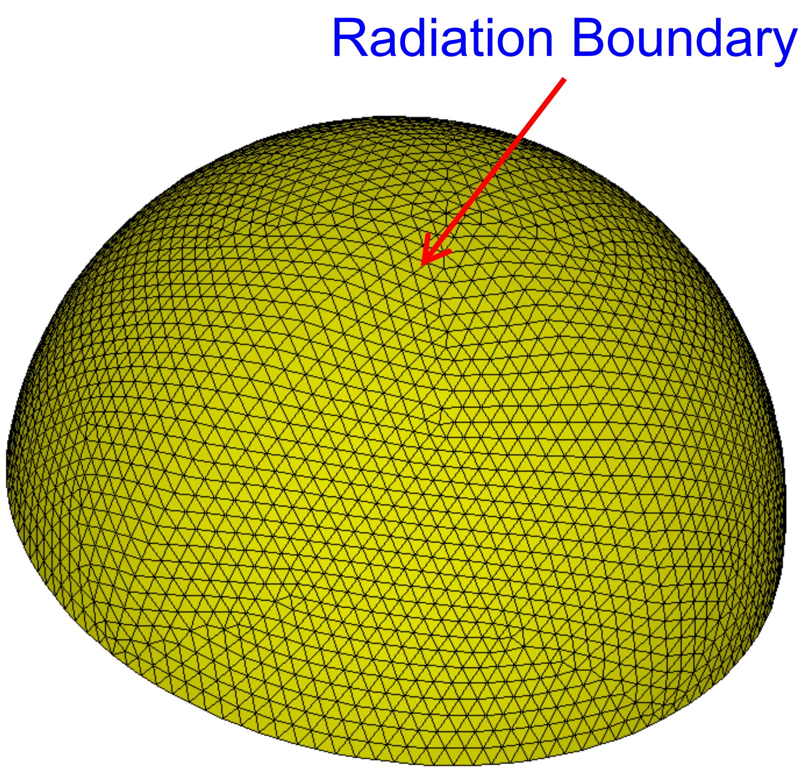
- Infinite Elements
- Perfectly Matched Layer (PML)
- Automatically Matched Layer (AML)

# Radiation Boundary Condition

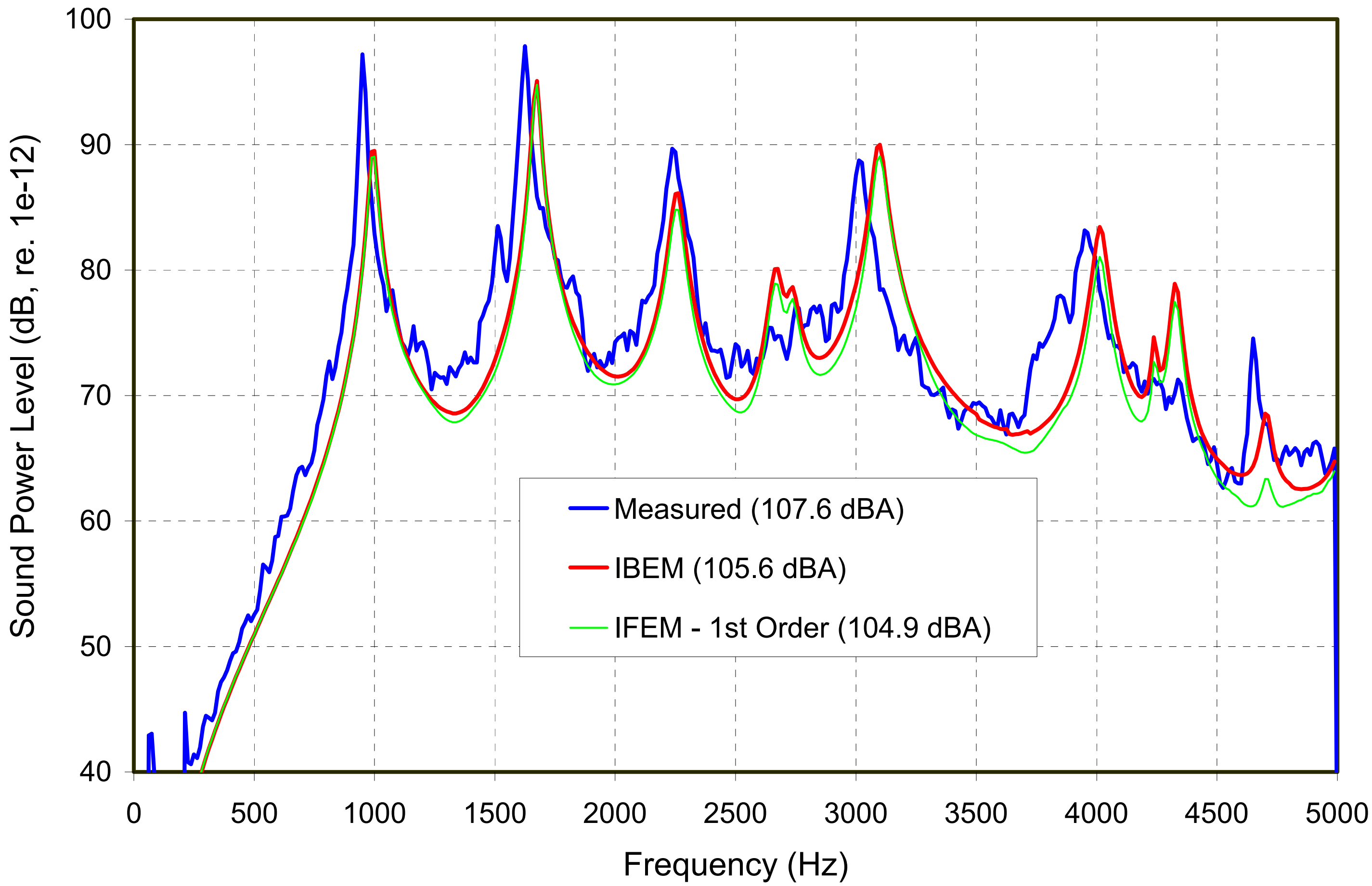



AML automates PML volume generation adjusting for needs on a frequency by frequency basis

# Infinite Elements



# Sound Power Comparison



# Overview

- Fundamentals
- Example 1 – Engine Cover
- Example 2 – HVAC Duct
- Example 3 – Concentric Perforated Tube Muffler

# Sound Absorptive Materials

## Local Reacting Model

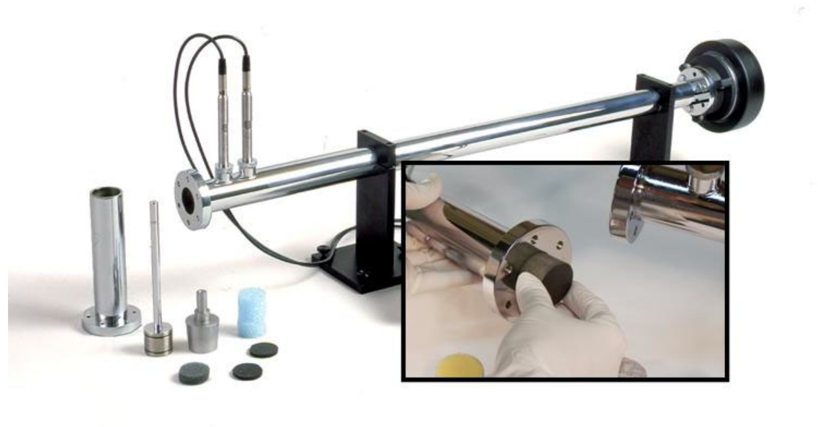
- Specific Boundary Impedance

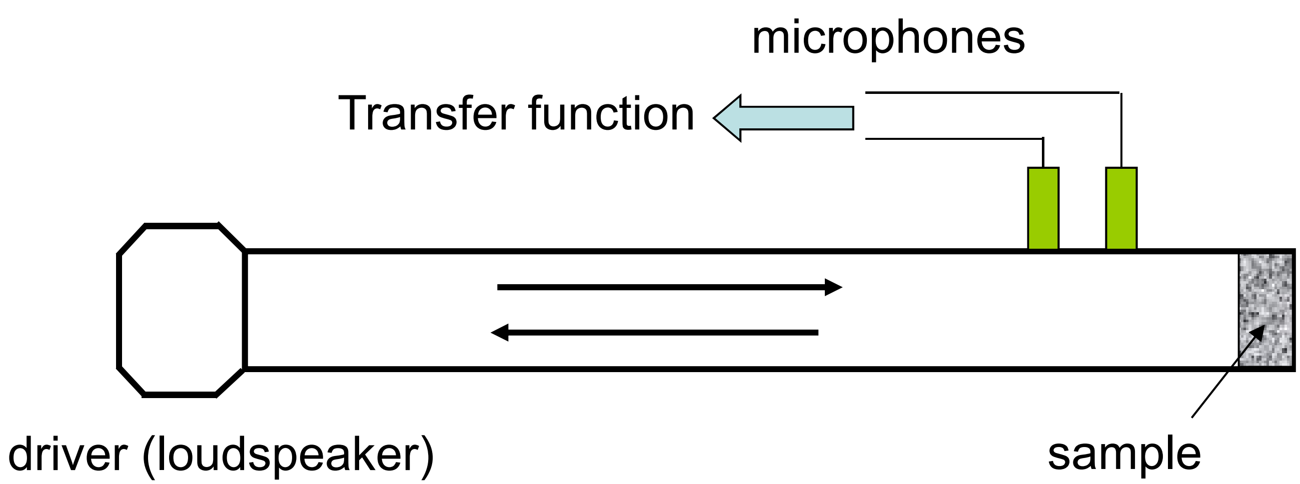
## Bulk Reacting Model

- Complex Speed of Sound and Density
- Johnson Champoux Allard (JCA) parameters

# Specific Boundary Impedance

ASTM E1050

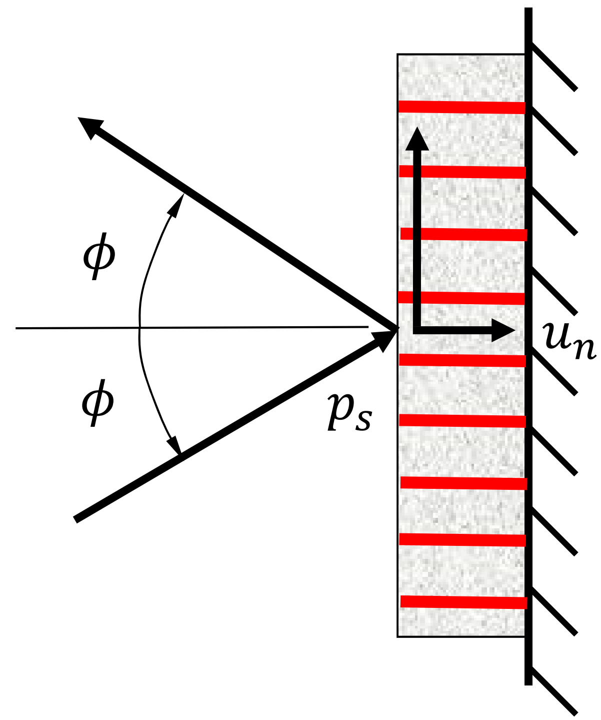




# Specific Boundary Impedance

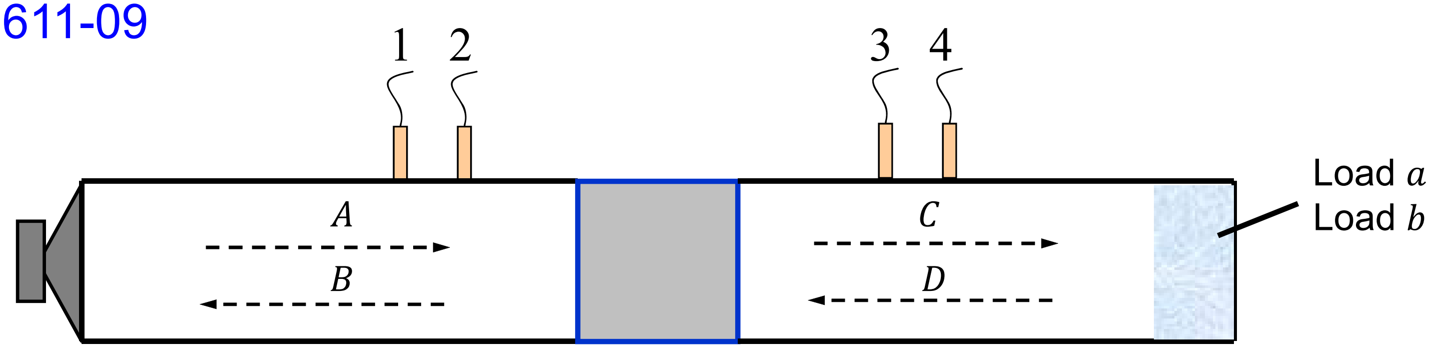
## Local Reacting Model

$$Z_n = \frac{p_s}{u_n} \quad \text{(independent of } p_i\text{)}$$



# Complex Speed of Sound and Density

ASTM E2611-09



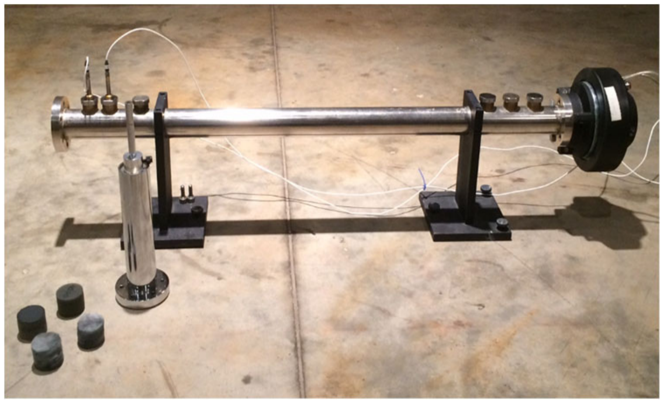

- Complex Speed of Sound
- Complex Density

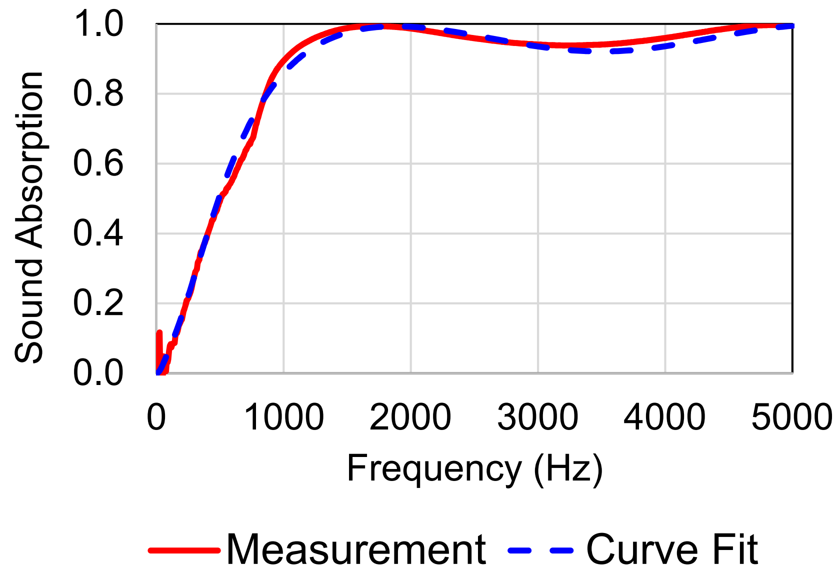
# JCA Model

## Johnson-Champoux-Allard Model

- Flow Resistivity
- Porosity
- Tortuosity
- Viscous Characteristic Length
- Thermal Characteristic Length
- Static Thermal Permeability
- Static Viscous Permeability

# Sound Absorptive Material Properties

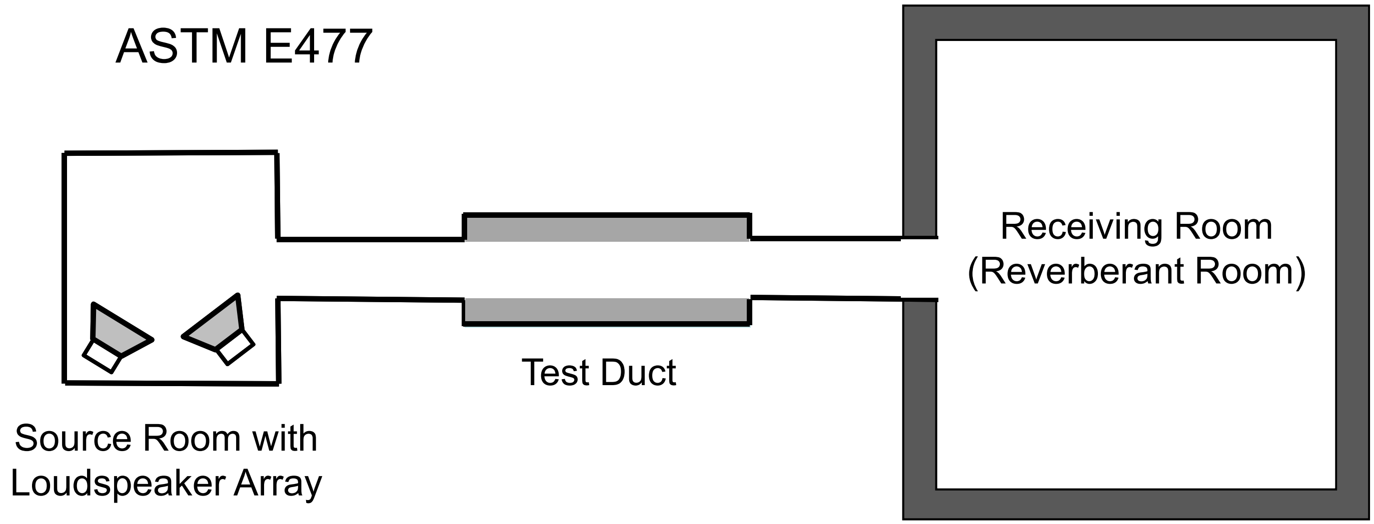




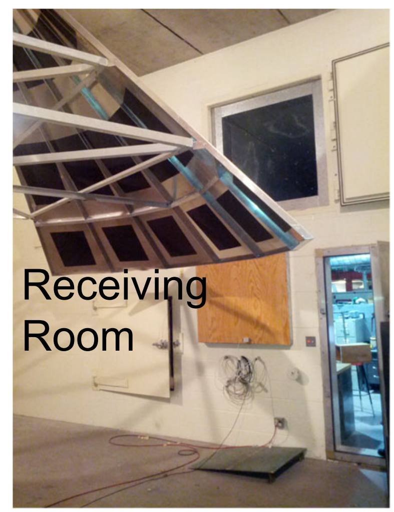

- Johnson-Champoux-Allard model (Allard and Atalla 2009)
- Curve fit (using ESI Foam-X software) used to determine JCA parameters.

# RP-1408 Campaign

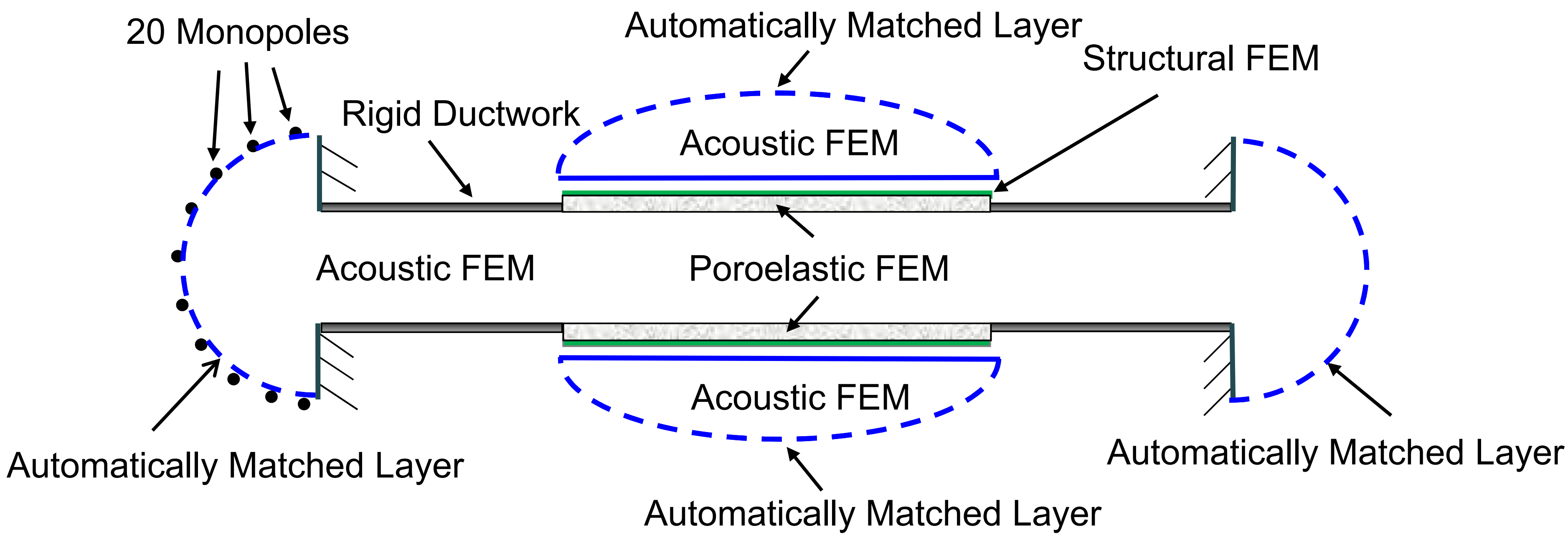
ASTM E477



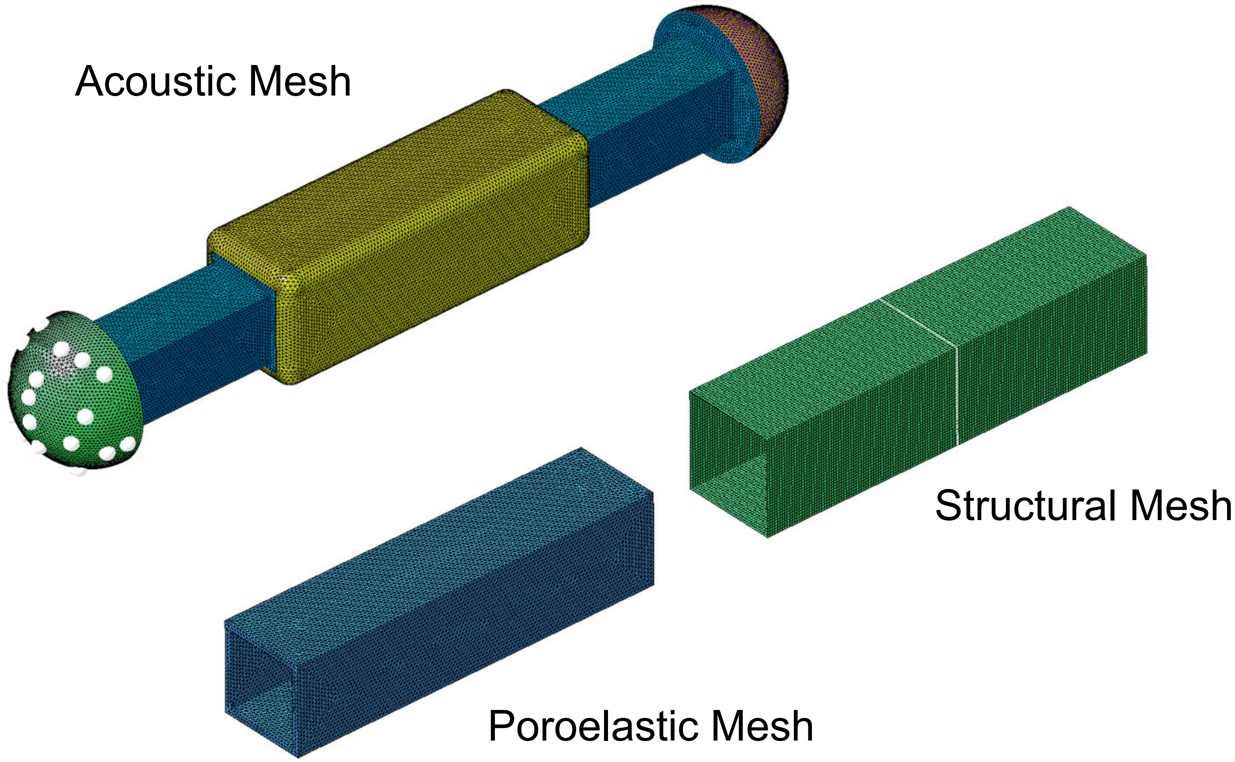




# Modeling Approach

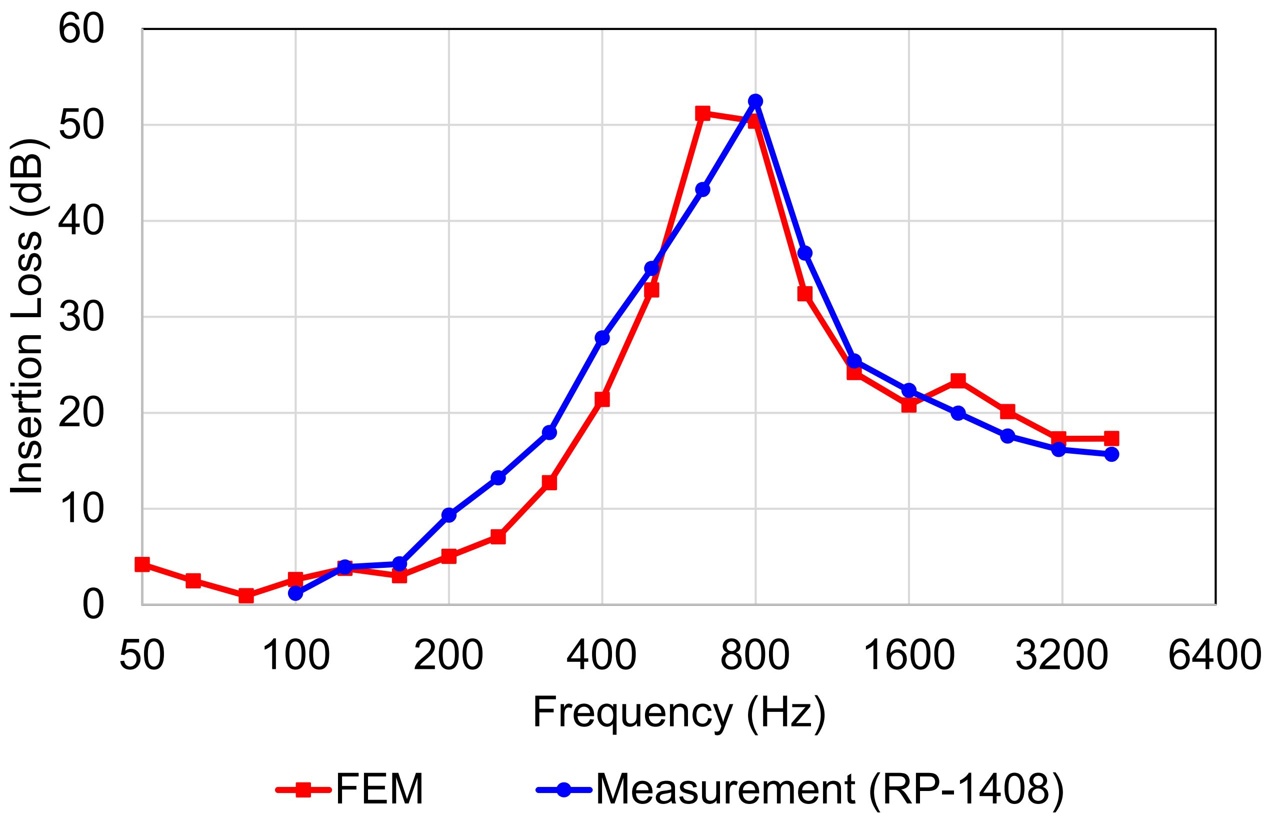


# Modeling Approach



# Insertion Loss – Lined Duct

24 in x 24 in (0.61 m x 0.61 m), 10 ft (3.05 m) Length Square Duct
2 in (5 cm) fiber lining)



# Overview

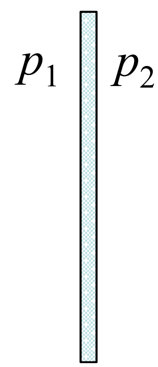
- Fundamentals
- Example 1 – Engine Cover
- Example 2 – HVAC Duct
- Example 3 – Concentric Perforated Tube Muffler

# Transfer Impedance

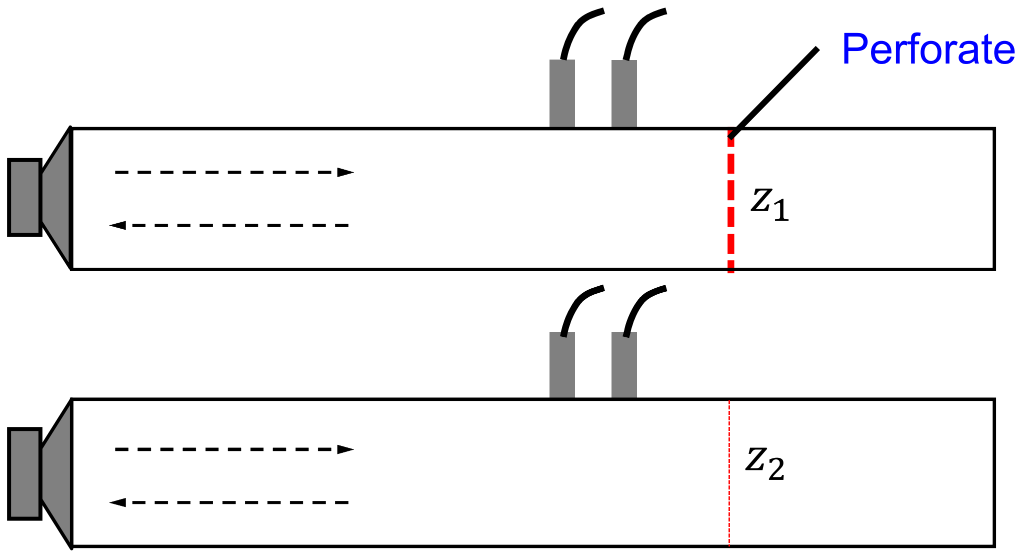
A transfer impedance is commonly used to model perforates, covers and source impedance.  Particle velocity is assumed to be continuous across the layer.

$$z_{tr} = \frac{p_1 - p_2}{u}$$

$$u_1 = u_2 = u$$



# Transfer Impedance Measurement




$$z_{tr} = z_1 - z_2$$

# Transfer Relation

## Transfer Relations

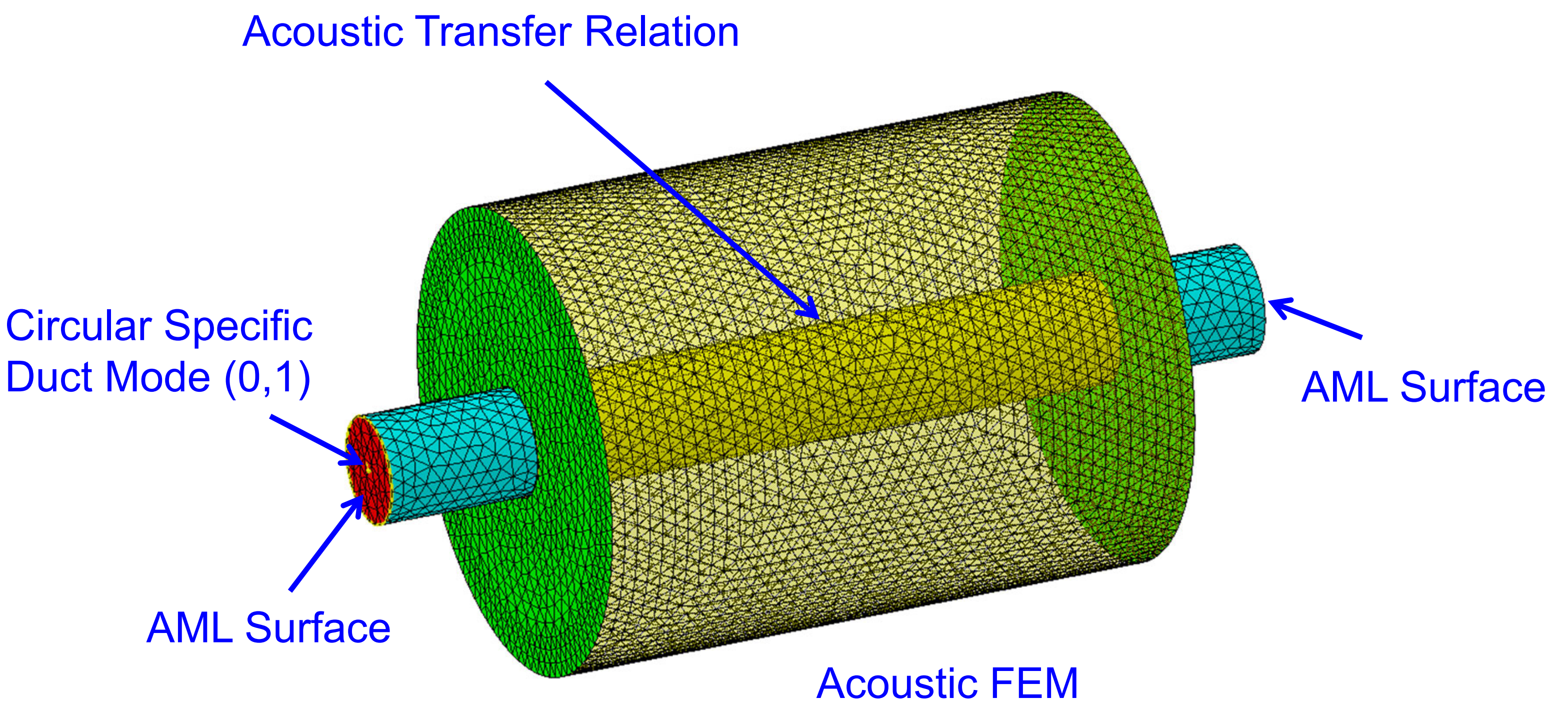
$$\begin{Bmatrix} u_{n1} \\ u_{n2} \end{Bmatrix} = \begin{bmatrix} \alpha_1 & \alpha_2 \\ \alpha_4 & \alpha_5 \end{bmatrix} \begin{Bmatrix} p_1 \\ p_2 \end{Bmatrix} + \begin{Bmatrix} \alpha_3 \\ \alpha_6 \end{Bmatrix}$$

Used for modeling perforates and catalysts.

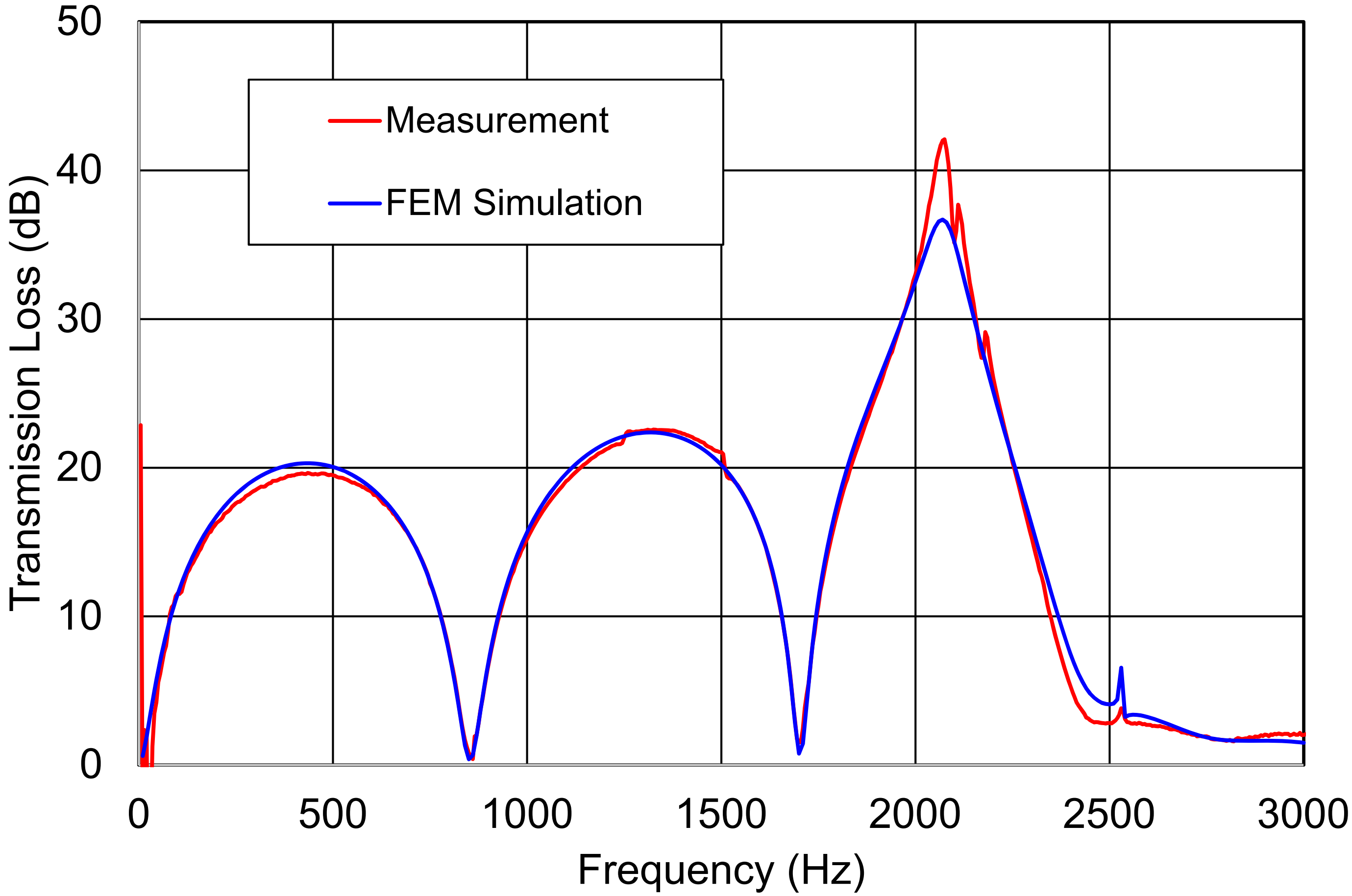
## Transfer Impedance

$$\begin{Bmatrix} u_{n1} \\ u_{n2} \end{Bmatrix} = \begin{bmatrix} \frac{1}{z_{tr}} & -\frac{1}{z_{tr}} \\ \frac{1}{z_{tr}} & -\frac{1}{z_{tr}} \end{bmatrix} \begin{Bmatrix} p_1 \\ p_2 \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

# FEM Model


Acoustic Transfer Relation
Circular Specific
Duct Mode (0,1)
AML Surface
AML Surface
Acoustic FEM

# Transmission Loss



# Summary

We have numerous simulation tools that are easy to use and can solve acoustic problems quickly.