Notes on Damping

Vibro-Acoustics Consortium Web Meeting
University of Kentucky

Organization

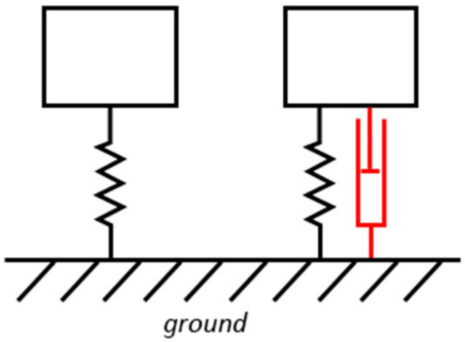
Notes on Damping

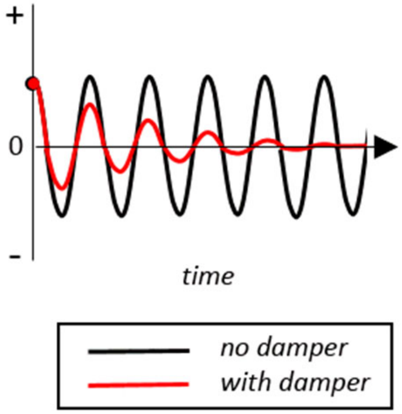
- The Basics
- Material Damping
- Damping Metrics
- Damping with Viscoelastic Materials

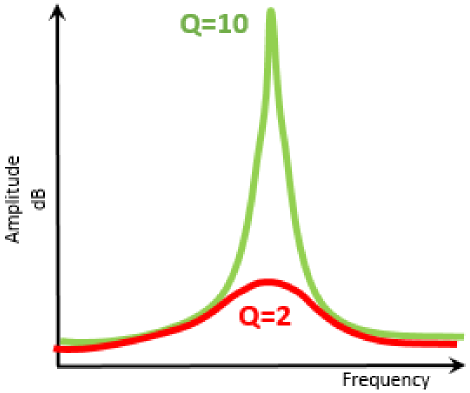
What is Damping?

Notes on Damping

Damping is energy dissipation by friction and other resistances.





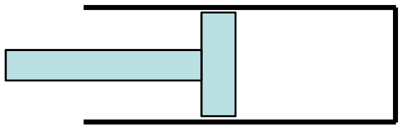


https://community.plm.automation.siemens.com/t5/Testing-Knowledge-Base/How-to-calculate-damping-from-a-FRF/ta-p/355050

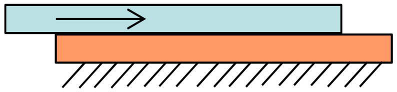
Damping Mechanisms

Notes on Damping

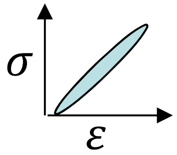
- Viscous Damping



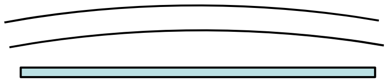
- Coulomb or Dry Damping



- Material Damping

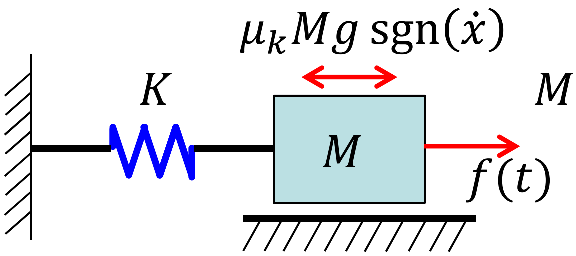


- Acoustic Damping

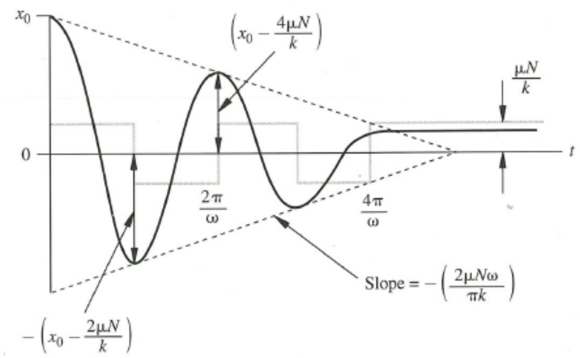


Coulomb Damping

Notes on Damping



$$M\ddot{x} + \mu_k M g\,\mathrm{sgn}(\dot{x}) + Kx = f(t)$$

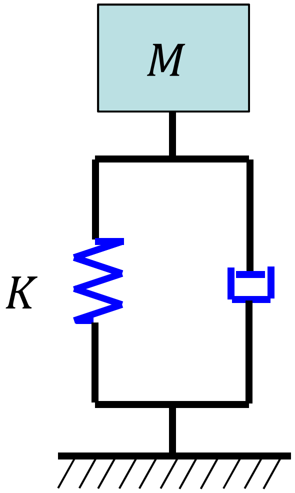


Inman, 2013

- Nonlinear differential equation
- Linear decay
- Decays to rest

Viscous Damping

Notes on Damping



$$M\ddot{x} + B\dot{x} + Kx = f(t)$$

Assume force is harmonic so

$f(t) = Fe^{j\omega t}$ and $x(t) = Xe^{j\omega t}$

$$(-M\omega^2 + jB\omega + K)X = F$$

$$\left(-M\omega^2 + K\left(1 + j\frac{\omega B}{K}\right)\right)X = F$$

$$K' = K\left(1 + j\frac{\omega B}{K}\right) = K(1 + j\eta)$$

Models for Damped Structures

Notes on Damping

Viscous Damping Representation

$$M\ddot{x} + \underline{B\dot{x}} + Kx = f(t)$$

$$\ddot{x} + 2\frac{B}{2\sqrt{KM}}\sqrt{\frac{K}{M}}\,\dot{x} + \frac{K}{M}x = \frac{1}{M}f(t)$$

$$(-\omega^2 + \underline{2\xi\omega_n(j\omega)} + \omega_n^2)X = \frac{1}{M}F$$

Loss Factor Representation

$$M\ddot{x} + K\left(1 + \frac{j\omega B}{K}\right)x = f(t)$$

$$\ddot{x} + \frac{K}{M}(1 + j\eta)x = \frac{1}{M}f(t)$$

$$(-\omega^2 + \underline{j\eta\omega_n^2} + \omega_n^2)X = \frac{1}{M}F$$

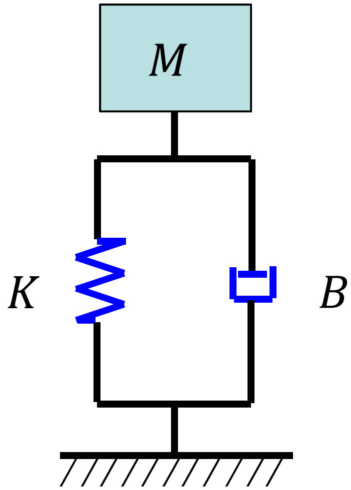
if $\omega = \omega_n \ldots \eta = 2\xi$

To be more precise

if $\omega = \omega_d = \omega_n\sqrt{1-\xi^2}$ then $\eta = \dfrac{2\xi}{\sqrt{1-\xi^2}}$

Viscous Damping

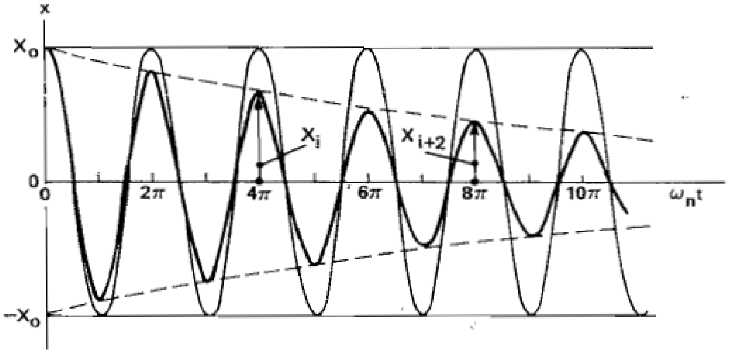
Notes on Damping



$$M\ddot{x} + B\dot{x} + Kx = f(t)$$

$$x(t) = X_0 e^{-\xi\omega_n t}\cos(\omega_d t + \varphi)$$

$$\omega_n = \sqrt{\frac{K}{M}} \qquad \omega_d = \omega_n\sqrt{1 - \xi^2}$$



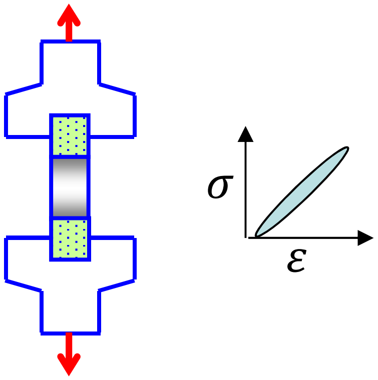
Ungar and Zapfe, 2006

- Linear differential equation.
- Exponential decay.
- Never quite reaches rest.

Damping Mechanisms

Notes on Damping

Material



$\eta \approx 0.0001$

Component

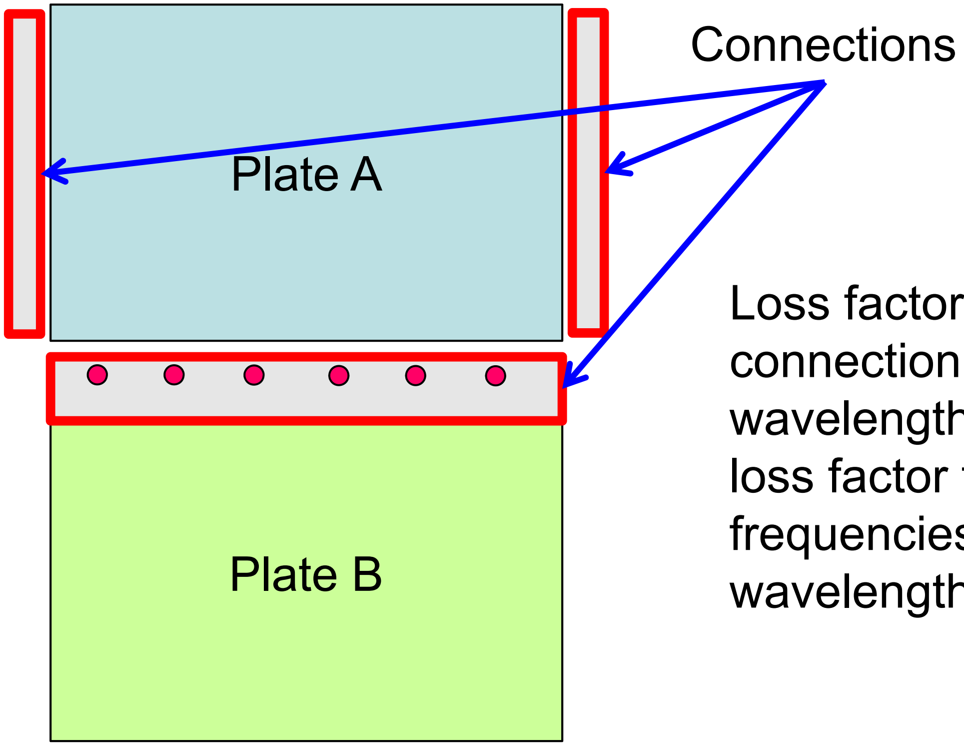
$\eta \approx 0.001$

Assembled Part

$\eta \approx 0.01$

Damping at Boundaries

Notes on Damping



Loss factor is roughly proportional to connection length and the bending wavelength of the plate. Hence, the loss factor tends to be lower at higher frequencies because the bending wavelength is lower.

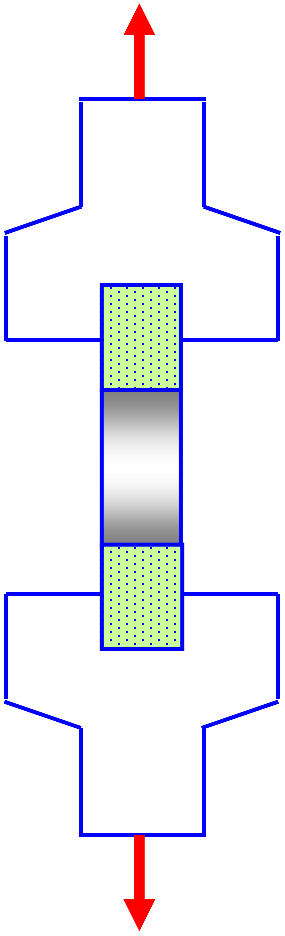
Organization

Notes on Damping

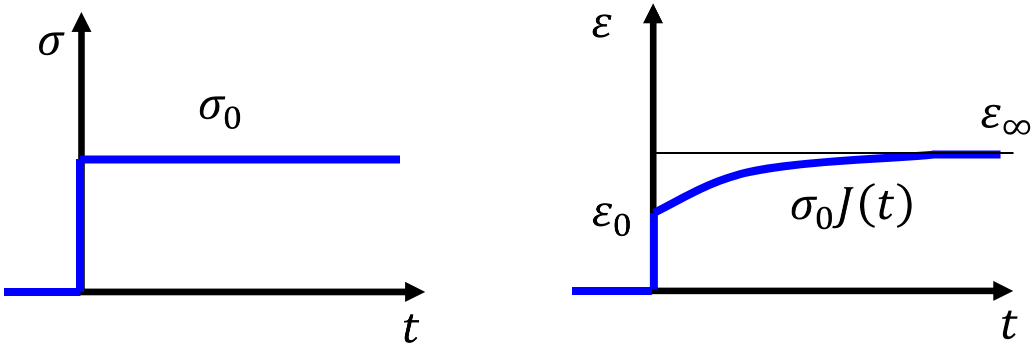
- The Basics
- Material Damping
- Damping Metrics
- Damping with Viscoelastic Materials

Material Damping

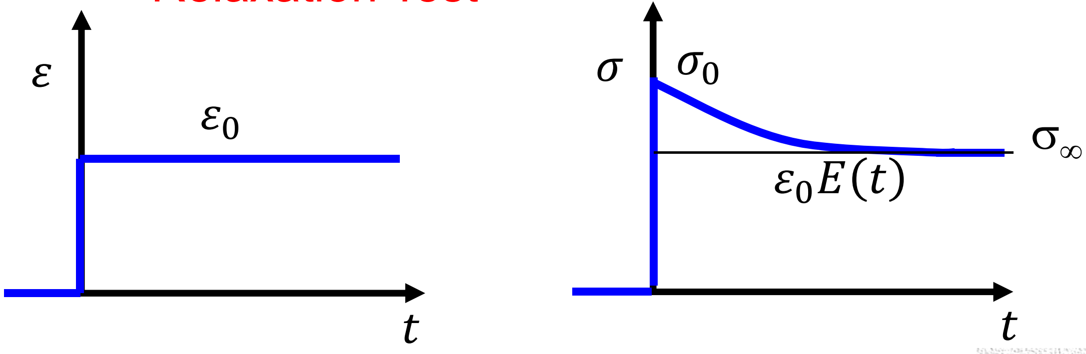
Notes on Damping



Creep Test



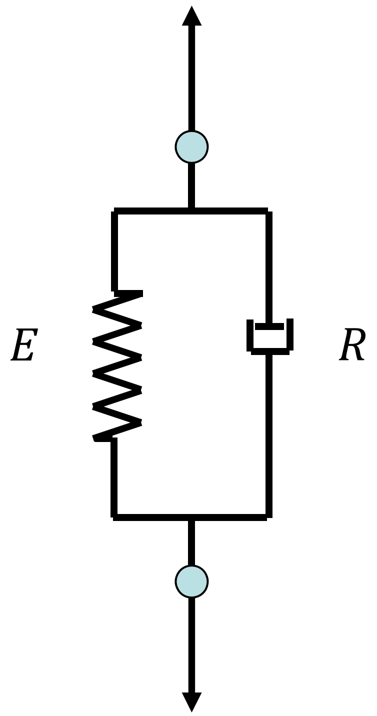
Relaxation Test



Material Damping

Notes on Damping

The Kelvin-Voigt 2-Parameter Model



Creep Compliance

$$\varepsilon(t) = \frac{\sigma_0}{E}\left(1 - e^{-\frac{E}{R}t}\right)$$

Stress Relaxation

$$\sigma(t) = \varepsilon_0 E$$

Material Damping

Notes on Damping

For Low Frequency Harmonic Processes the Kelvin-Voigt Model is Applicable

$$E(\omega) \approx E_0 + j\omega R = E_0(1 + j\eta)$$

The Material Loss Factor (Steel ~ .0001)

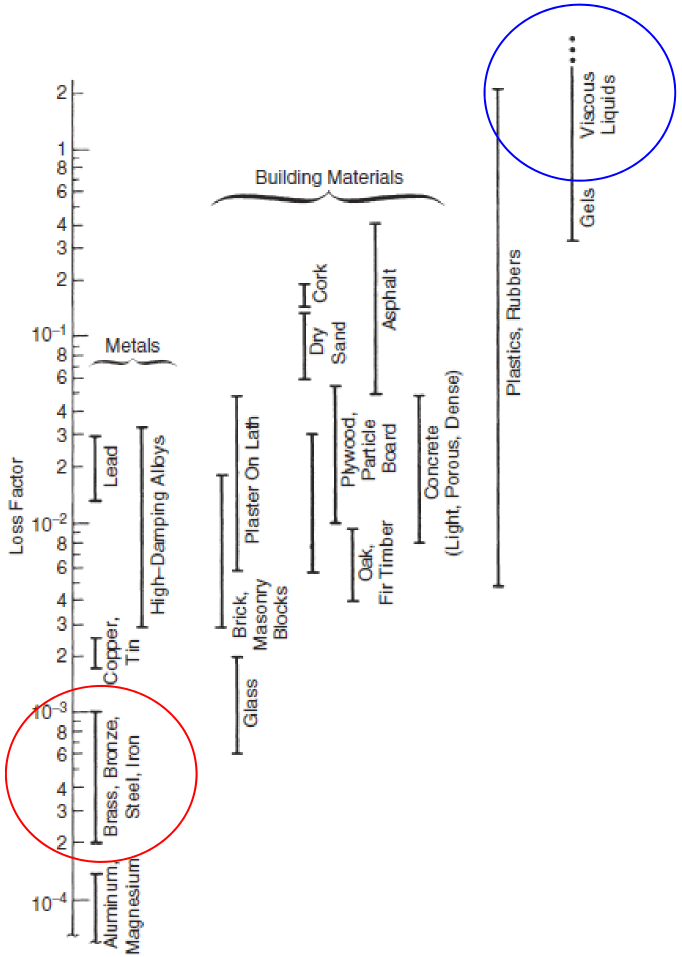
Energy Dissipated per Cycle

$$\eta = \frac{E_{dis}/2\pi}{\max\left(E_{pot}\right)} = \frac{\omega R}{E_0}$$

Maximum Potential Energy

Loss Factors of Common Materials

Notes on Damping



Ungar, 2007

Organization

Notes on Damping

- The Basics
- Material Damping
- Damping Metrics
- Damping with Viscoelastic Materials

Damping Metrics

Notes on Damping

Loss factor (η)

Note: normally used for SEA and room acoustics.

Viscous or critical damping ratio (ξ)

$$\eta = \frac{2\xi}{\sqrt{1-\xi^2}} \approx 2\xi \quad \text{if } \xi \text{ is low}$$

Note: normally used for harmonic FEA (especially mode superposition analyses).

Imaginary part of modulus of elasticity (E_i)

$$E' = E_r + jE_i = E(1 + j\eta)$$

$$\eta = \frac{E_i}{E}$$

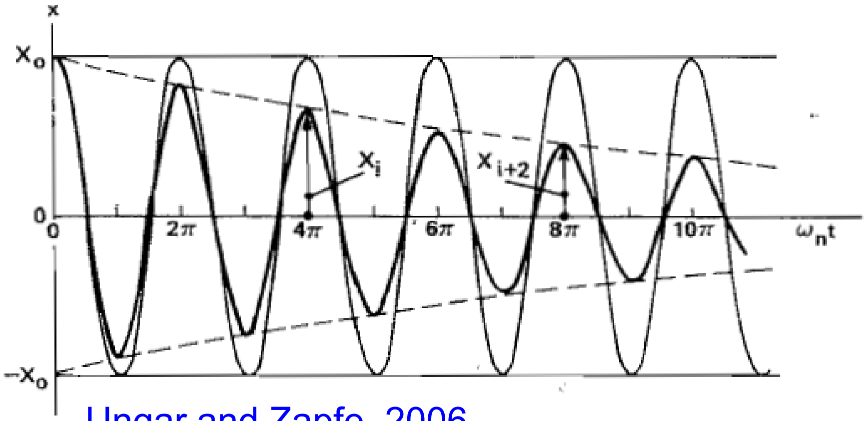
Note: normally used for transient analyses (i.e., not harmonic analyses).

Damping Metrics

Notes on Damping

Logarithmic decrement (δ)

$$\eta = \frac{\delta}{\pi}$$



Ungar and Zapfe, 2006

$$\delta = \frac{1}{N}\ln\left(\frac{X_i}{X_{i+N}}\right)$$

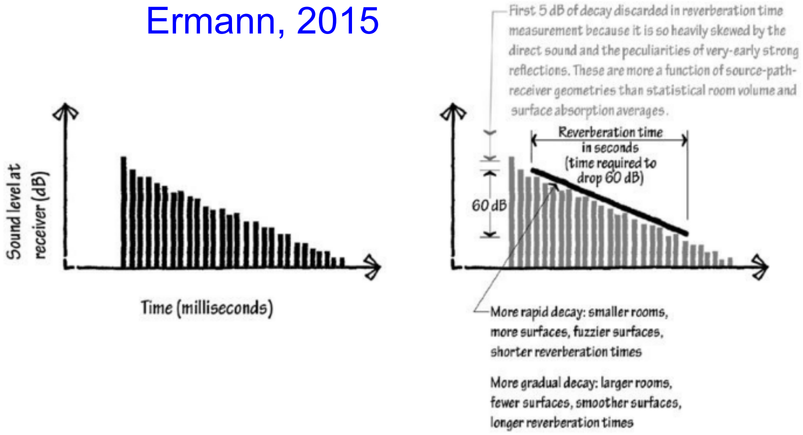
Note: often used in classwork and perhaps in some analysis codes.

Damping Metrics

Notes on Damping

60 dB reverberation time (T_{60})

$$\eta = \frac{2.2}{f T_{60}}$$


Ermann, 2015

Note: often used experimentally in reverberation rooms to determine the loss factor of a room.

Decay rate in dB/sec

$$\eta = \frac{DR}{27.3 f}$$

Note: used when background noise is too high to identify T_{60} to determine loss factor.

Damping Metrics

Notes on Damping

Diffuse field sound absorption (α_{diff})

$$\eta = \frac{cS\langle\alpha_{diff}\rangle}{4\omega V}$$

$$S\langle\alpha_{diff}\rangle = \sum_i \alpha_{diff,i} S_i$$

$$T_{60} = \frac{(24 \cdot \ln 10)V}{c\langle\alpha_{diff}\rangle S}$$

Note: diffuse field sound absorption is measured in a reverberation room and is used in room acoustics theory and SEA software.

Steady State Harmonic Response

Notes on Damping

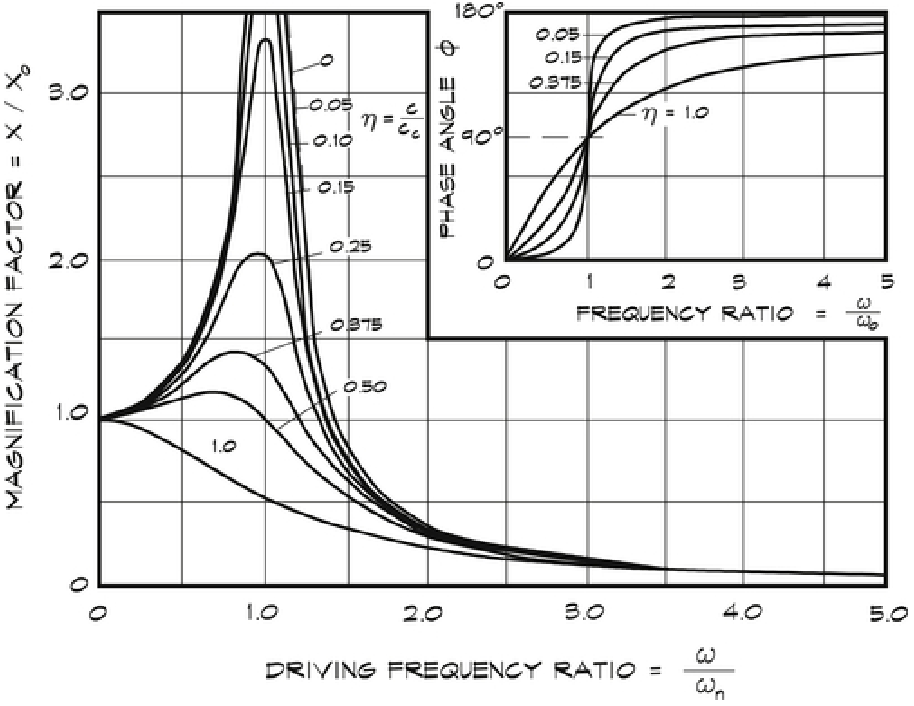
Magnification (or Quality Factor)

$$\left|\frac{X}{X_0}\right| = \frac{1}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}}$$

At resonance (i.e. $\omega = \omega_n$)

$$\left|\frac{X}{X_0}\right| = Q = \frac{1}{2\xi} = \frac{1}{\eta}$$

Long, 2014

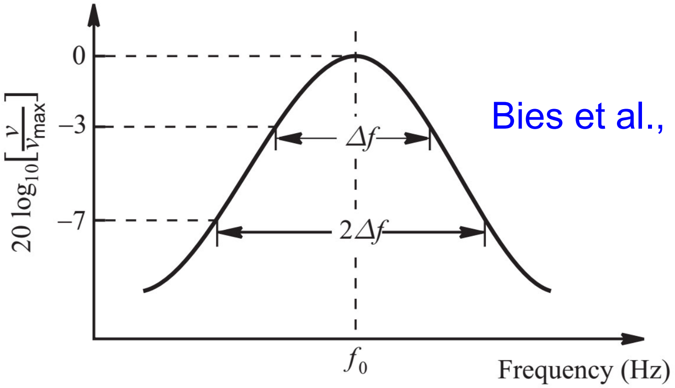


Measurement of Damping

Notes on Damping

Half Power Method

$$\eta = 2\xi = \frac{\Delta f_{3dB}}{f_0} = \frac{\Delta f_{7dB}}{2f_0}$$



Bies et al., 2018

Note: used experimentally to determine damping ratio at modal frequencies.

Quality Factor

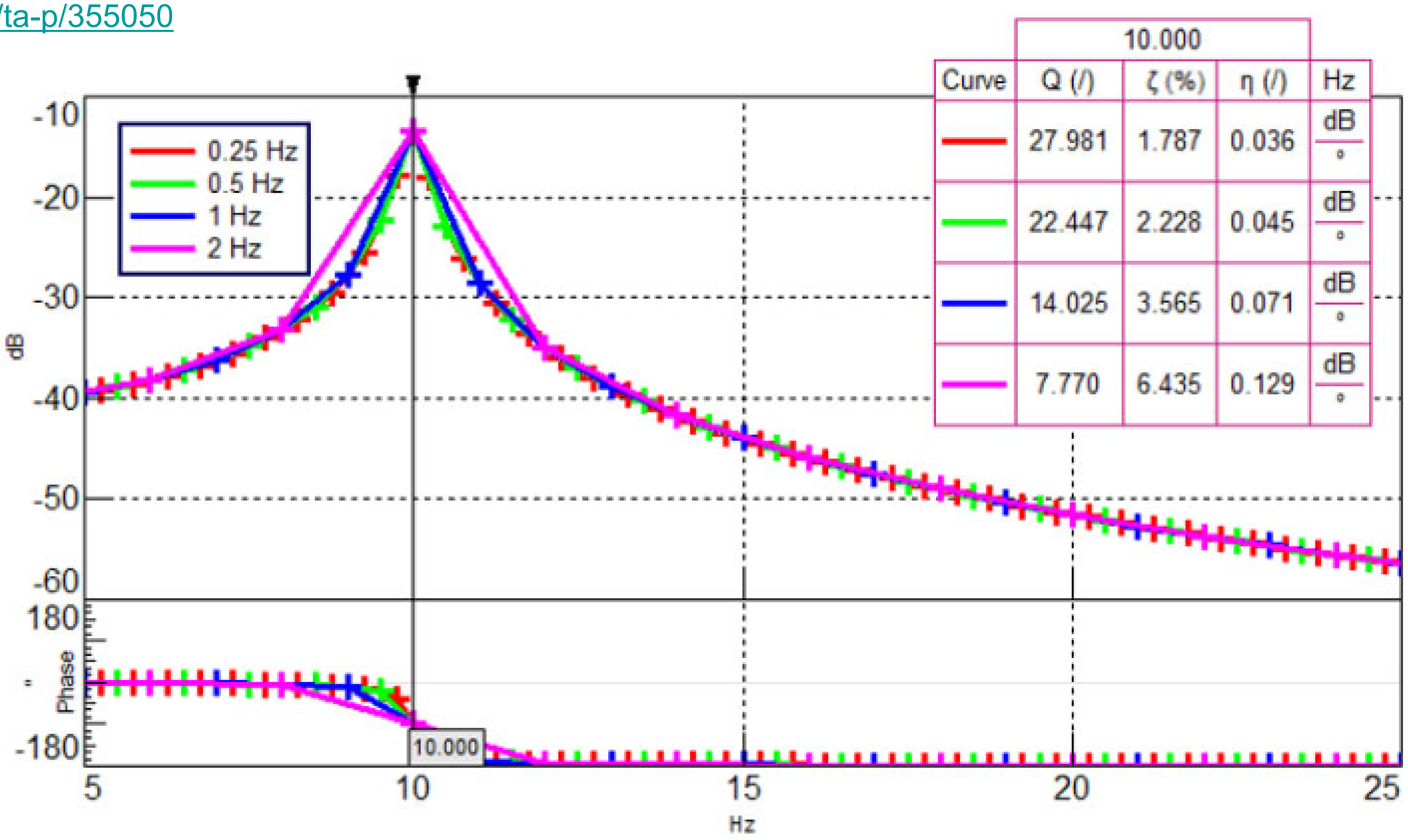
$$\eta = \frac{1}{Q}$$

$$Q = \frac{f_0}{\Delta f_{3\,\mathrm{dB}}} = \frac{X(\omega_n)}{X_{st}}$$

Dependent on Frequency Spacing

Notes on Damping

https://community.plm.automation.siemens.com/t5/Testing-Knowledge-Base/How-to-calculate-damping-from-a-FRF/ta-p/355050



	10.000			
Curve	Q (/)	ζ (%)	η (/)	Hz
🟥	27.981	1.787	0.036	dB °
🟩	22.447	2.228	0.045	dB °
🟦	14.025	3.565	0.071	dB °
🟪	7.770	6.435	0.129	dB °

Organization

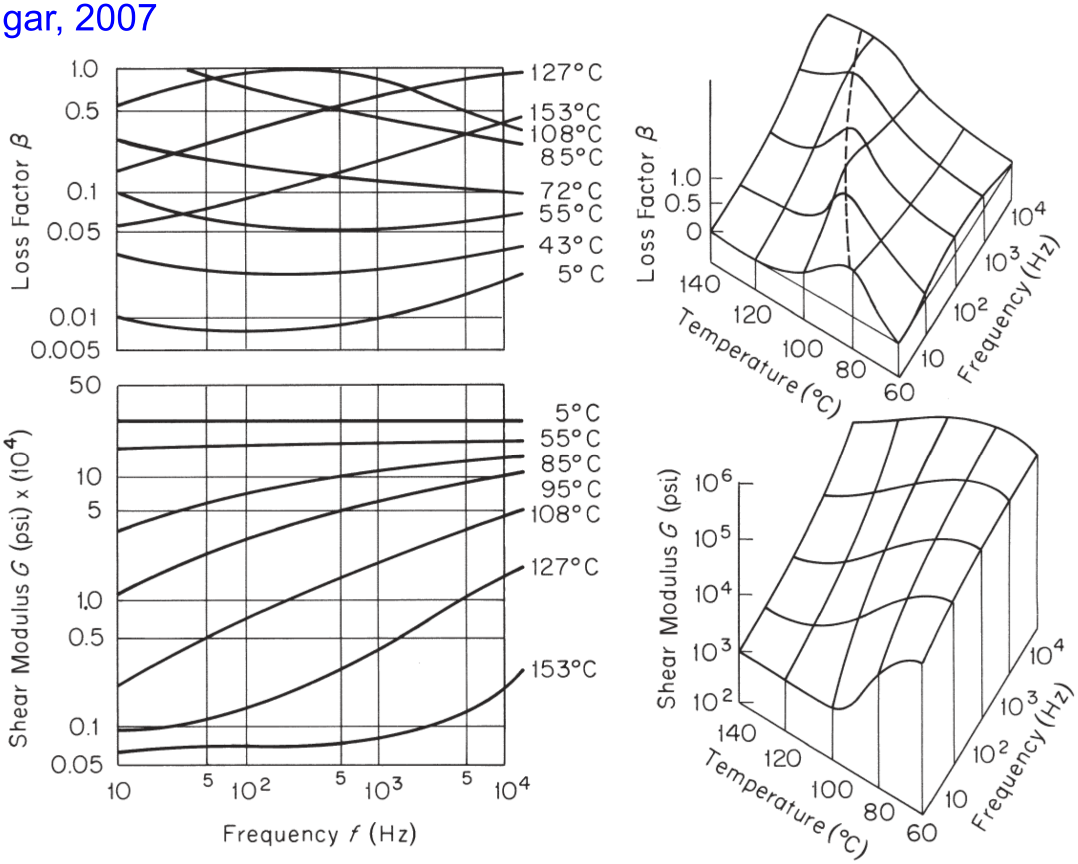
Notes on Damping

- The Basics
- Material Damping
- Damping Metrics
- Damping with Viscoelastic Materials

Polyester Plastic Material Properties

Notes on Damping

Ungar, 2007



(a) (b)

Properties of Damping Materials

Notes on Damping

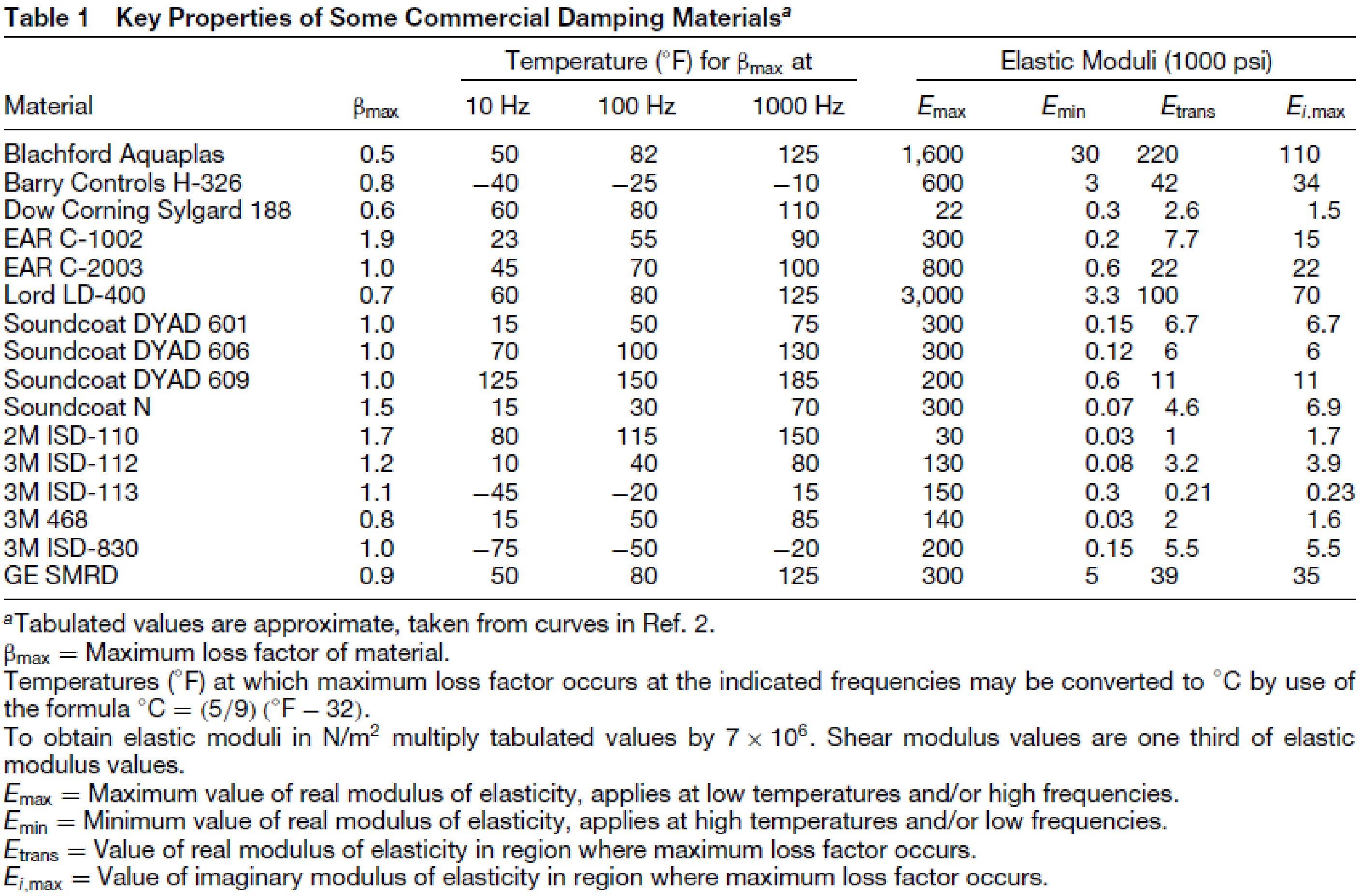


Table 1 Key Properties of Some Commercial Damping Materials[a]

Material	β_{max}	Temperature (°F) for β_{max} at 10 Hz	100 Hz	1000 Hz	Elastic Moduli (1000 psi) E_{max}	E_{min}	E_{trans}	$E_{i,max}$
Blachford Aquaplas	0.5	50	82	125	1,600	30	220	110
Barry Controls H-326	0.8	−40	−25	−10	600	3	42	34
Dow Corning Sylgard 188	0.6	60	80	110	22	0.3	2.6	1.5
EAR C-1002	1.9	23	55	90	300	0.2	7.7	15
EAR C-2003	1.0	45	70	100	800	0.6	22	22
Lord LD-400	0.7	60	80	125	3,000	3.3	100	70
Soundcoat DYAD 601	1.0	15	50	75	300	0.15	6.7	6.7
Soundcoat DYAD 606	1.0	70	100	130	300	0.12	6	6
Soundcoat DYAD 609	1.0	125	150	185	200	0.6	11	11
Soundcoat N	1.5	15	30	70	300	0.07	4.6	6.9
2M ISD-110	1.7	80	115	150	30	0.03	1	1.7
3M ISD-112	1.2	10	40	80	130	0.08	3.2	3.9
3M ISD-113	1.1	−45	−20	15	150	0.3	0.21	0.23
3M 468	0.8	15	50	85	140	0.03	2	1.6
3M ISD-830	1.0	−75	−50	−20	200	0.15	5.5	5.5
GE SMRD	0.9	50	80	125	300	5	39	35

[a] Tabulated values are approximate, taken from curves in Ref. 2.

β_{max} = Maximum loss factor of material.

Temperatures (°F) at which maximum loss factor occurs at the indicated frequencies may be converted to °C by use of the formula $°C = (5/9)(°F - 32)$.

To obtain elastic moduli in N/m^2 multiply tabulated values by 7×10^6. Shear modulus values are one third of elastic modulus values.

E_{max} = Maximum value of real modulus of elasticity, applies at low temperatures and/or high frequencies.

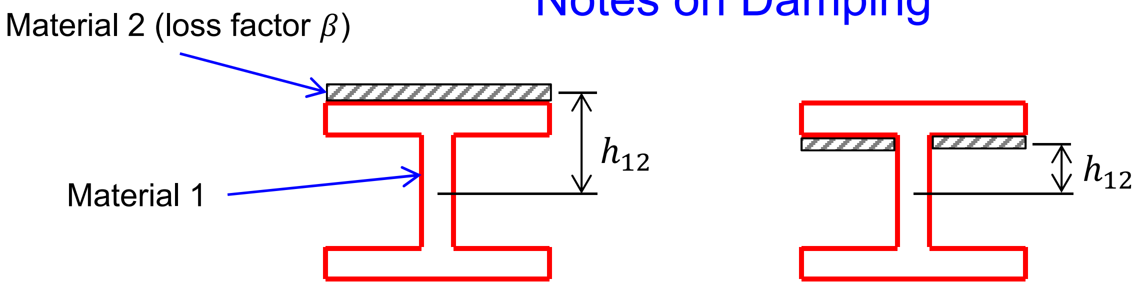
E_{min} = Minimum value of real modulus of elasticity, applies at high temperatures and/or low frequencies.

E_{trans} = Value of real modulus of elasticity in region where maximum loss factor occurs.

$E_{i,max}$ = Value of imaginary modulus of elasticity in region where maximum loss factor occurs.

Beams with Viscoelastic Inserts

Notes on Damping



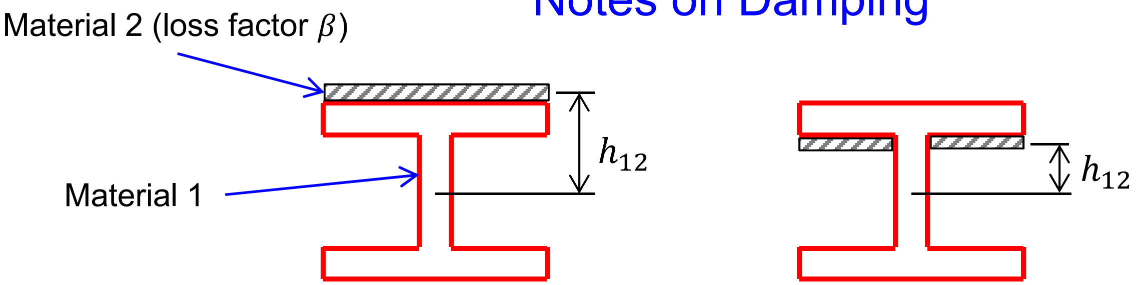
$$\frac{\eta}{\beta} = \left(1 + \frac{k^2(1+\beta^2) + (r_1/h_{12})^2\alpha}{k(1+(r_2/h_{12})^2\alpha}\right)^{-1}$$

$$k = \frac{E_2 S_2}{E_1 S_1} \qquad \alpha = (1+k)^2 + (\beta k)^2 \qquad r_i = \sqrt{\frac{I_i}{S_i}}$$

E_i	elastic modulus of ith material
S_i	cross-sectional area of ith material
I_i	moment of inertia of ith material
β	loss factor of viscoelastic material

Beams with Viscoelastic Inserts

Notes on Damping



Simplification if $k \ll 1$

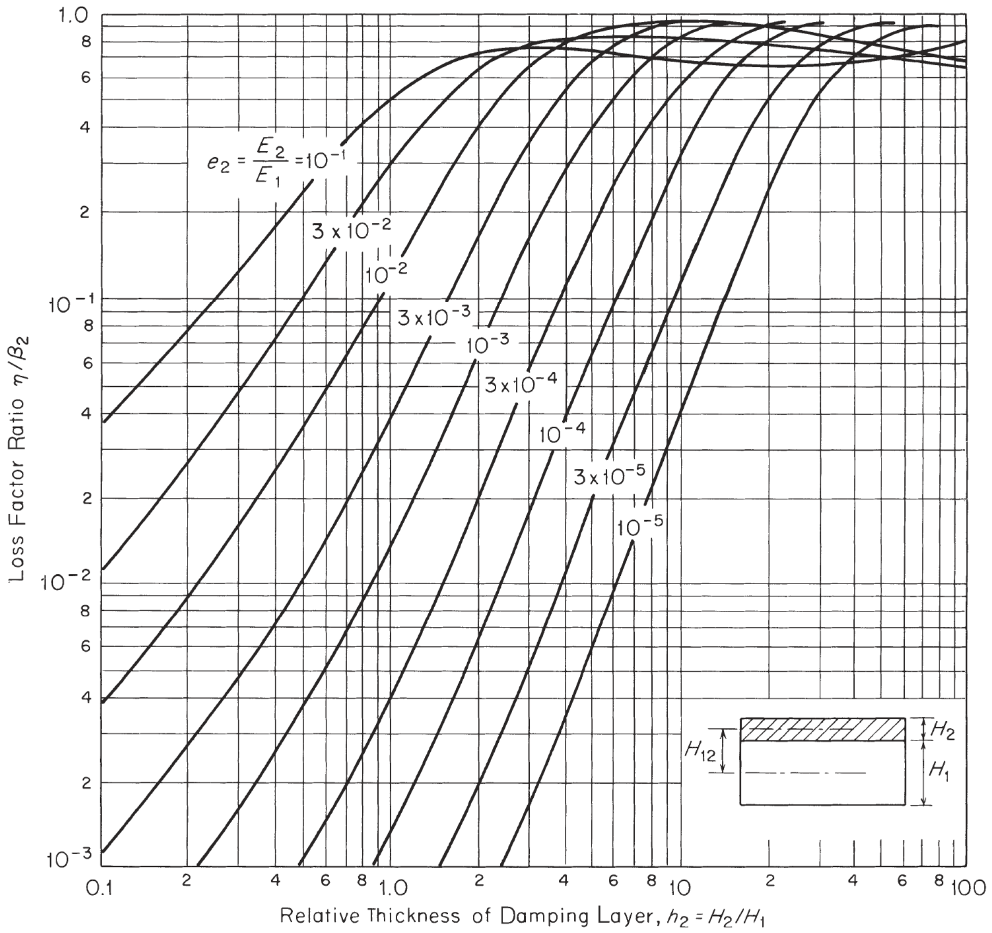
$$\frac{\eta}{\beta} \approx \frac{E_2}{E_1}\frac{(I_2 + S_2 h_{12}^2)}{I_1}$$

E_i	elastic modulus of ith material
S_i	cross-sectional area of ith material
I_i	moment of inertia of ith material
β	loss factor of viscoelastic material

Ungar, 2007

Properties of Damping Materials

Notes on Damping



$$\eta \approx \frac{\beta_2 E_2}{E_1} h_2 (3 + 6h_2 + 4h_2^2)$$

$$h_2 = \frac{H_2}{H_1}$$

β_2 loss factor for viscoelastic material

Summary

Notes on Damping

- The Basics
- Material Damping
- Damping Metrics
- Damping with Viscoelastic Materials