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Vibration Isolation
Measurement and Simulation

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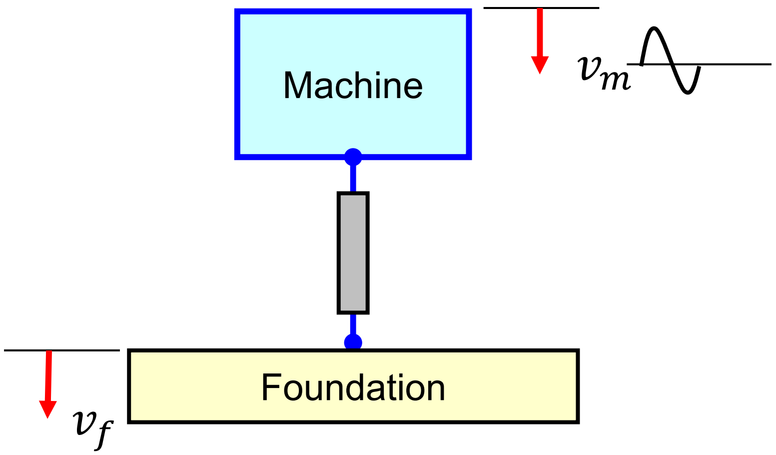
Overview

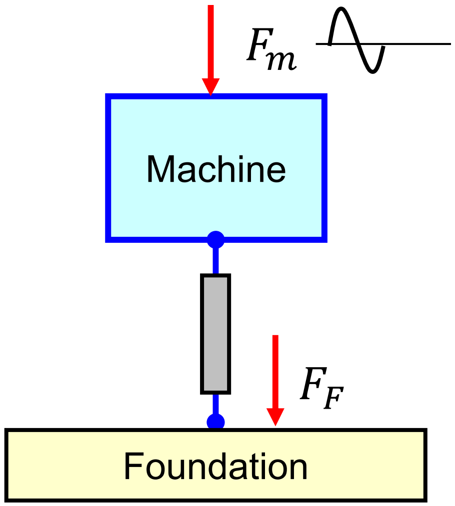
- Basics
- Simulation
  - Method 1 Mobility Matrix
  - Method 2 Impedance Matrix
- Measurement
  - Method 1 Direct Measurement
  - Method 2 Indirect Measurement
- Correlation

Transmissibility



Motion Transmissibility

$$T_{\text{motion}} = \frac{v_F}{v_m}$$

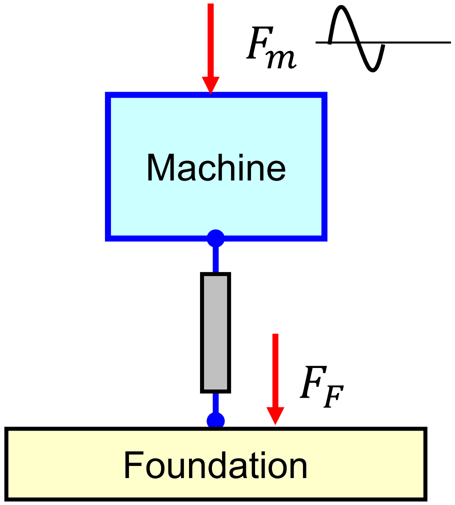


Force Transmissibility

$$T_{\text{force}} = \frac{F_F}{F_m}$$

Note: Transmissibility does not account for changes in the excitation force or motion that may occur when a more flexible isolator is used. Most models using transmissibility assume the machine and foundation to be rigid and the mass of the isolator to be negligible.

Force Transmissibility



If $\xi = 0$

$$T_{\text{force}} = \frac{1}{1 - \left(\frac{\omega}{\omega_n}\right)^2}$$

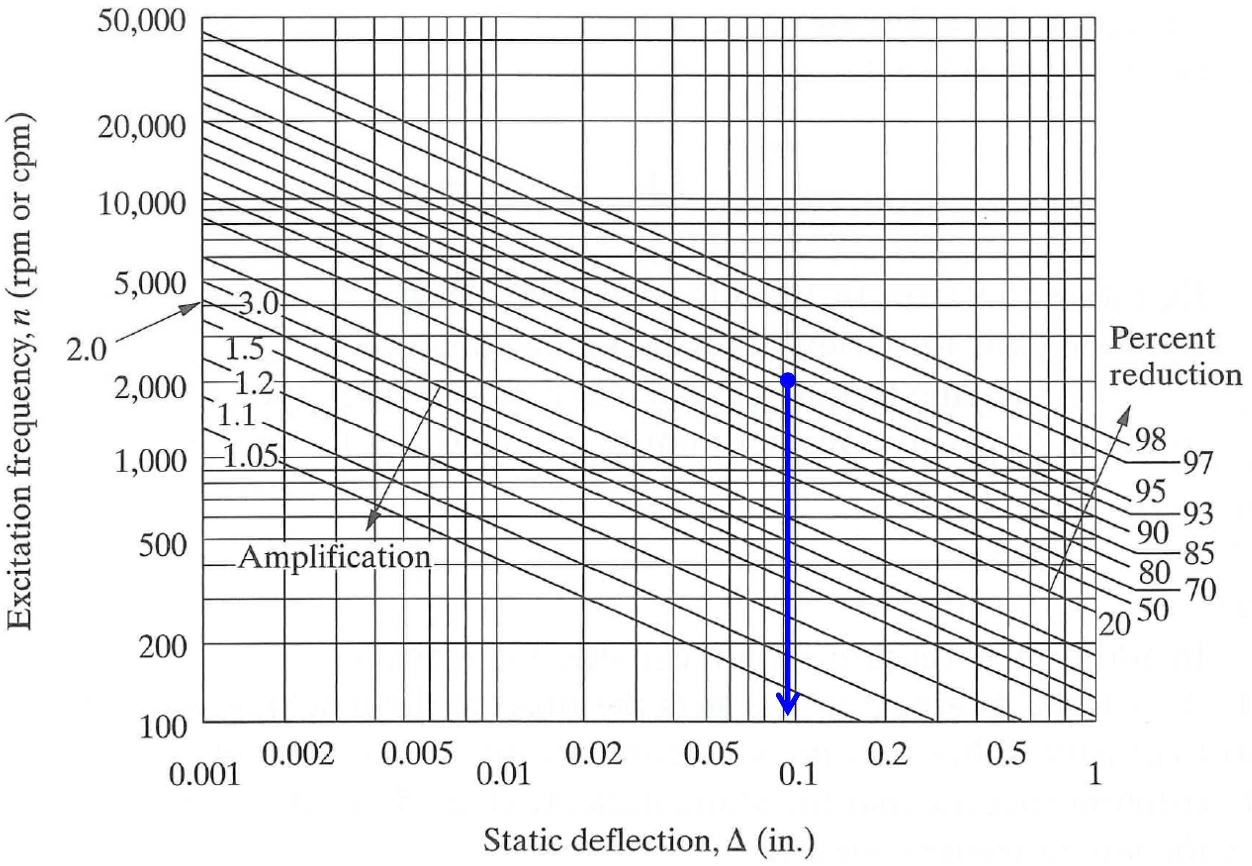
$$\omega = \frac{2\pi n}{60} \qquad \Delta = \frac{mg}{k}$$

$$R = 1 - T_{\text{force}}$$

$$\frac{\omega}{\sqrt{k/m}} = \sqrt{\frac{2 - R}{1 - R}}$$

$$n = \frac{30}{\pi}\sqrt{\frac{g(2 - R)}{\Delta(1 - R)}} = 29.9\sqrt{\frac{(2 - R)}{\Delta(1 - R)}}$$

Design Curves

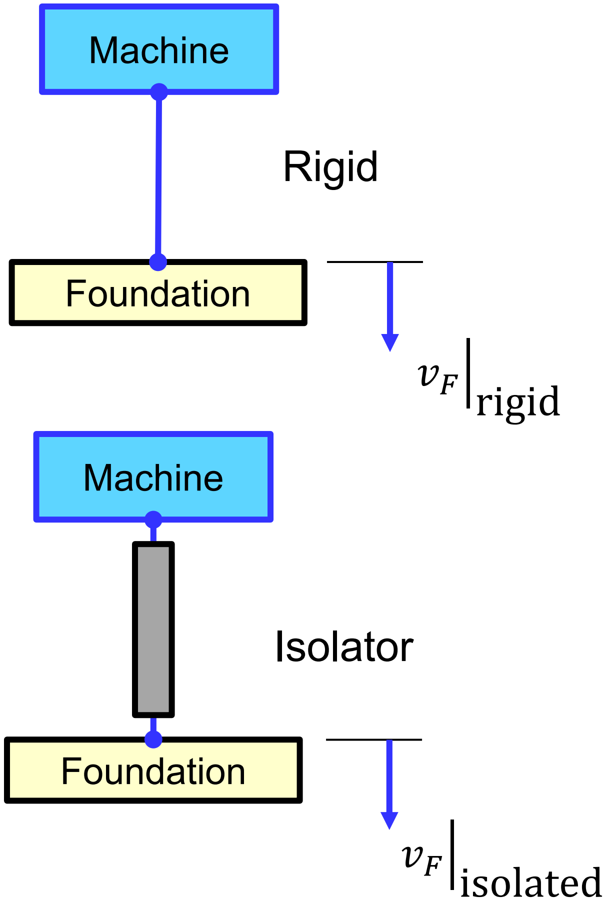


1. Identify static deflection using design curve.
2. Calculate spring stiffness.

$$k = \frac{mg}{\Delta_{static}}$$

3. Clearance between machine and foundation should be more than twice the static deflection of the spring.

Introduction Characterization of Isolator



$$\begin{Bmatrix} F_1 \\ v_1 \end{Bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{Bmatrix} F_2 \\ v_2 \end{Bmatrix}$$

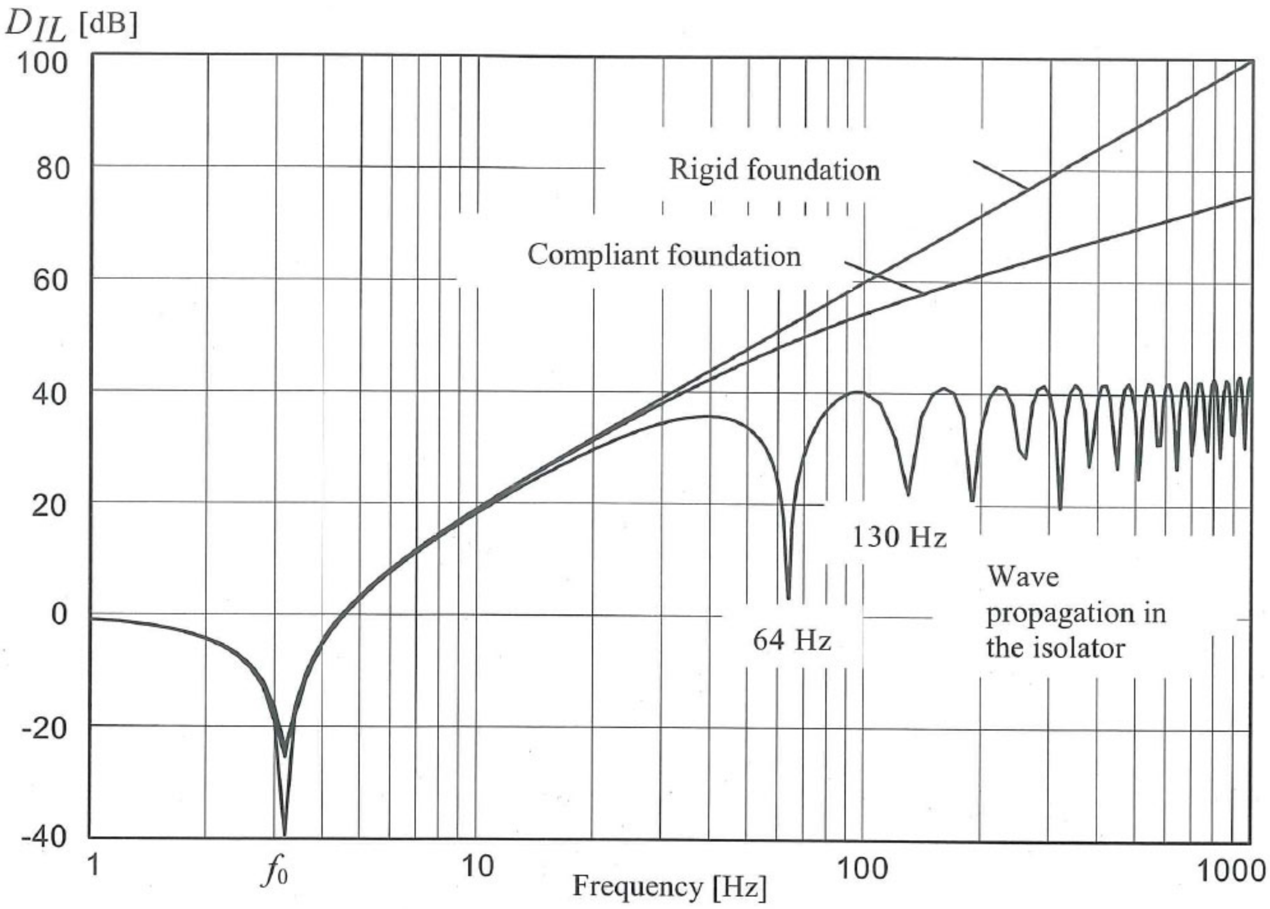
The effectiveness of an isolator can be described using isolator insertion loss:

$$IL = 20 \cdot \log_{10} \left| \frac{v_F|_{\text{rigid}}}{v_F|_{\text{isolated}}} \right|$$

$$= 20 \cdot \log_{10} \left| \frac{a_{11} Z_F + a_{12} + a_{21} Z_F Z_S + a_{22} Z_S}{Z_S + Z_F} \right|$$

Z_S and Z_F are the mechanical impedances at the isolator mounting point on source and foundation sides, respectively.

Effect of Wave Propagation in Isolator



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Analysis Steps

- Static Analysis to pre-load mount (nonlinear, large deformation analysis)
- Modal Analysis to find loaded/pre-stressed modes
- Forced Response Analysis to find the transfer matrix

Static Analysis

Modal Analysis

Modal Superposition / Forced Response Analysis

Boundary conditions depend upon the method used.

Method 1 Mobility Matrix

Reconfigure into mobility matrix

$$\begin{Bmatrix} v_1 \\ v_2 \end{Bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix}$$

Solve model twice

Solve 1: $F_1 = 1; F_2 = 0$ Solve 2: $F_1 = 0; F_2 = 1$

$$b_{11} = \left.\frac{v_1}{F_1}\right|_{F_1=1, F_2=0} \qquad b_{12} = \left.\frac{v_1}{F_2}\right|_{F_1=0, F_2=1}$$

$$b_{21} = \left.\frac{v_2}{F_1}\right|_{F_1=1, F_2=0} \qquad b_{22} = \left.\frac{v_2}{F_2}\right|_{F_1=0, F_2=1}$$

Method 1 Mobility Matrix

Convert to traditional four-poles

$$\begin{Bmatrix} v_1 \\ v_2 \end{Bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix}$$

$$a_{11} = -\frac{b_{22}}{b_{21}} \qquad a_{12} = \frac{1}{b_{21}}$$

$$a_{21} = b_{12} - \frac{b_{11}b_{22}}{b_{21}} \qquad a_{22} = \frac{b_{11}}{b_{21}}$$

$$\begin{Bmatrix} F_1 \\ v_1 \end{Bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{Bmatrix} F_2 \\ v_2 \end{Bmatrix}$$

Method 2 Impedance Matrix

Reconfigure into impedance matrix

$$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} \begin{Bmatrix} v_1 \\ v_2 \end{Bmatrix}$$

Solve model twice

Solve 1: $F_1 = 1 \, ; v_2 = 0$
Solve 2: $F_2 = 1; v_1 = 0$

$$c_{11} = \left. \frac{F_1}{v_1} \right|_{v_2=0} \qquad c_{12} = \left. \frac{F_1}{v_2} \right|_{F_1=0}$$

$$c_{21} = \left. \frac{F_2}{v_1} \right|_{F_2=0} \qquad c_{22} = \left. \frac{F_2}{v_2} \right|_{F_1=0}$$

Method 2 Impedance Matrix

Convert to traditional four-poles

$$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} \begin{Bmatrix} v_1 \\ v_2 \end{Bmatrix}$$

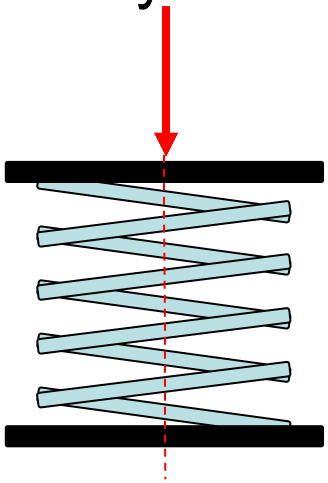
$$a_{11} = \frac{c_{11}}{c_{21}} \qquad a_{12} = c_{12} - \frac{c_{11}c_{22}}{c_{21}}$$

$$a_{21} = \frac{1}{c_{21}} \qquad a_{22} = -\frac{c_{22}}{c_{21}}$$

$$\begin{Bmatrix} F_1 \\ v_1 \end{Bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{Bmatrix} F_2 \\ v_2 \end{Bmatrix}$$

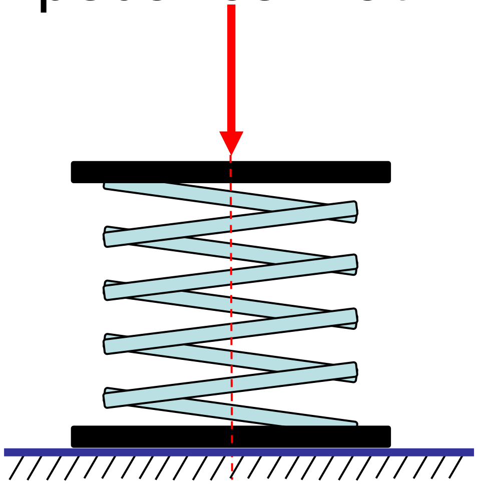
Difference Between Methods

Mobility Matrix



Isolator is free-free
after static analysis.

Impedance Matrix



Isolator is fixed on
one side.

The isolator was constrained in the lateral direction in each case. The difference in boundary conditions leads to slight differences.

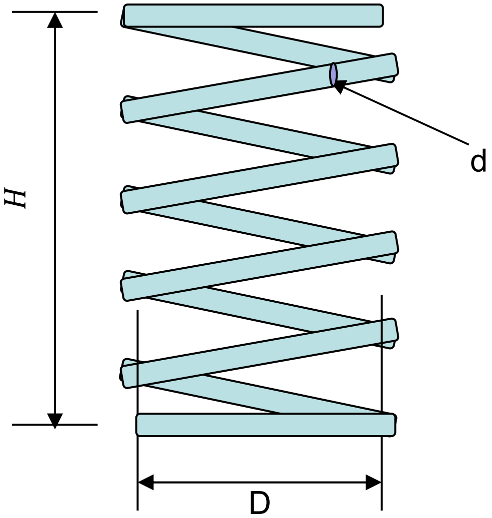
Simple Spring Properties

Spring Stiffness (Ungar, 2007)

$$k = \frac{Gd^4}{8nD^3}$$

Spring Mass

$$m = \rho\sqrt{H^2 + (n\pi D)^2}\,\frac{\pi d^2}{4}$$



ρ	density of material
G	shear modulus of material
d	diameter of the spring wire
H	height of spring
D	average diameter of the spring
n	number of active coil turns

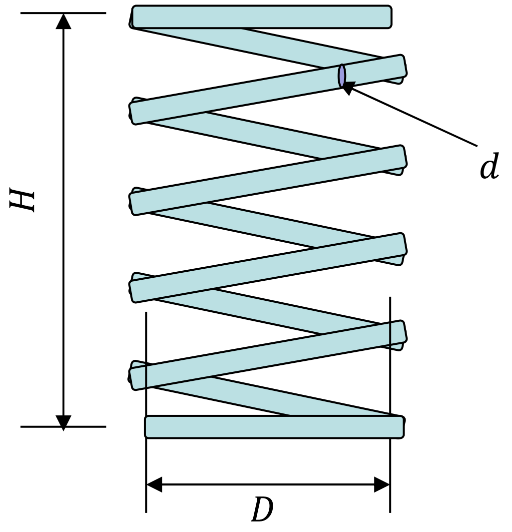
Simple Relationships

Insertion loss proportional to

$$IL \propto 20\log_{10}\left|\frac{\omega n D^3}{G d^4}\right|$$

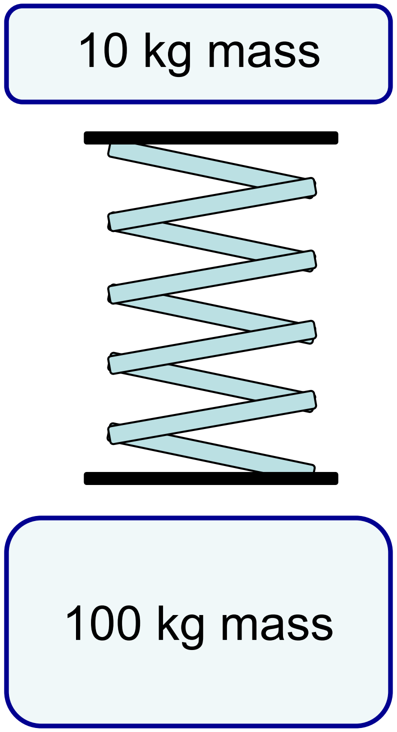
First surge frequency

$$f_1 \propto \frac{d}{nD^2}\sqrt{\frac{G}{\rho}}$$

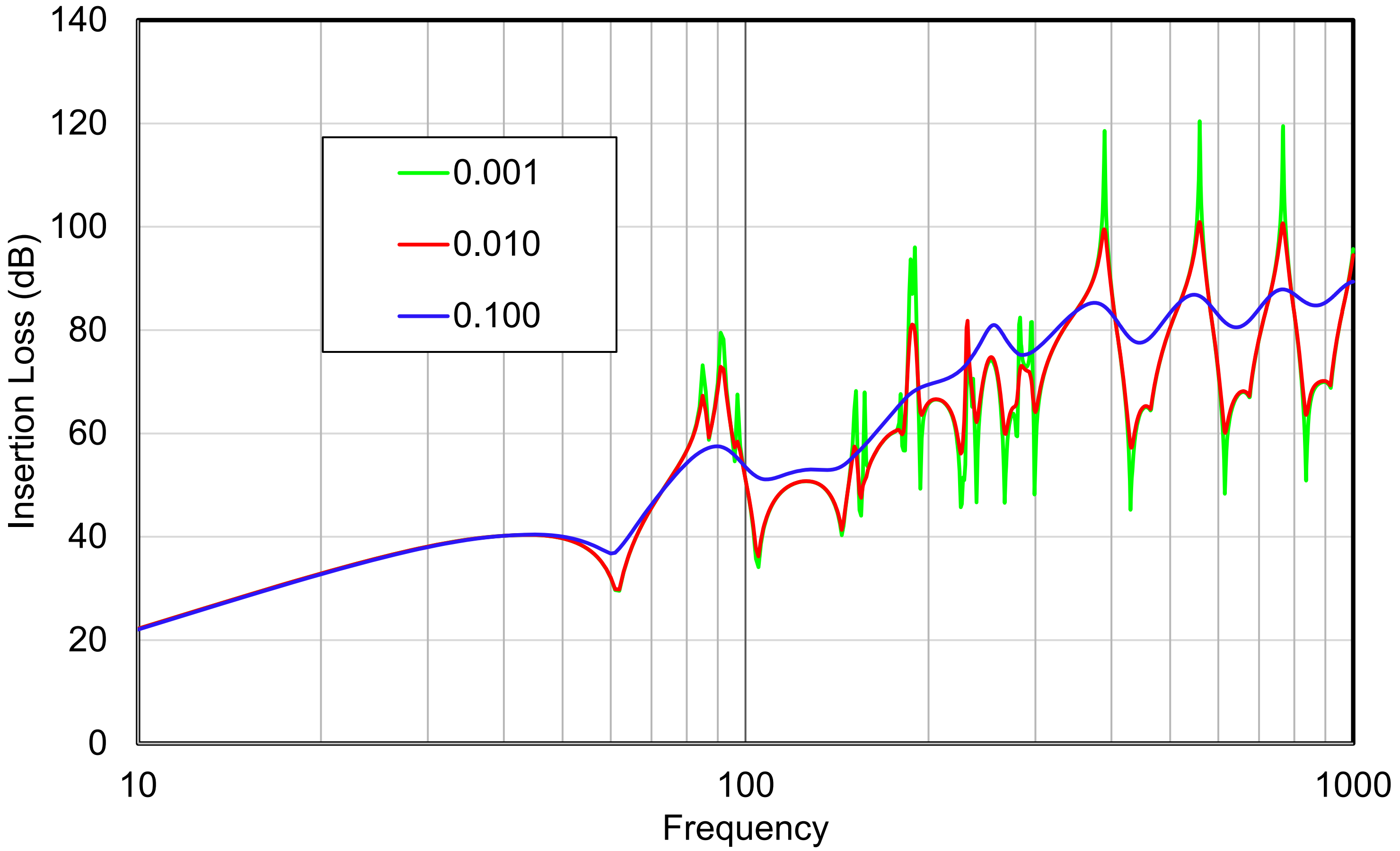


ρ	density of material
G	shear modulus of material
d	diameter of the spring wire
H	height of spring
D	average diameter of the spring
n	number of active coil turns

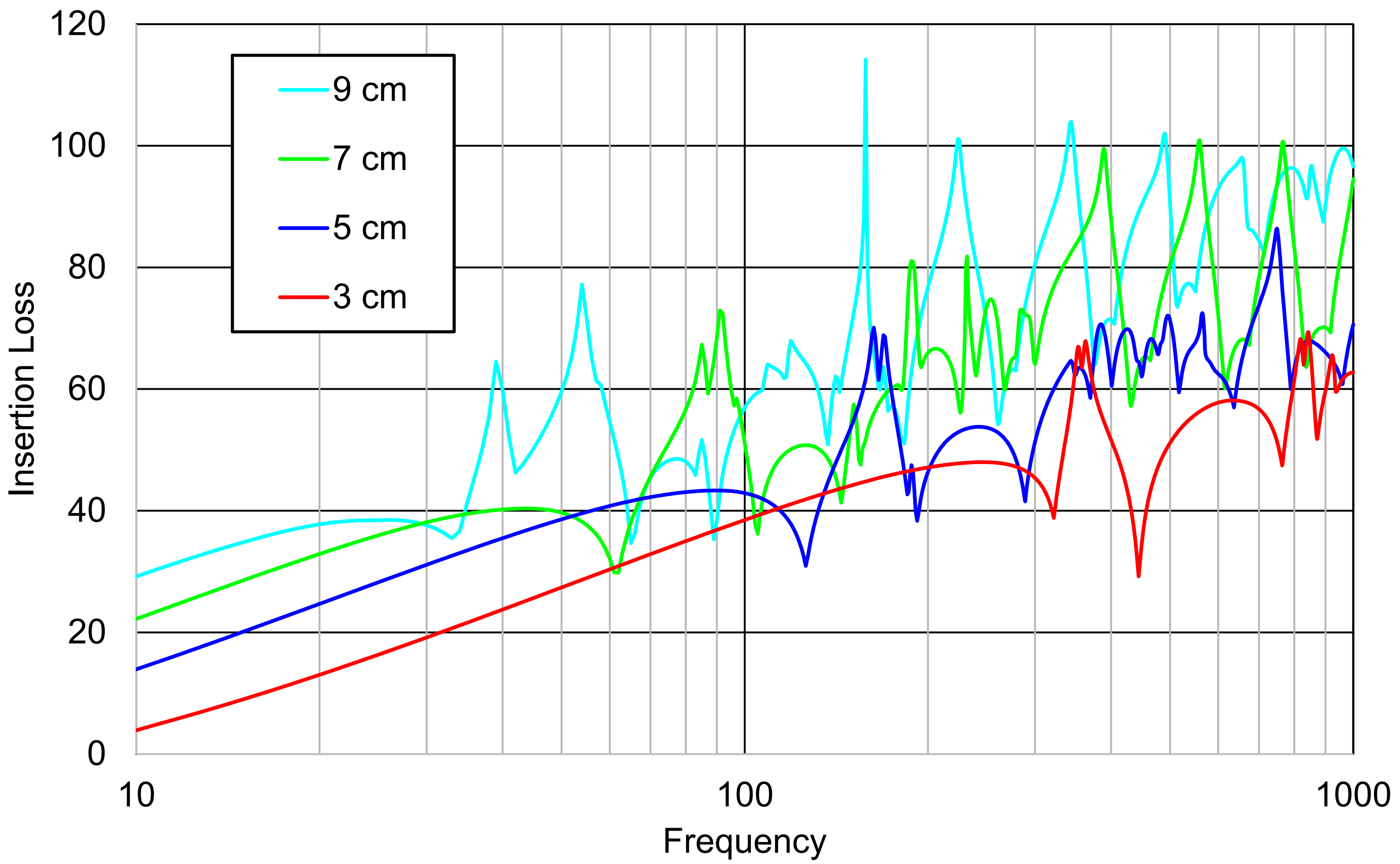
Case 1 Isolator Between Two Masses



Effect of Damping



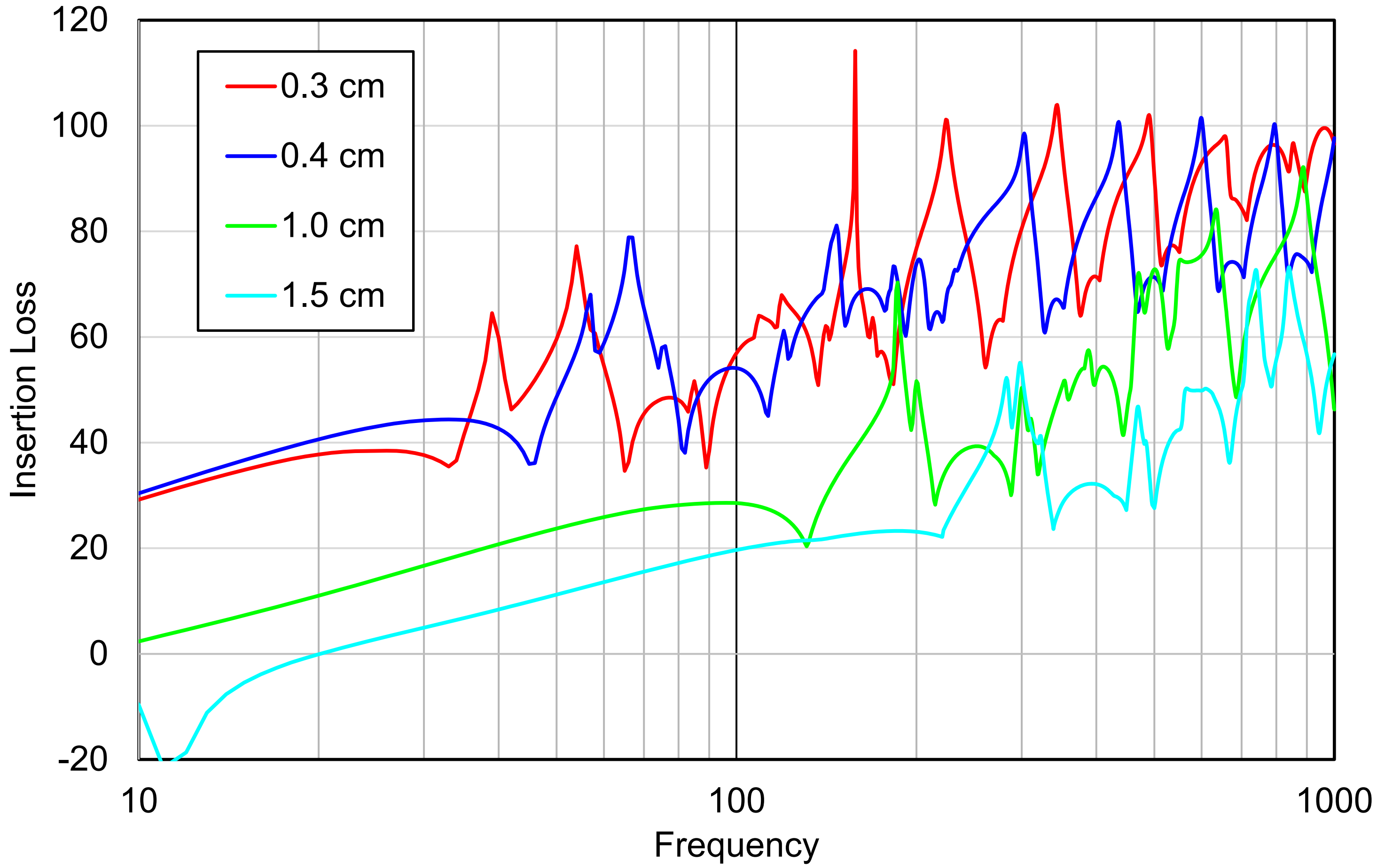
Insertion Loss Vary Spring Diameter *D*



$$IL \propto 20\log_{10}|D^3|$$

$$f_1 \propto \frac{1}{D^2}$$

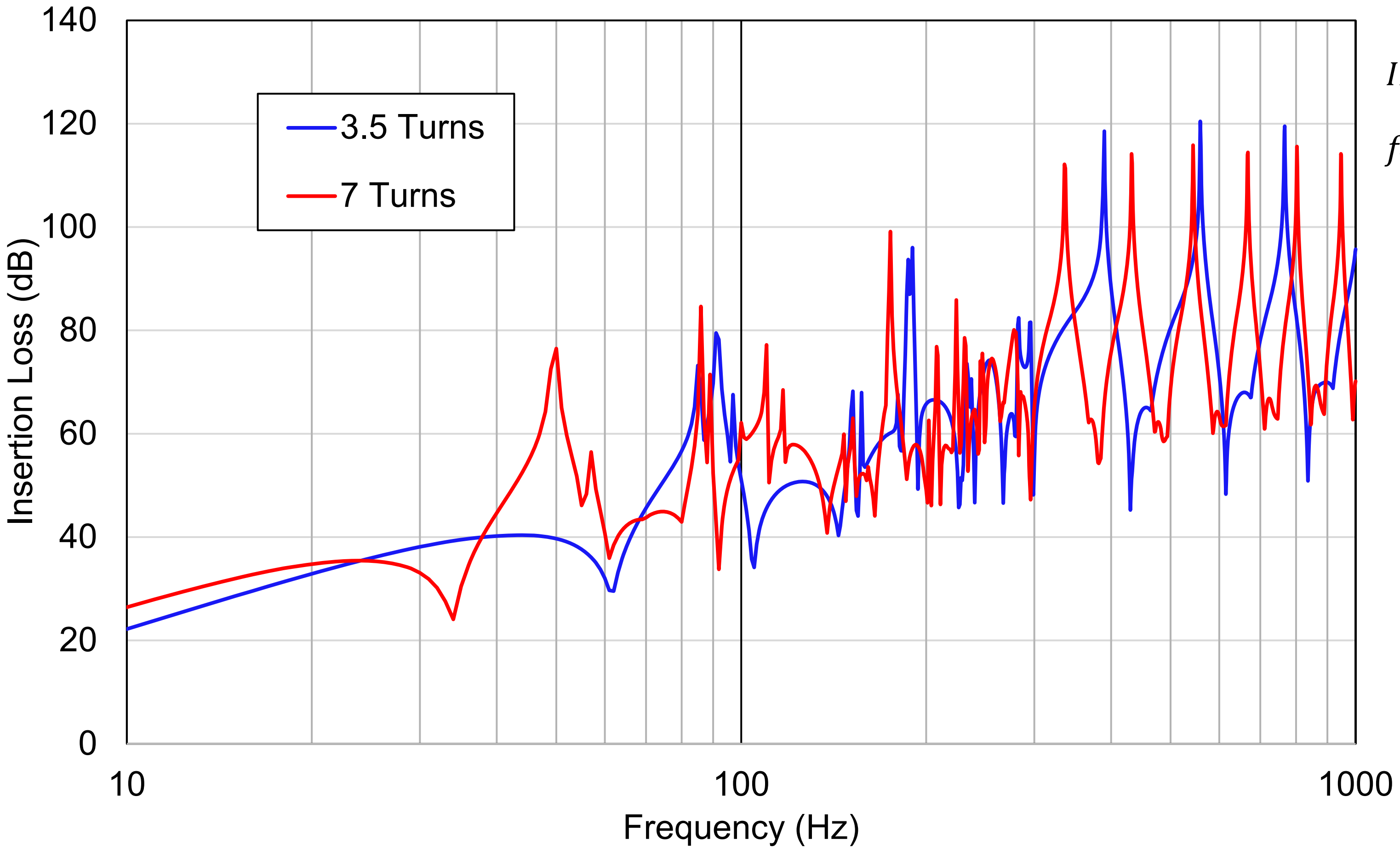
Insertion Loss Vary Wire Diameter d



$$IL \propto 20\log_{10}\left|\frac{1}{d^4}\right|$$

$$f_1 \propto d$$

Insertion Loss Vary Number of Turns n



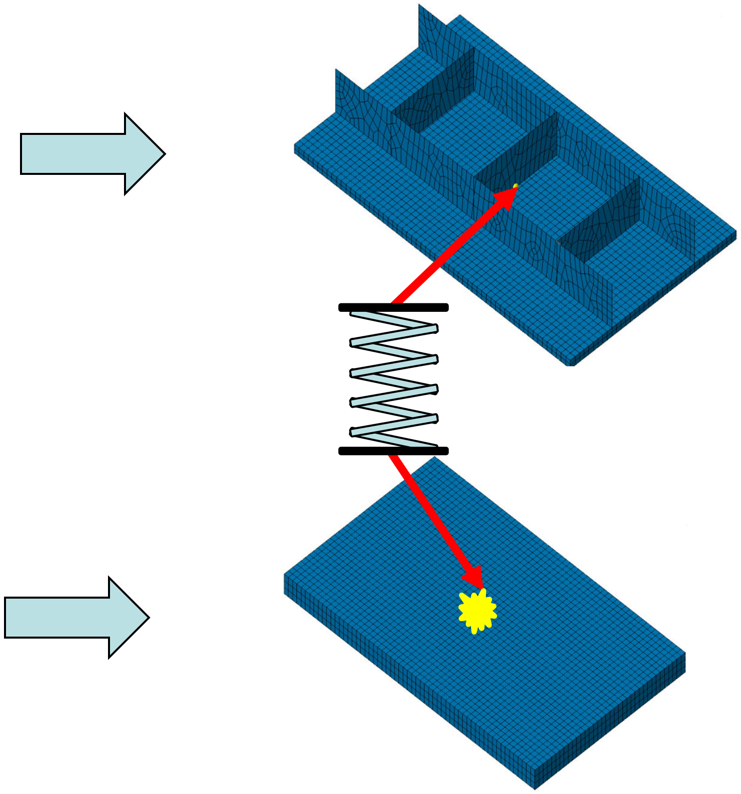
$IL = \propto 20\log_{10}|n|$

$f_1 \propto \frac{1}{n}$

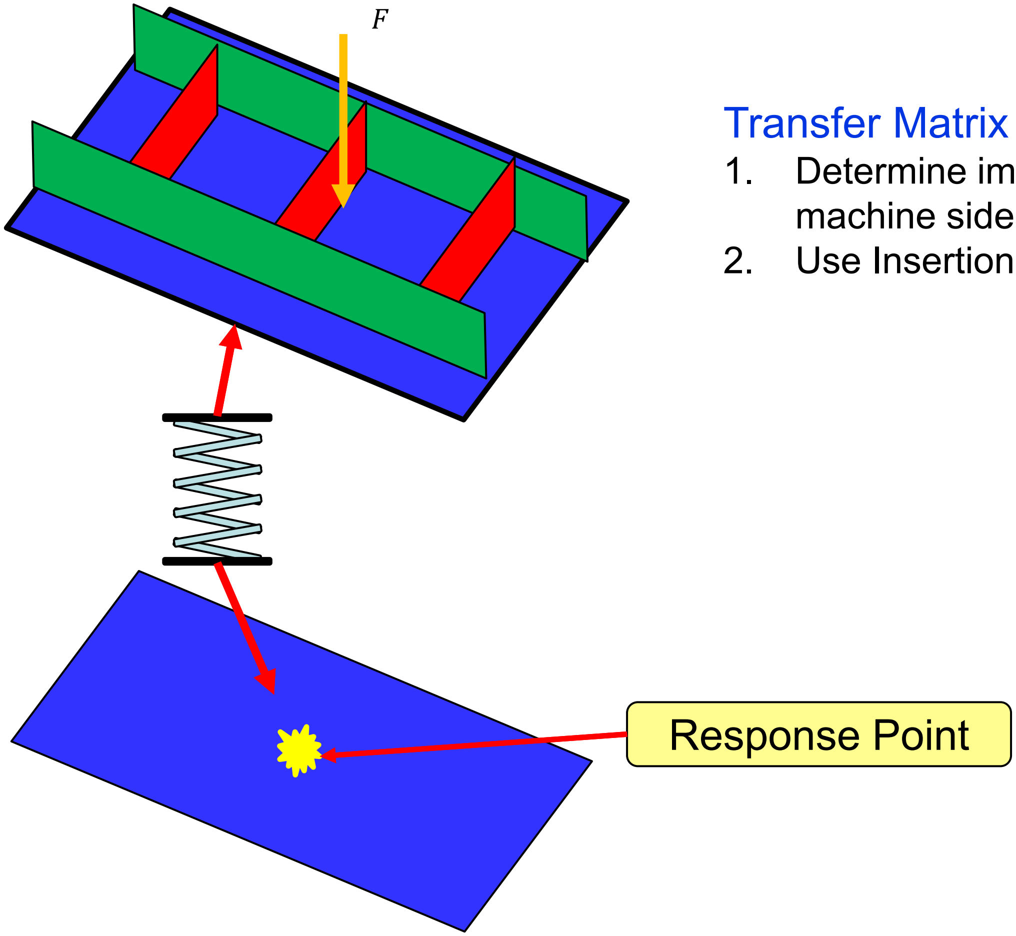
Case 2 Isolator Between Two Structures

Machine Side
- 1 cm thick ribbed steel structure

Foundation Side
- 50 cm X 30 cm
- 5 cm thick steel plate



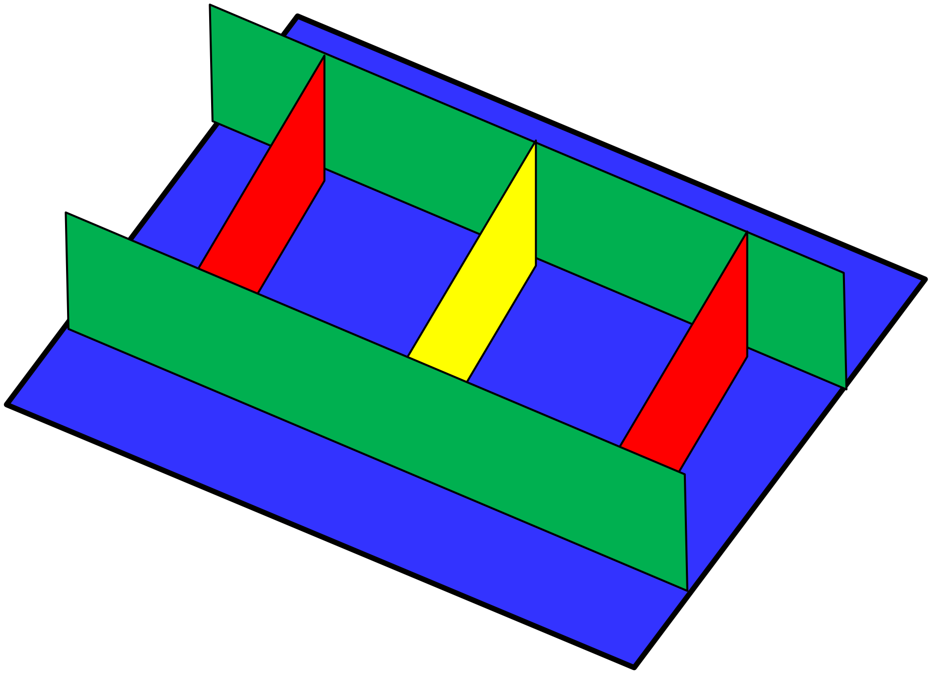
Local Model of Isolator



Transfer Matrix Approach

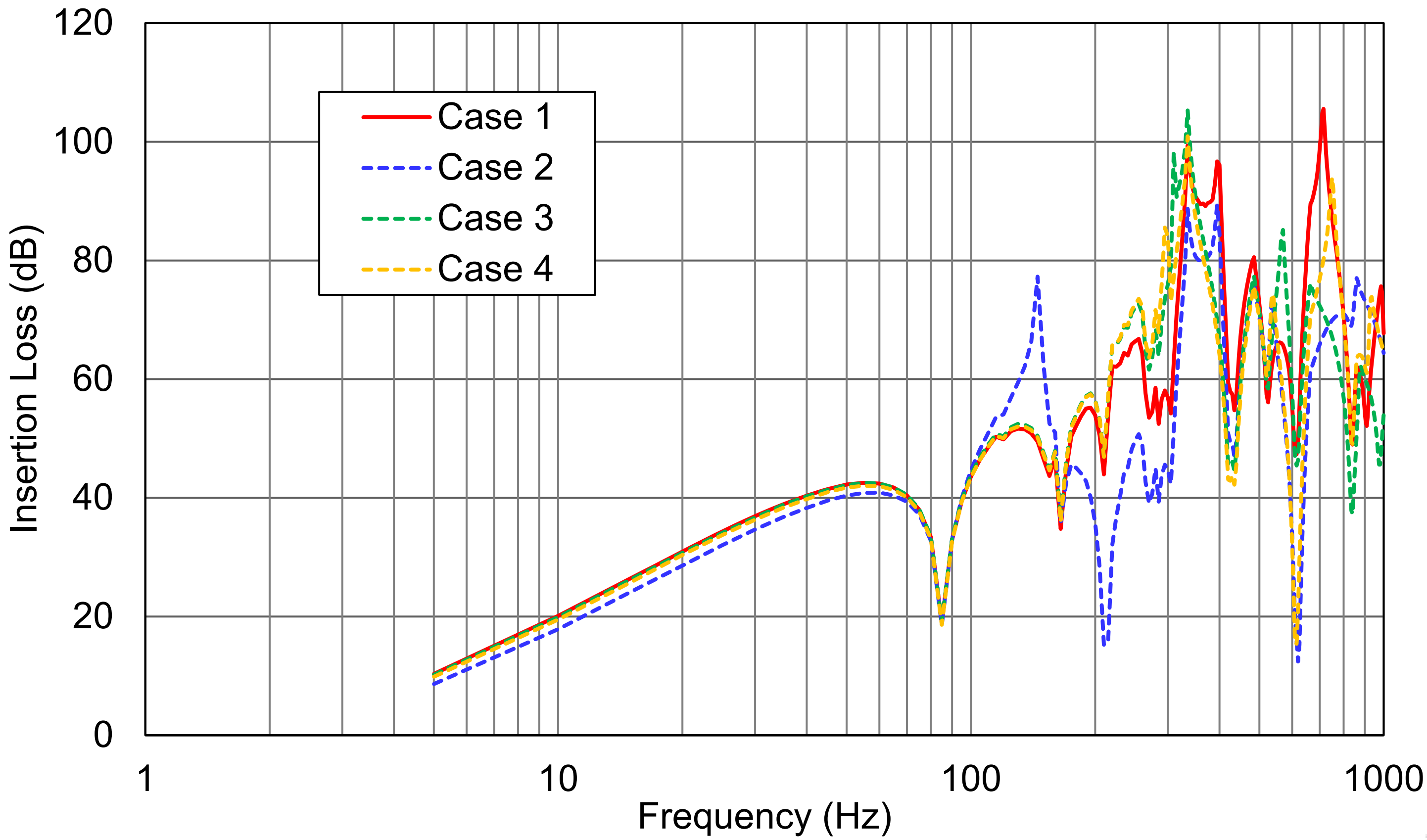
1. Determine impedances of foundation and machine sides.
2. Use Insertion loss equation.

Sensitivity Study



- Case 1 – All Ribs
- Case 2 – No Ribs
- Case 3 – Remove Yellow
- Case 4 – Remove Yellow and Red

Insertion Loss Comparison



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ISO 10846

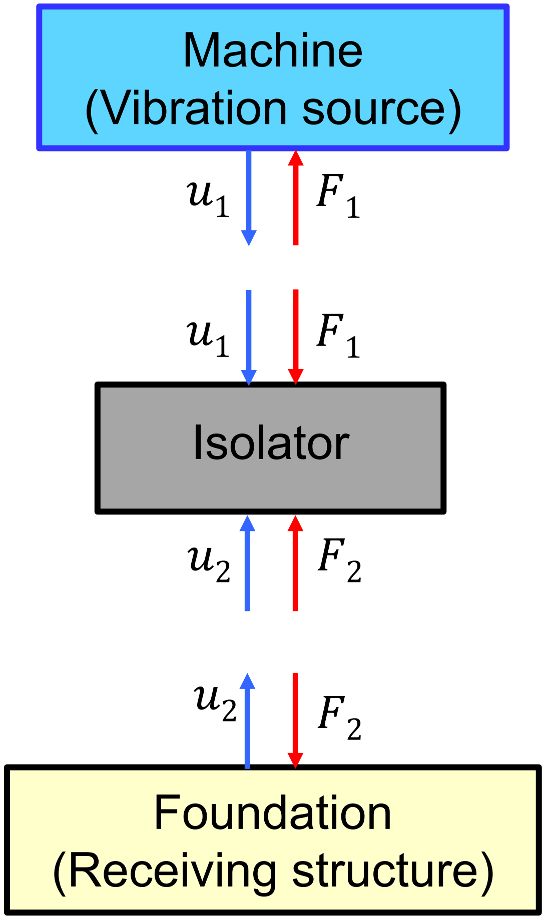
Acoustics and vibration – Laboratory measurement of vibro-acoustic transfer elements of resilient elements

Part 1 (2008): Principles and guidelines

Part 2 (2008): Direct method for determination of the dynamic stiffness of resilient supports for translator motion

Part 3 (2002): Indirect method for determination of the dynamic stiffness of resilient supports for translator motion

ISO 10846-1 General Principles



Assume

1. Linearity for vibrational behavior under a static preload.
2. Contact interfaces can be considered point contacts.

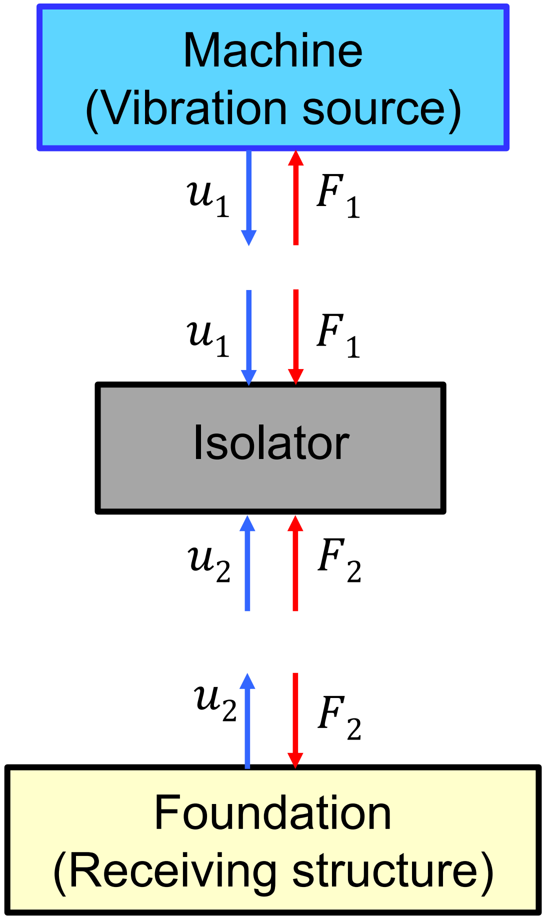
$$F_1 = k_{11}u_1 + k_{12}u_2$$
$$F_2 = k_{21}u_1 + k_{22}u_2$$

$$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

k_{11} and k_{22} indicate dynamic driving point stiffness when the output/input is blocked (k_{11} = k_{22} at low frequencies).

k_{12} and k_{21} indicate dynamic transfer stiffness ($k_{12} \approx k_{21}$ if inertial forces can be neglected).

ISO 10846-1 General Principles



Machine
(Vibration source)

u_1 F_1

u_1 F_1

Isolator

u_2 F_2

u_2 F_2

Foundation
(Receiving structure)

Foundation Dynamic Stiffness

$$k_t = -\frac{F_2}{u_2}$$

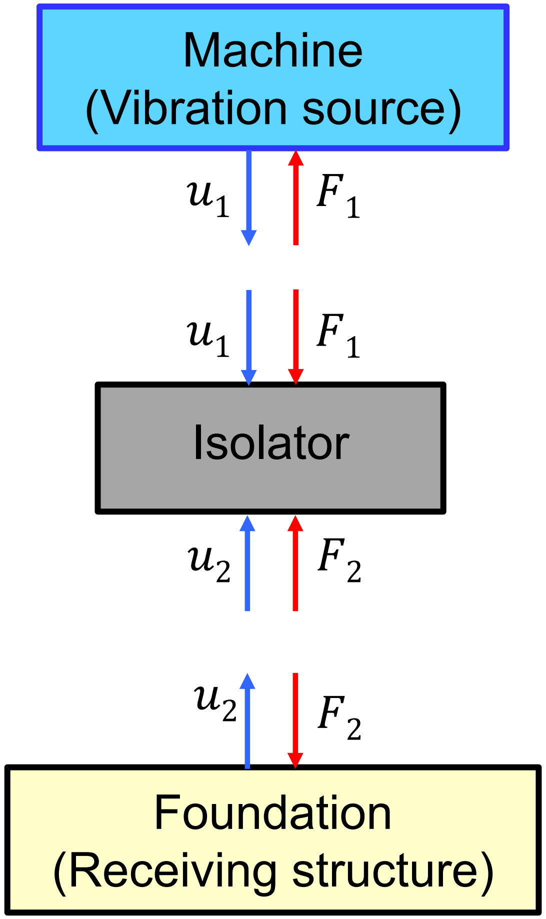
$$F_2 = \frac{k_{21}}{1 + \frac{k_{22}}{k_t}} u_1$$

when $k_{22} < 0.1k_t$

$$F_2 \approx F_2^{\text{blocked}} = k_{21} u_1$$

$$k_{21} = \frac{F_2}{u_1}$$

ISO 10846-1 General Principles



Assume $k_t \gg k_{21}$

$F_1 = k_{11} u_1$

$F_2 = k_{21} u_1$

At low frequencies

$k \approx k_{11} \approx k_{21}$

Complex low-frequency dynamic stiffness

$k = k_0 (1 + j\eta)$

$\eta = \tan \psi$

k_0	real part of dynamic stiffness
η	loss factor
ψ	phase angle of the dynamic stiffness

ISO 10846-2 Direct Method

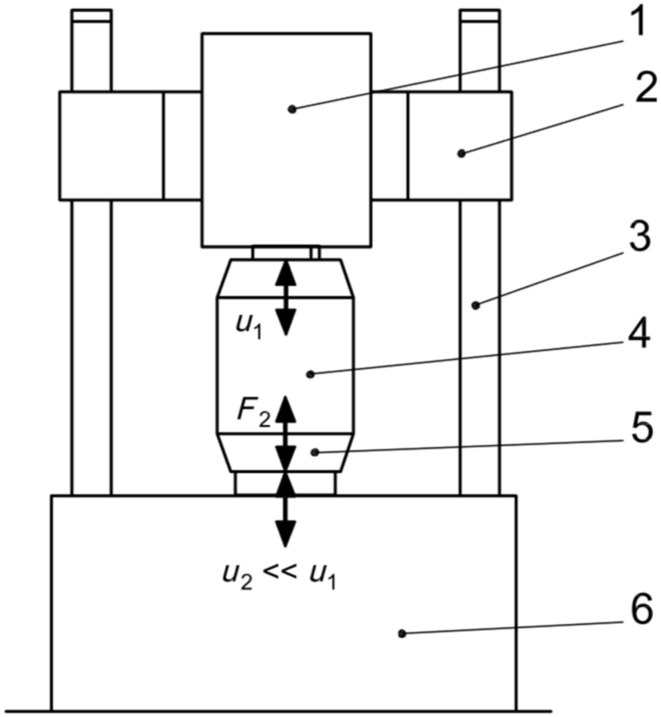


Image from ISO 10846-1

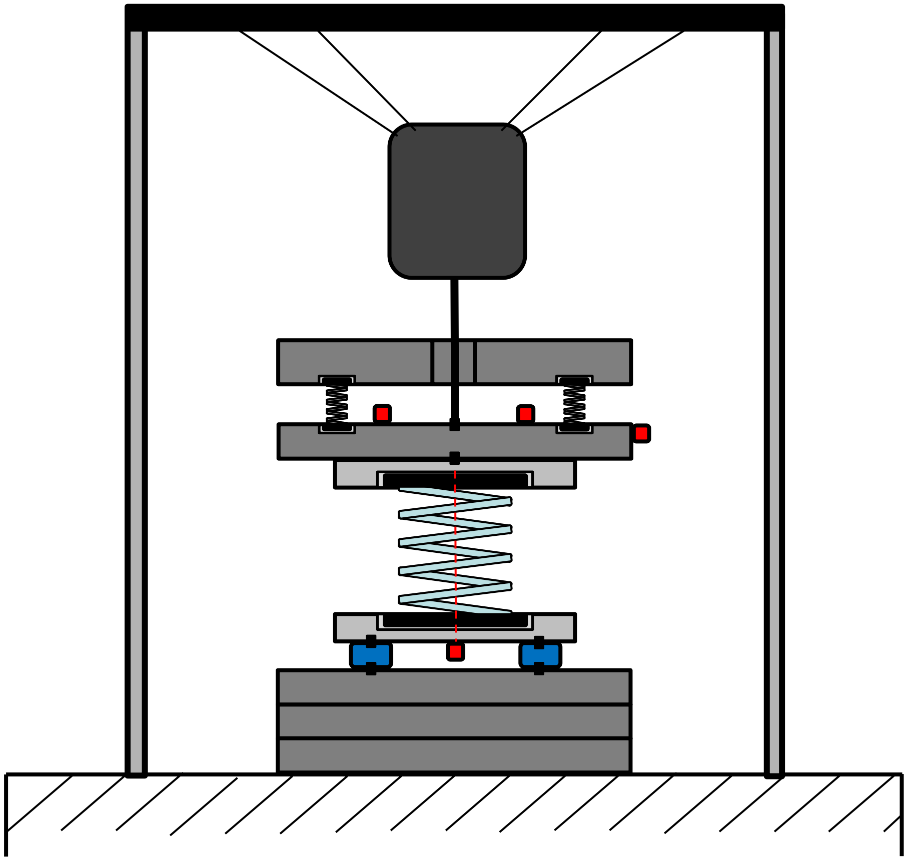
$$k_{21} = \frac{F_2}{u_1}$$

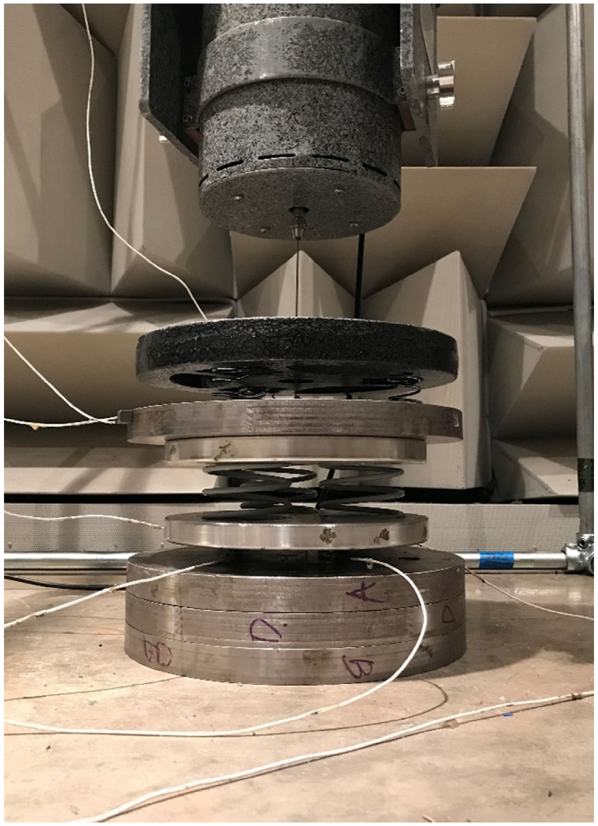
Assume $u_1 \gg u_2$

Schematic of typical test rig

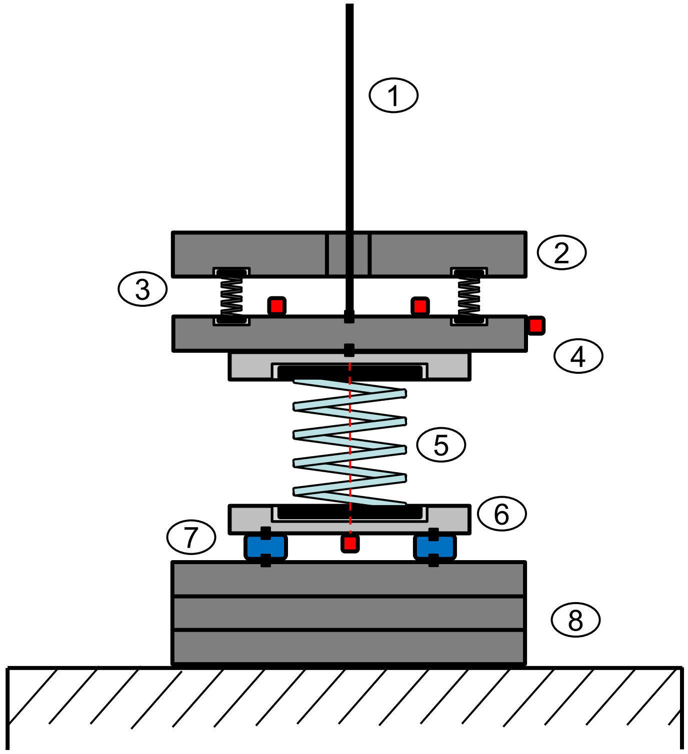
1. Static preload and dynamic excitation (shaker)
2. Moveable traverse
3. Columns (guide rods, frame)
4. Test element (isolator)
5. Force measurement (load cells)
6. Rigid foundation (Blocking mass)

Test Rig Design





Direct Method Test Rig Design



Direct measurement

$$k_{21} = \frac{F_2}{u_1}$$

assume $u_1 \gg u_2$

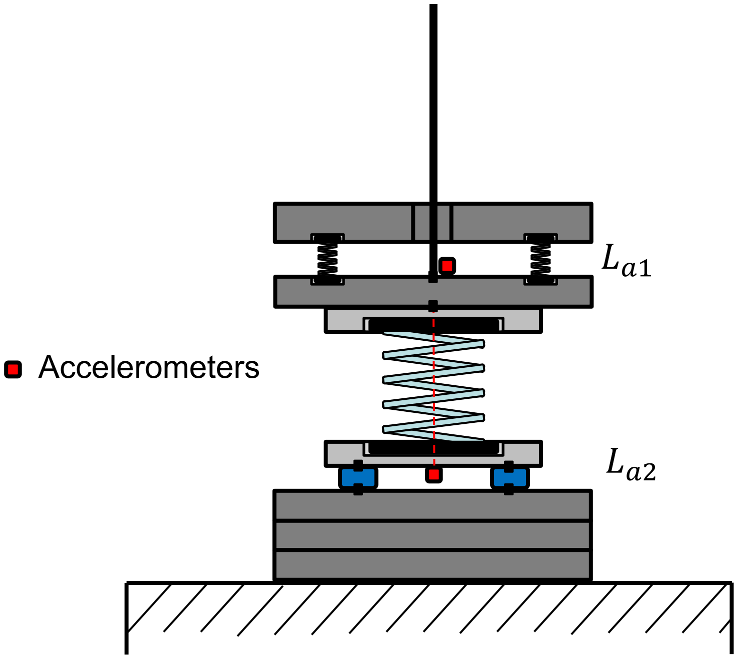
Schematic of test rig for Direct Method

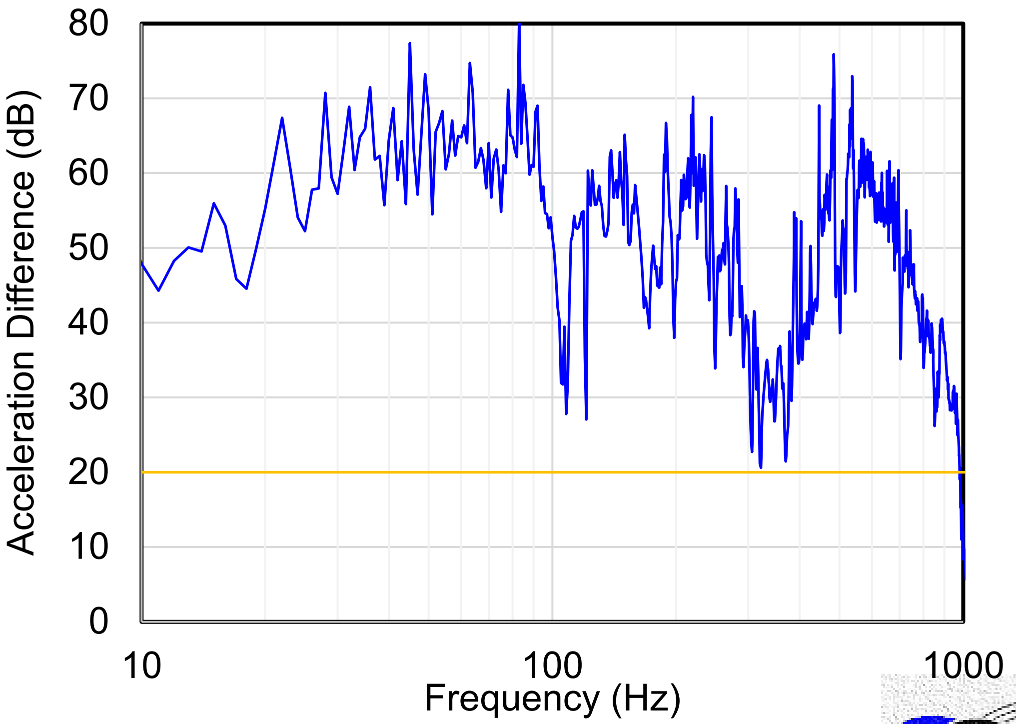
1. Dynamic excitation (shaker)
2. Static preload
3. Decoupling springs
4. Excitation mass (m_1)
5. Test element (isolator)
6. Lower force distribution flange
7. Force measurement (load cells)
8. Rigid foundation

ISO 10846-2 Direct Method

Valid Frequency Range:

$$\Delta L_{12} = L_{a_1} - L_{a_2} \geq 20\ \mathrm{dB}$$

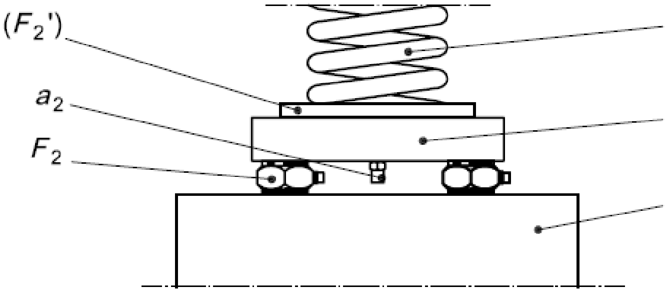




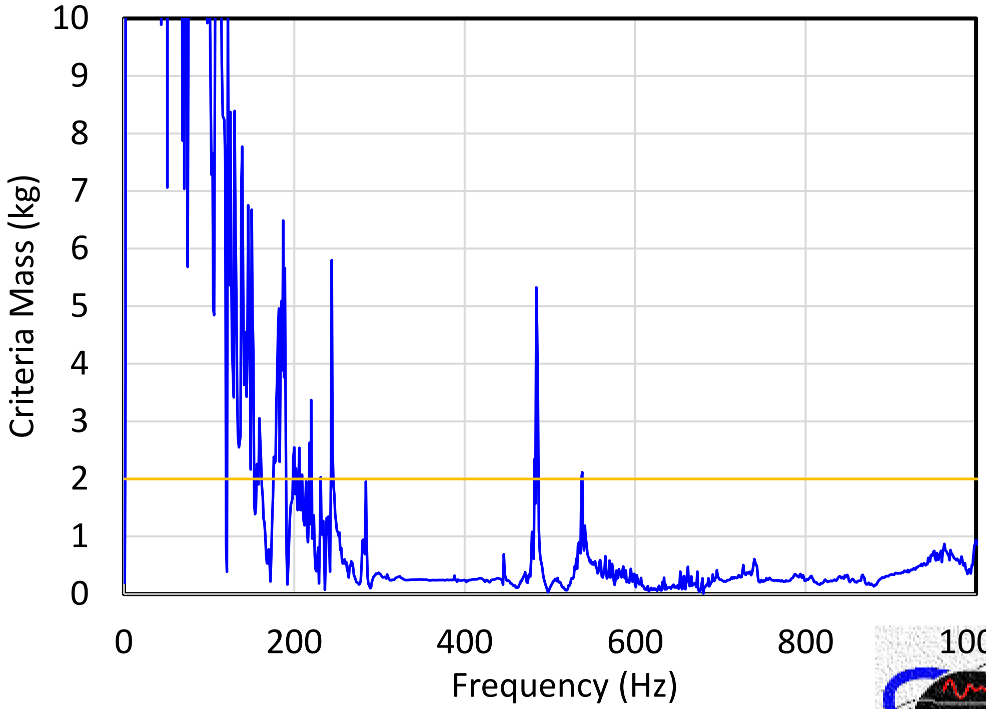
ISO 10846-2 Direct Method

Adequacy of blocking force measurement

$$m_2 \leq 0.06 \times \frac{10^{\frac{L_{F_2}}{20}}}{10^{\frac{L_{a_2}}{20}}} \ \text{kg}$$



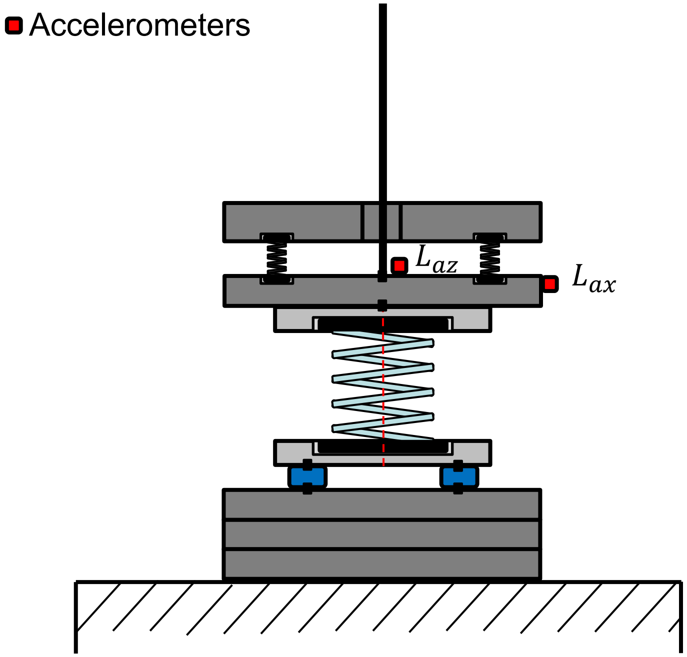
2 is the force distribution plate
(image from ISO-10846-2)

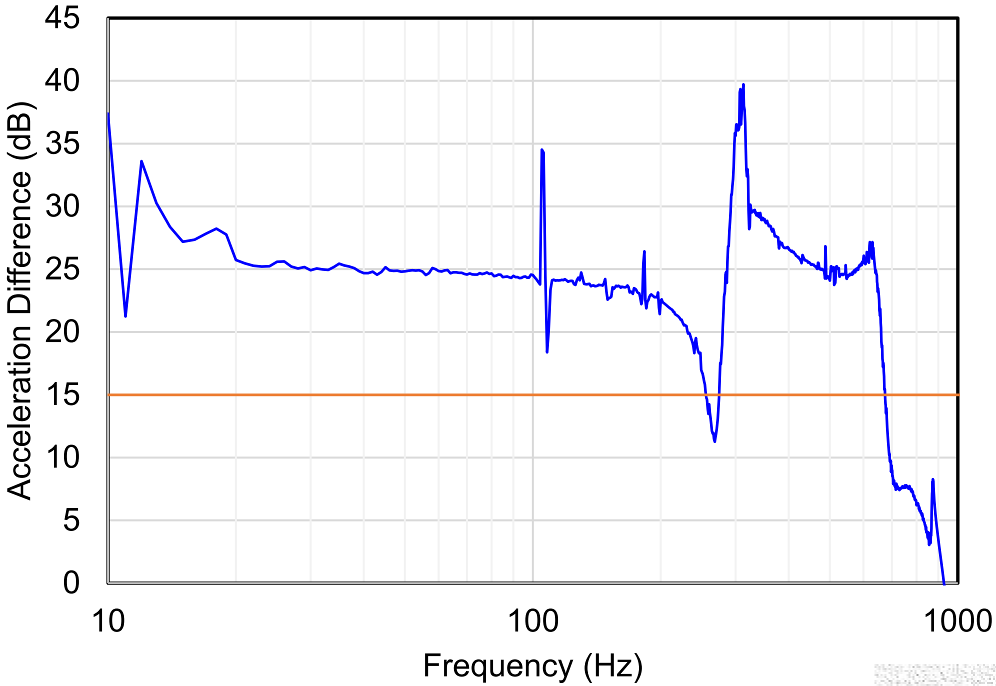


ISO 10846-2 Direct Method

Unwanted input vibration 1:

$$L_{az} - L_{ax} \geq 15 \text{ dB}$$

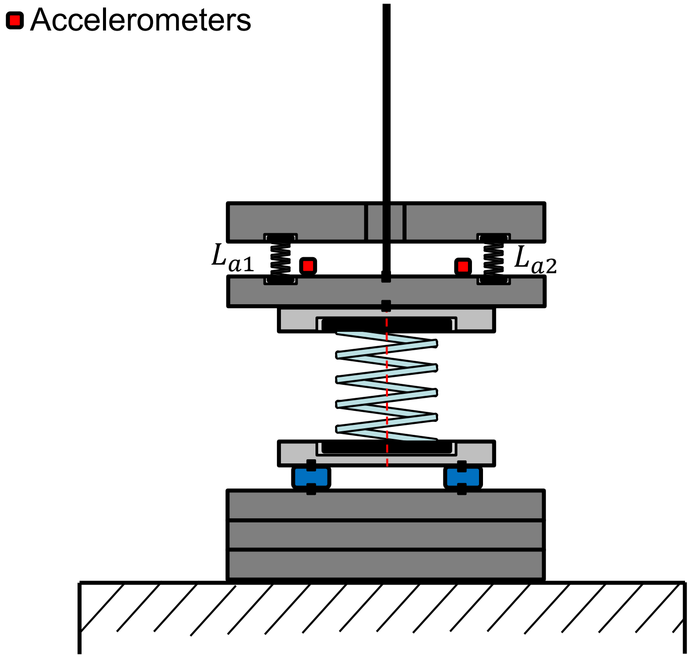


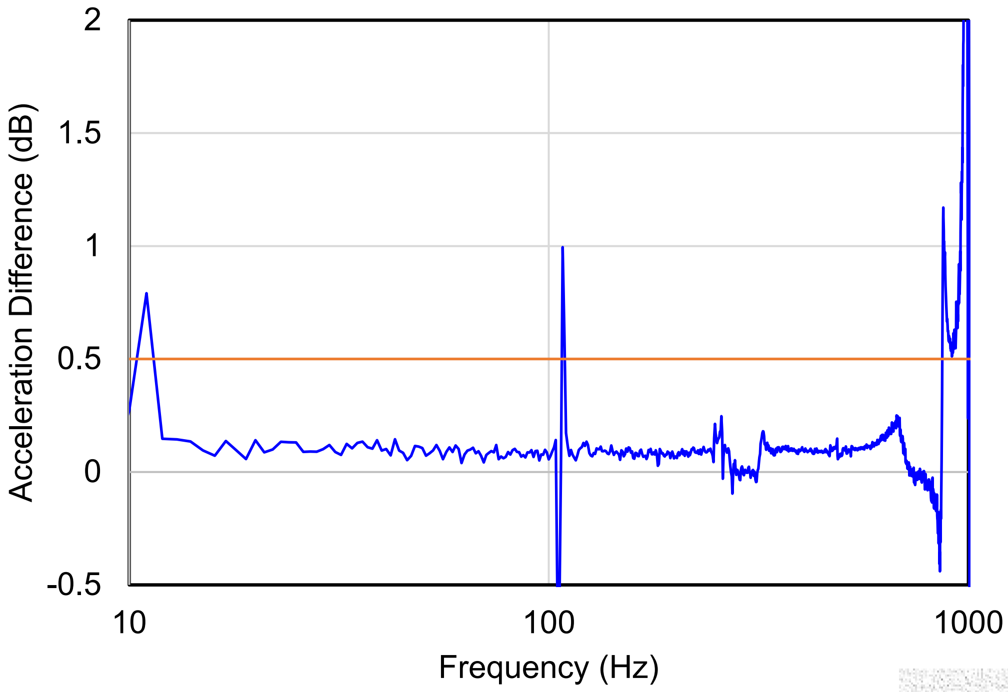


ISO 10846-2 Direct Method

Unwanted input vibration 2:

$$L_{a1} - L_{a2} \leq 0.5 \text{ dB}$$



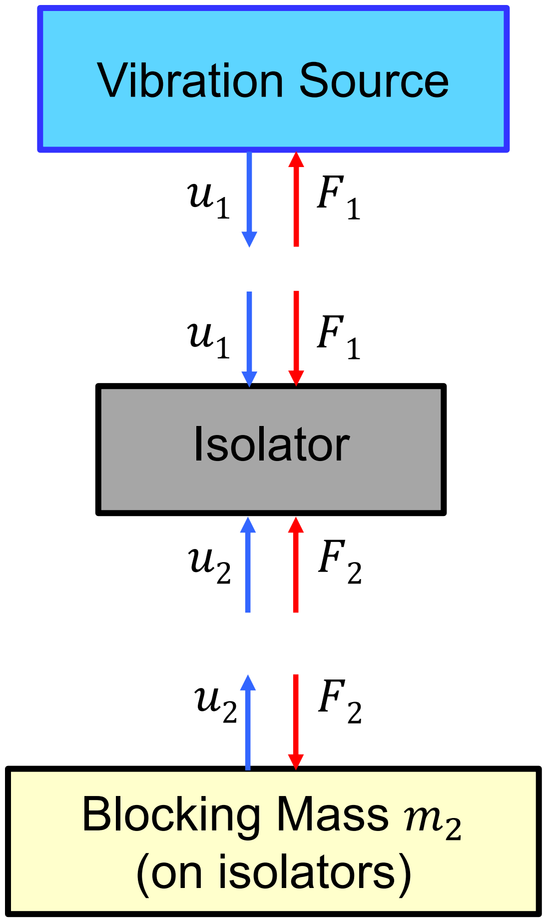


ISO 10846-2 Direct Method

Other Notes

1. Dynamic stiffness can be averaged in 1/3 octave bands using a minimum of 5 frequencies per 1/3 octave band.
2. Results should be presented in dB with a reference of 1 N/m.
3. Vibration levels should be similar to those in practice.
4. Linearity check is required. Reduce input by 10 dBA to ensure that the dynamic stiffness dB levels do not differ by more than 1.5 dB.

ISO 10846-3 Indirect Method



Vibration Source

u_1 F_1

u_1 F_1

Isolator

u_2 F_2

u_2 F_2

Blocking Mass m_2
(on isolators)

Indirect measurement of force

$$k_{2,1} \approx \frac{F_2}{u_1} \approx -\omega^2 m_2 \frac{u_2}{u_1}$$

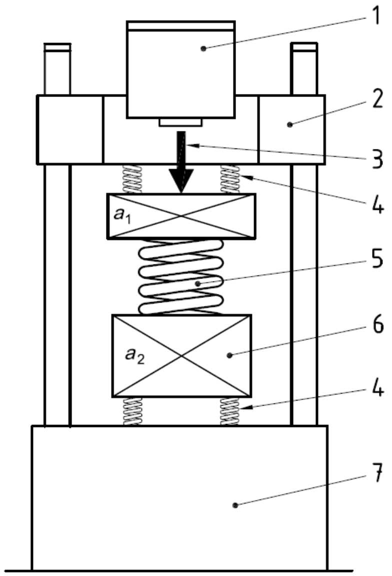
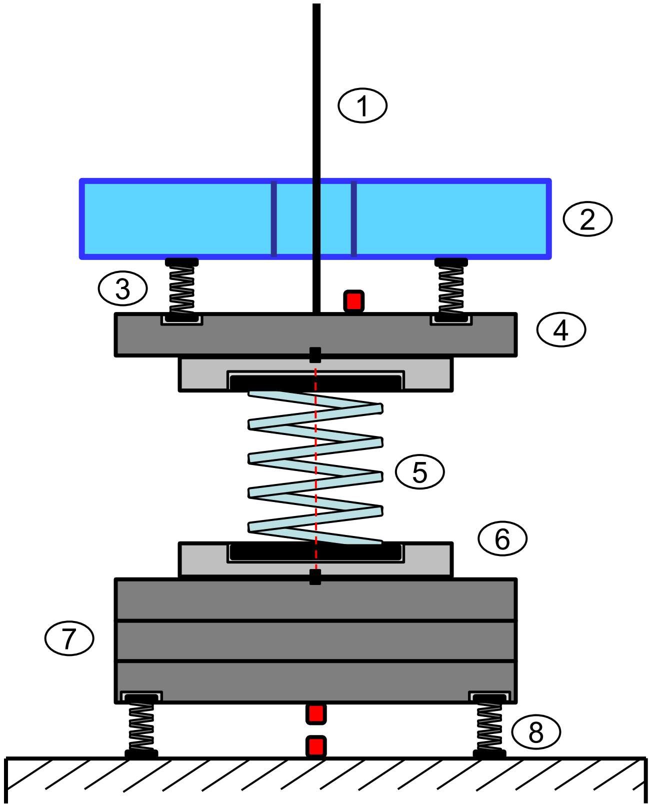


Image from ISO 10846-3

Parts

1. Exciter
2. Traverse
3. Connecting rod
4. Dynamic decoupling springs, static preload
5. Test element
6. Blocking mass
7. Rigid foundation

Indirect Method Test Rig Design



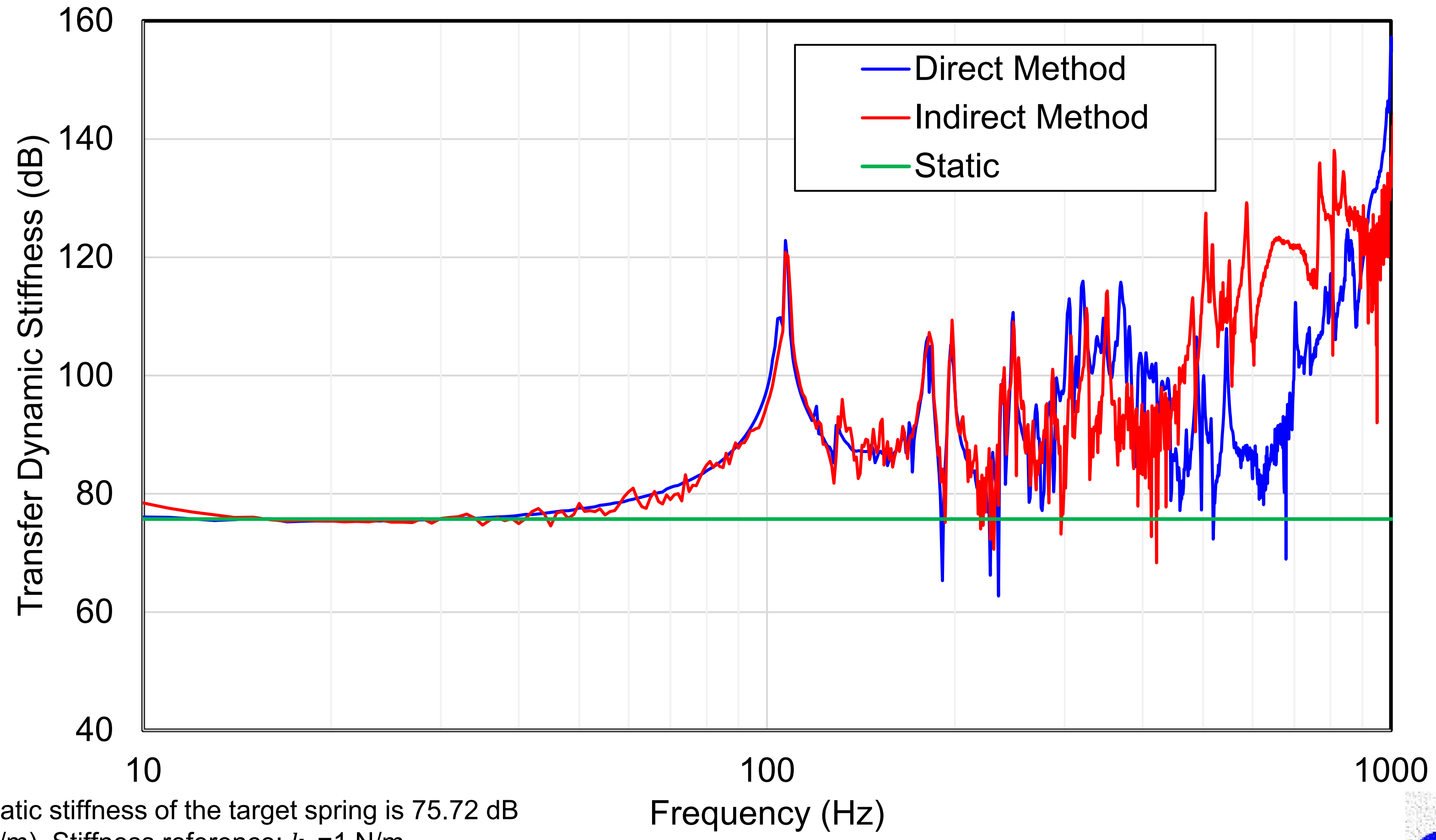
Indirect measurement

$$k_{2,1} \approx \frac{F_2}{u_1} \approx -\omega^2 m_2 \frac{u_2}{u_1}$$

Schematic of test rig for Indirect Method

1. Dynamic excitation (shaker)
2. Static preload
3. Decoupling springs
4. Excitation mass (m_1)
5. Test element (isolator)
6. Lower force distribution flange
7. Blocking mass (m_2)
8. Rigid foundation

Results Transfer Dynamic Stiffness



Note: Static stiffness of the target spring is 75.72 dB (6113 N/m). Stiffness reference: k_0=1 N/m

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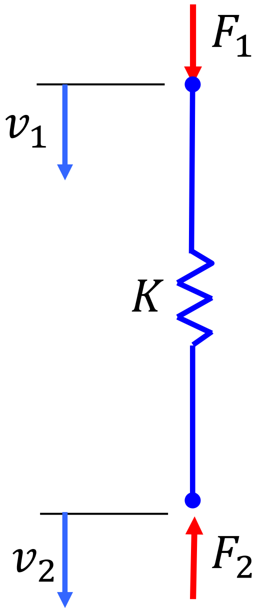
1D Spring Models Transfer Matrix

Transfer Matrix

$$\begin{Bmatrix} F_1 \\ v_1 \end{Bmatrix} = \begin{bmatrix} \cos(kL) & \rho_{eff} c_L S j \sin(kL) \\ \frac{1}{\rho_{eff} c_L S} j \sin(kL) & \cos(kL) \end{bmatrix} \begin{Bmatrix} F_2 \\ v_2 \end{Bmatrix}$$



May be rearranged in Impedance Matrix form

$$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \frac{\rho_{eff} c_L S}{j \sin(kL)} \begin{bmatrix} \cos(kL) & -1 \\ 1 & -\cos(kL) \end{bmatrix} \begin{Bmatrix} v_1 \\ v_2 \end{Bmatrix}$$

1D Spring Models Transfer Matrix

Model spring as an equivalent longitudinal force element.

Longitudinal Wave Speed

$$c_L = \sqrt{\frac{E_{eff}}{\rho_{eff}}} = L\sqrt{\frac{k_s L/S}{m_s/LS}} = L\sqrt{\frac{k_s}{m_s}}$$

Spring Stiffness

$$k_s = \frac{Gd^4}{8nD^3}$$

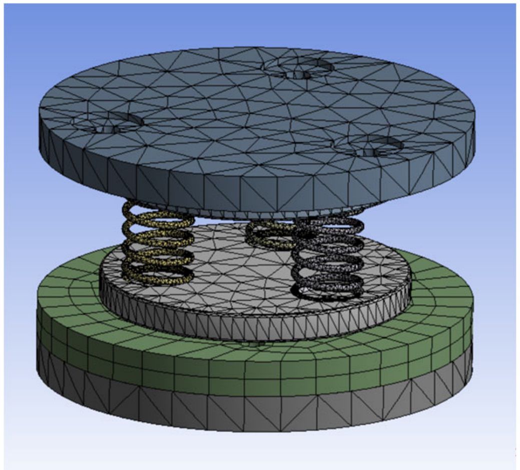
Spring Stiffness with Damping

$$k_s' = k_s(1 + j\eta)$$

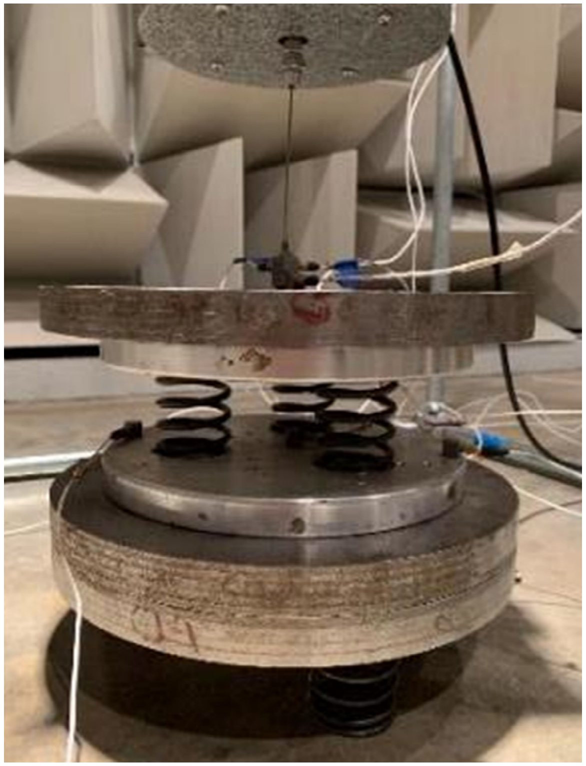
Spring Mass

$$m_s = \frac{\rho_s \pi d^2}{4}\sqrt{(n\pi D)^2 + L^2}$$

ANSYS FEM Simulation

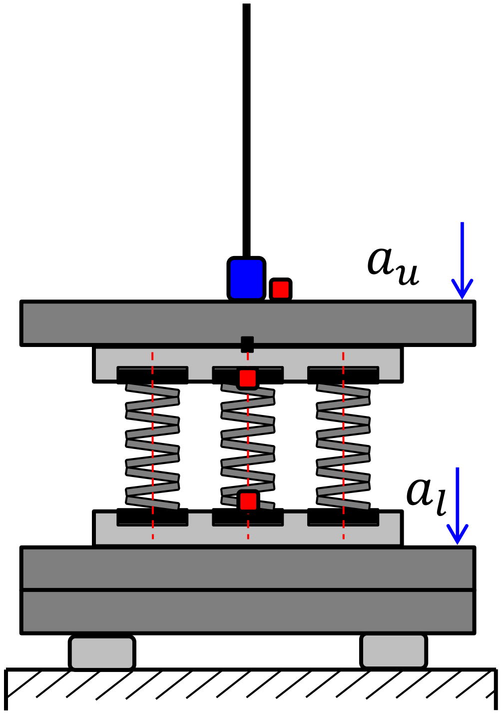


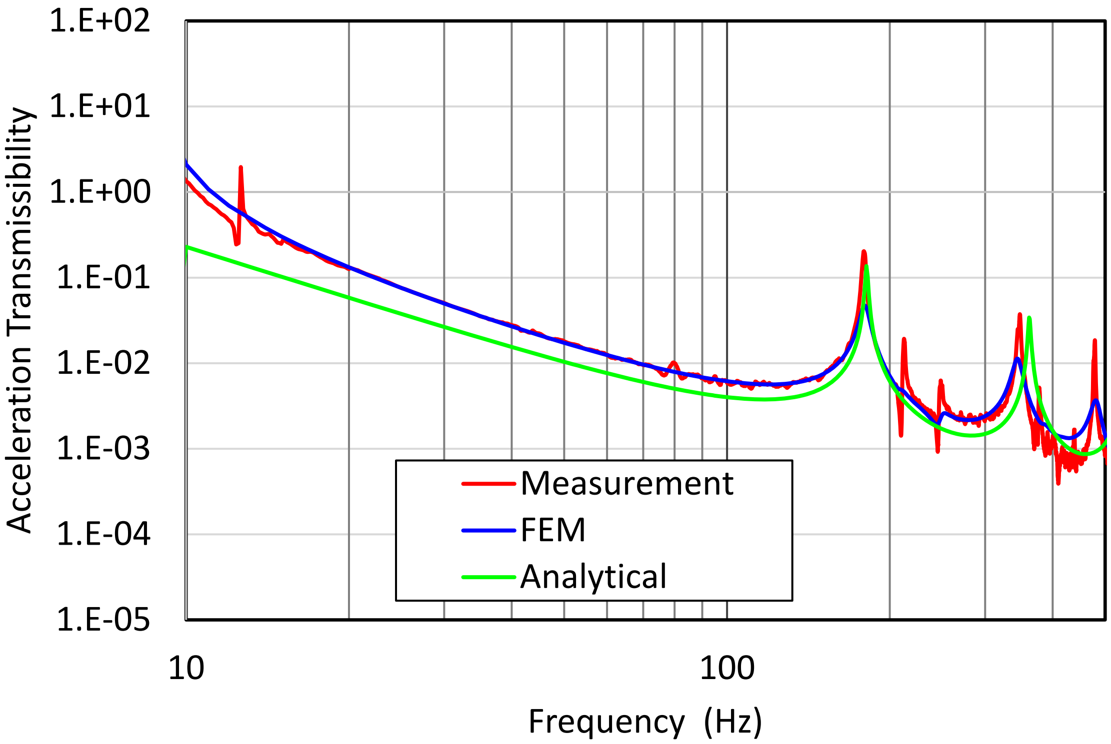
Measurement Setup



Material	Structural Steel	/
Young's Modulus	2.00E+11	Pa
Shear Modulus	7.69E+10	Pa
Number of Effective Coils	~4	/
Material Density	7850	kg m^-3
Wire Diameter	0.005	m
Outer Diameter	0.05	m
Length (Uncompressed)	0.075	m

Results Acceleration Transmissibility





$$\tau_{motion} = \frac{a_l}{a_u}$$

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