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Determination of the Transmission Loss of Mufflers above the Cutoff Frequency

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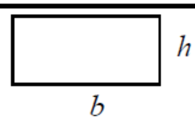
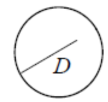
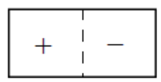
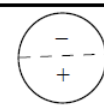
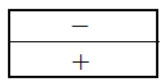
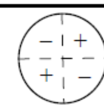
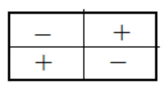
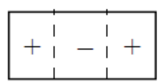
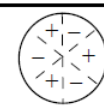
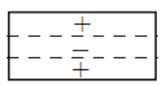
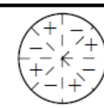
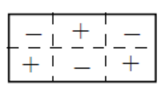
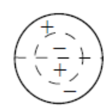
The Vibro-Acoustics Consortium

Cutoff Frequencies

Wallin et al., 2007

Rectangular cross section		**Circular cross section**	
$f_{10}^{c} = c/2b$		$f_{01}^{c} = 1.841 c/\pi D$	
$f_{01}^{c} = c/2h$		$f_{02}^{c} = 3.054 c/\pi D$	
$f_{11}^{c} = \frac{c}{2}\left(\frac{1}{b^2}+\frac{1}{h^2}\right)^{1/2}$		$f_{10}^{c} = 3.832 c/\pi D$	
$f_{02}^{c} = c/b$		$f_{03}^{c} = 4.201 c/\pi D$	
$f_{20}^{c} = c/h$		$f_{04}^{c} = 5.318 c/\pi D$	
$f_{21}^{c} = \frac{c}{2}\left(\frac{4}{b^2}+\frac{1}{h^2}\right)^{1/2}$		$f_{11}^{c} = 5.331 c/\pi D$	

Rectangular Section

$$f_{cutoff} = \frac{c}{2b}$$

Circular Section

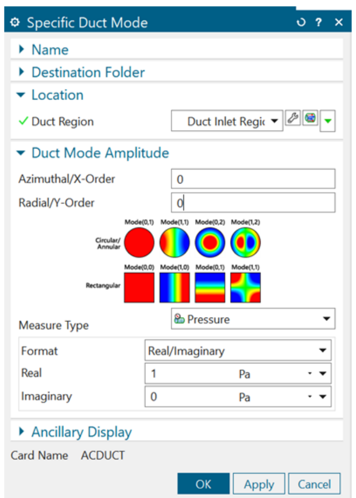
$$f_{cutoff} = \frac{c}{1.7D}$$

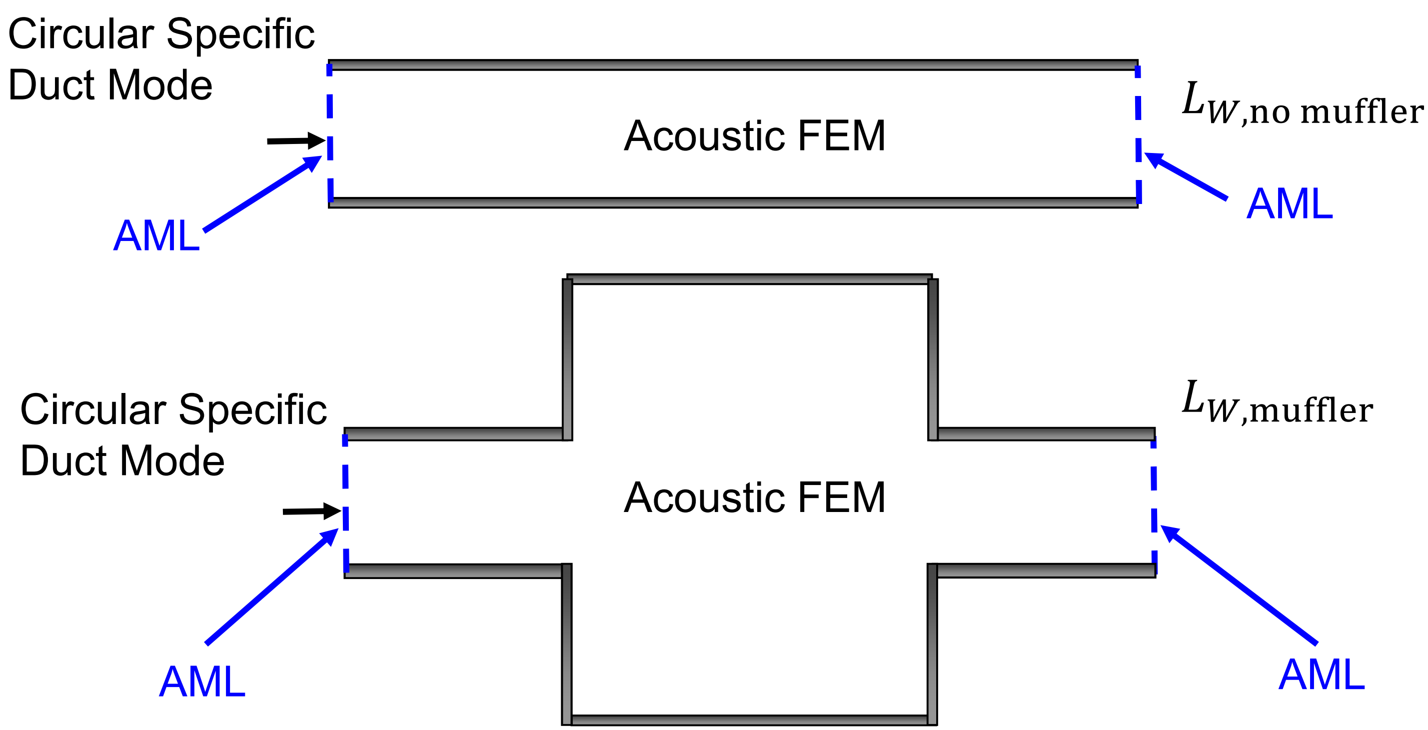
Motivation

1. Large silencers usually involve many higher-order modes at the inlet and outlet. Traditional methods for evaluating the transmission loss (TL) have their limits on how to define the true anechoic termination.
2. Two different numerical methods are used to calculate the transmission loss above the cut-off frequency.
   - Reciprocal work identity method in conjunction with BEM
   - Equivalent IL using automatically matched layer (AML) boundary condition (Siemens Virtual.Lab)

Equivalent Insertion Loss Method

Siemens Simcenter

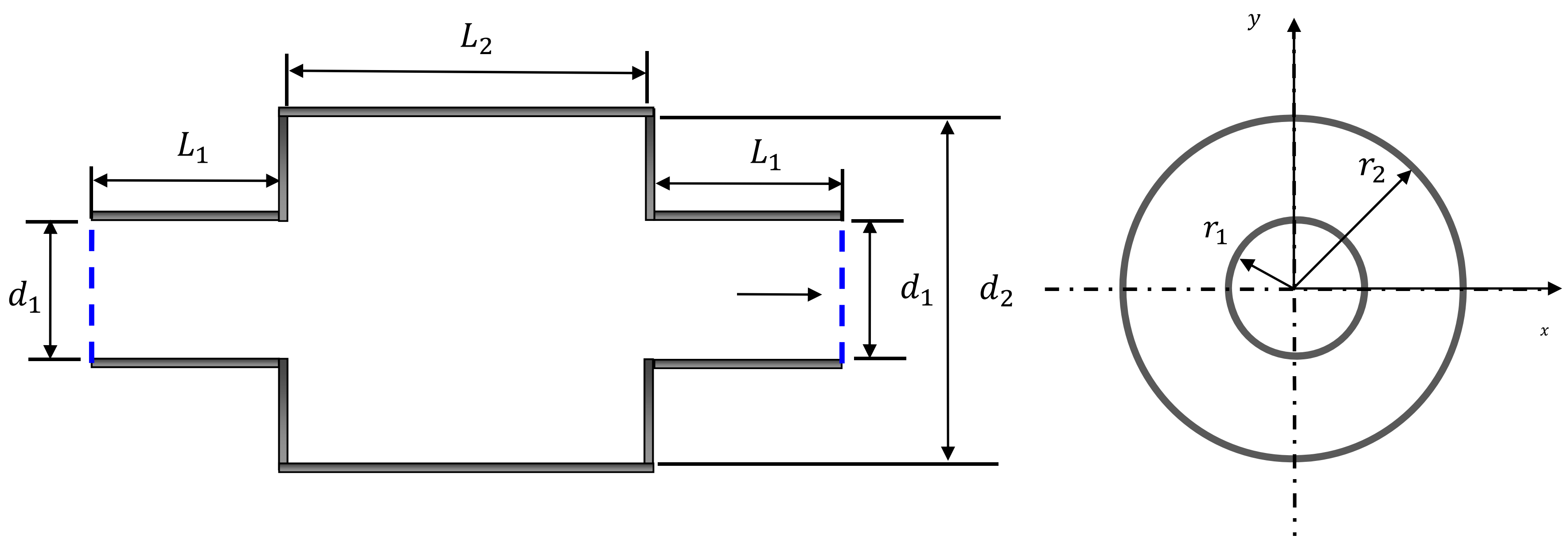




Given an anechoic source and termination

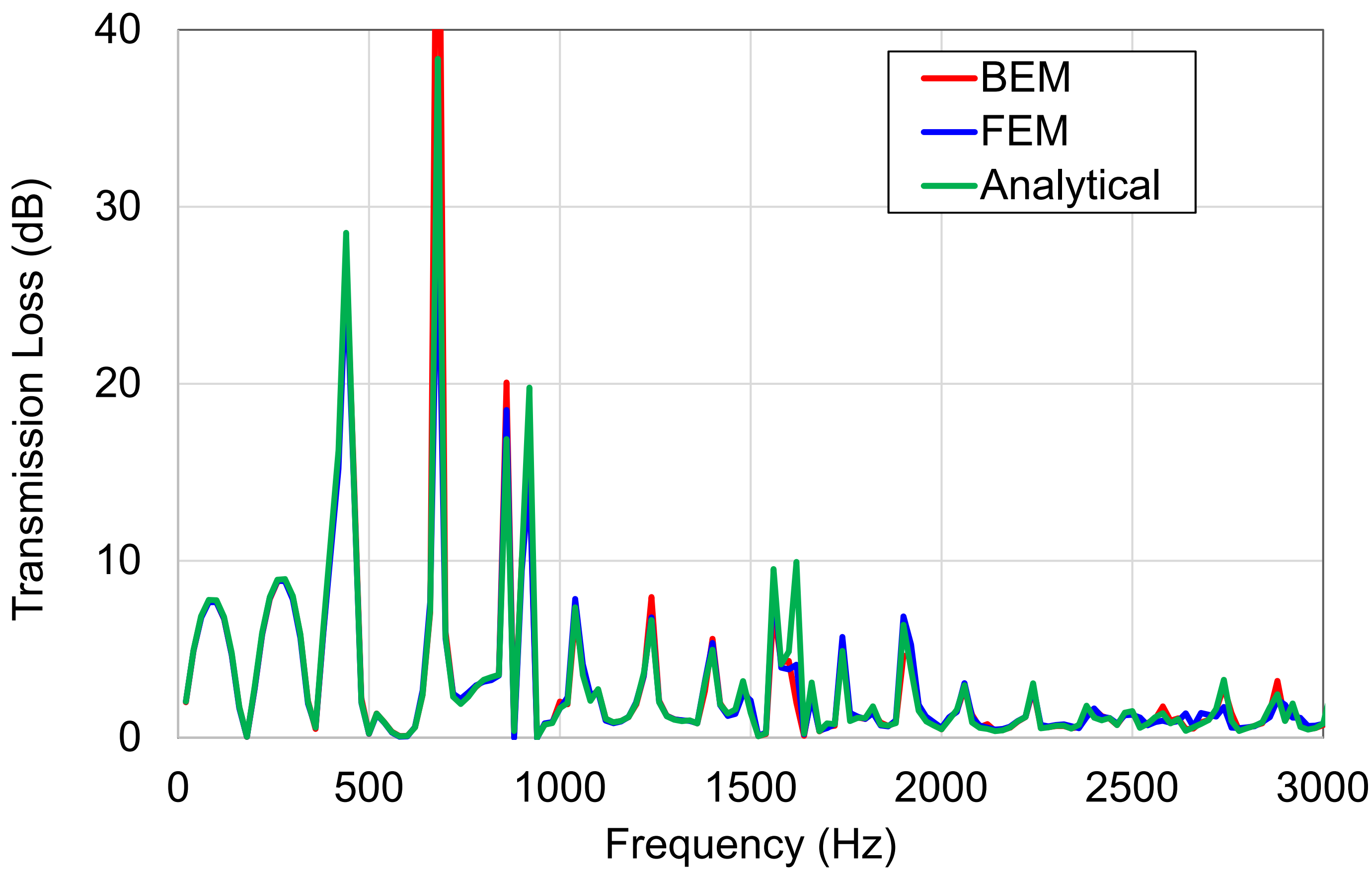
$$TL = IL = L_{W,\text{no muffler}} - L_{W,\text{muffler}}$$

Test Case 1 Circular Expansion Chamber



$d_1 = 0.42$ m; $d_2 = 0.90$ m; $L_1 = 0.5$ m; $L_2 = 1$ m

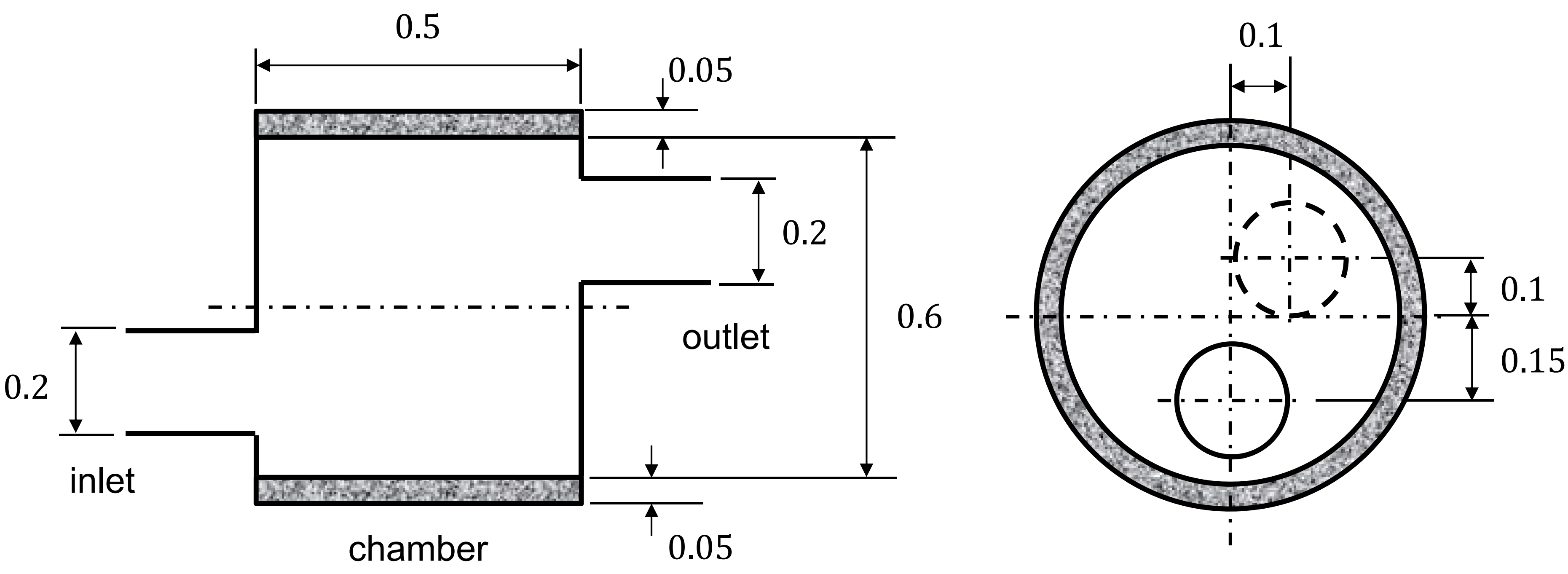
Test Case 1 Circular Expansion Chamber



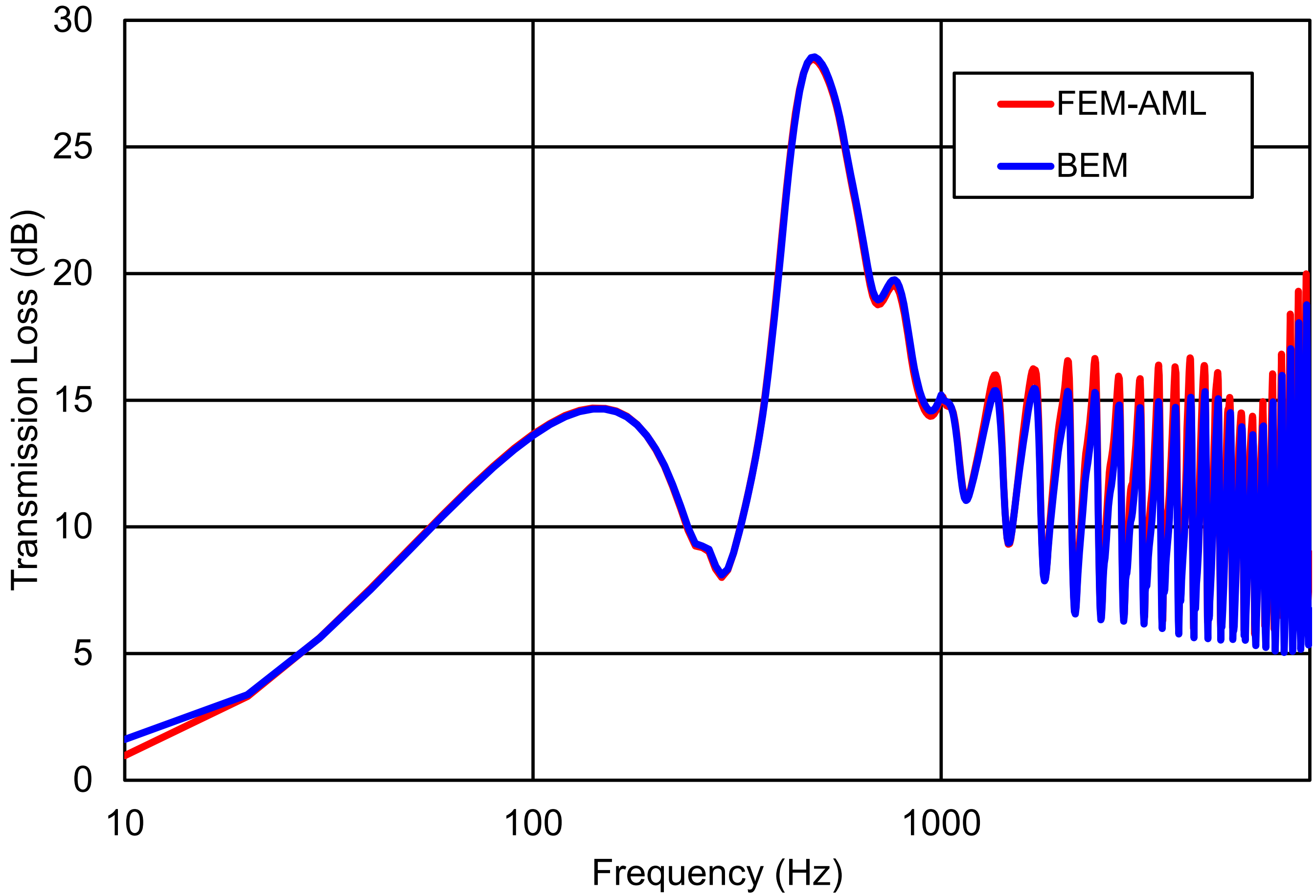
Test Case 2 Lined Circular Expansion Chamber

Offset Inlet and Outlet Ducts

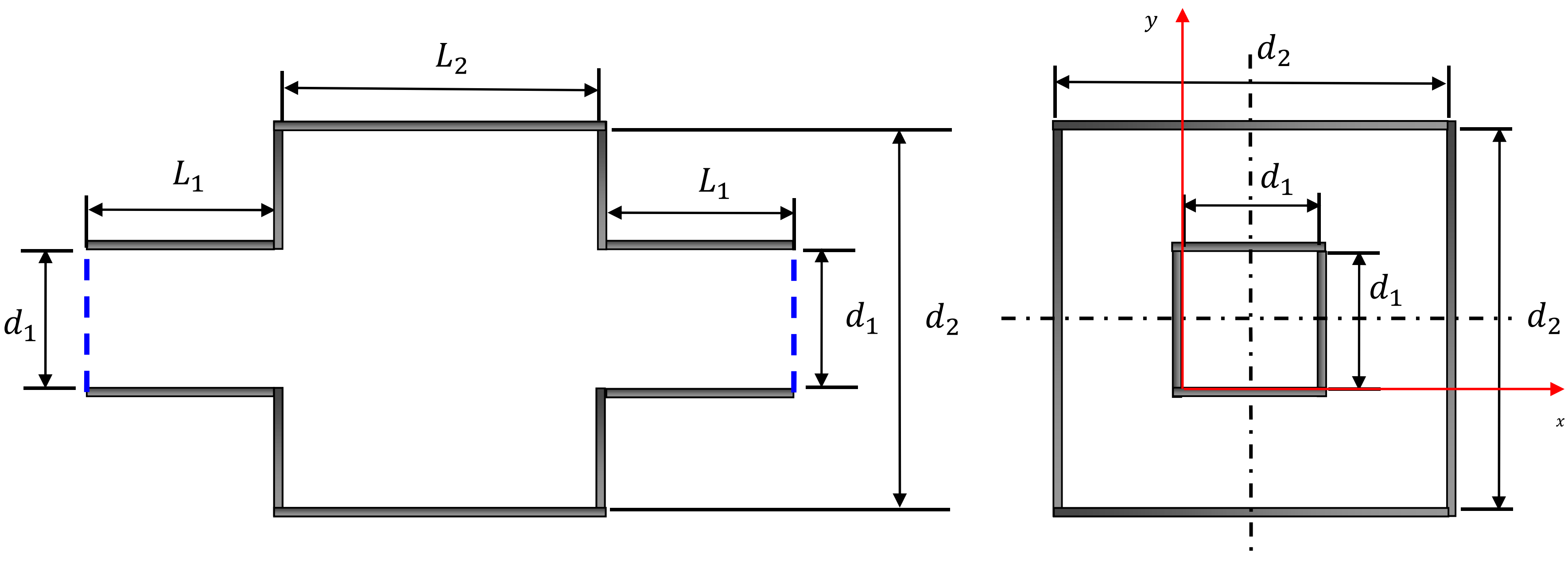
Unit: m



Test Case 2 Circular Expansion Chamber

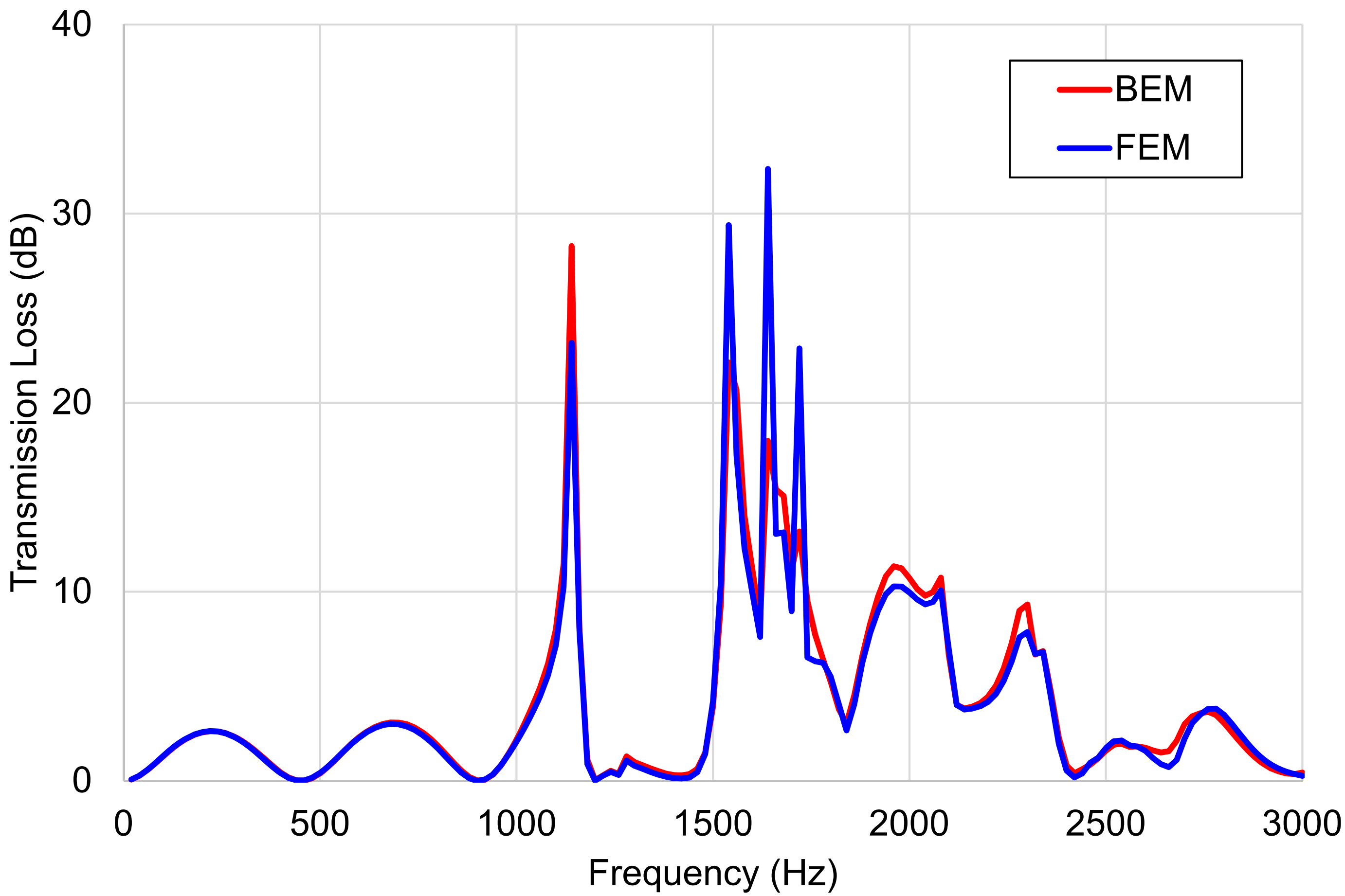


Test Case 3 Rectangular Expansion Chamber

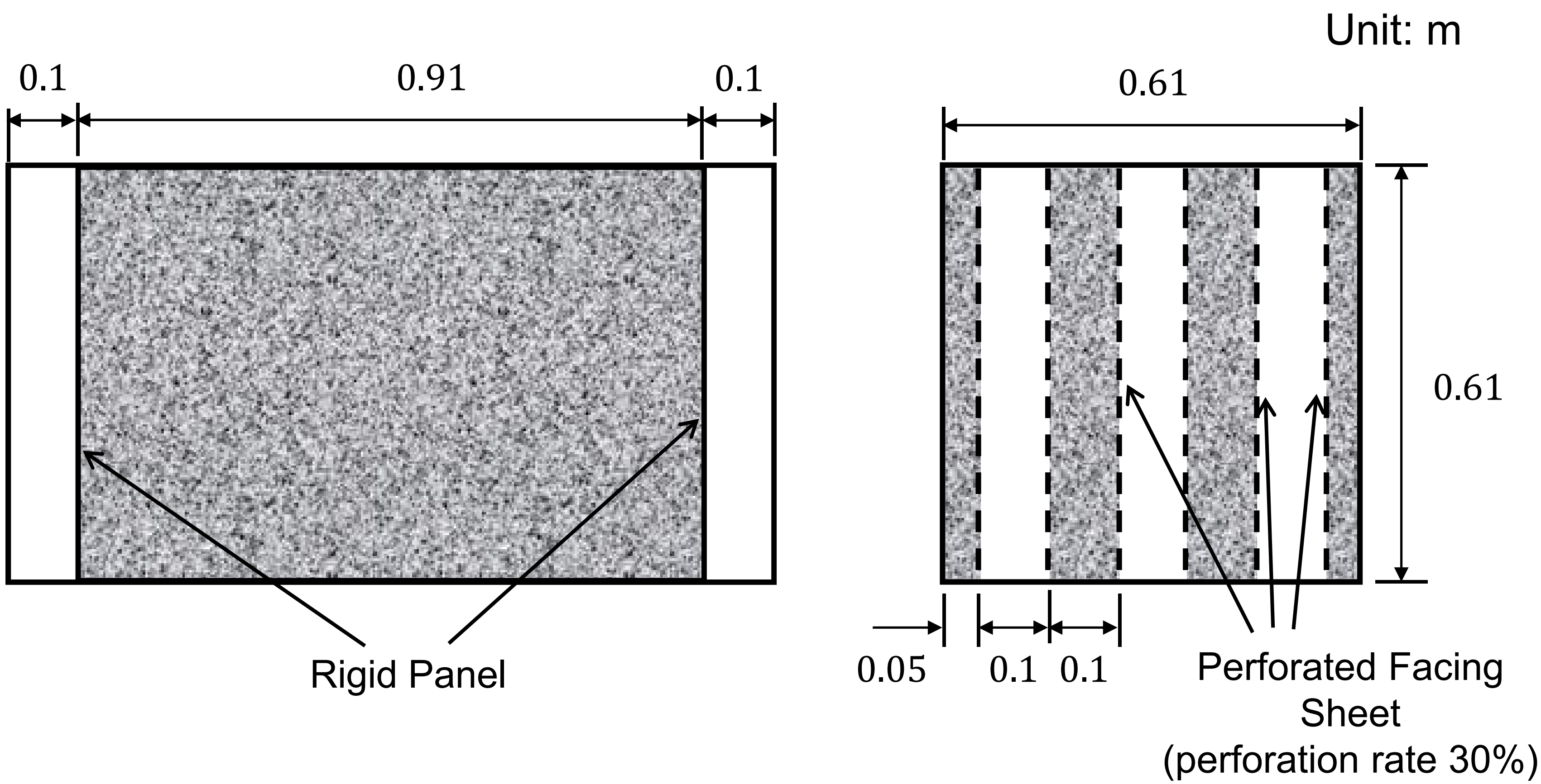


$d_1 = 0.2$ m; $d_2 = 0.3$ m; $L_1 = 0.2$ m; $L_2 = 0.4$ m

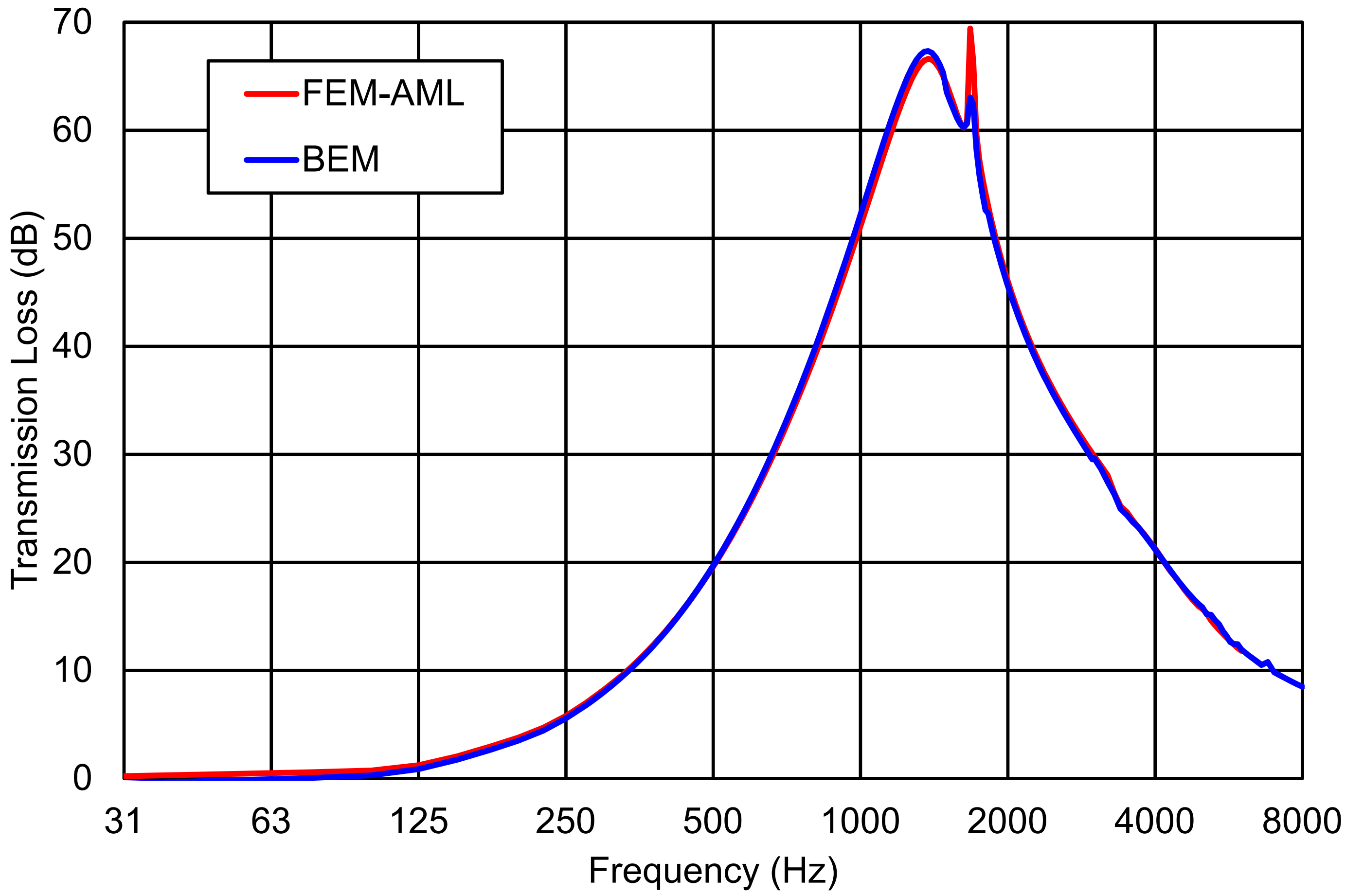
Test Case 3 Rectangular Expansion Chamber



Test Case 4 Parallel Baffle Silencer



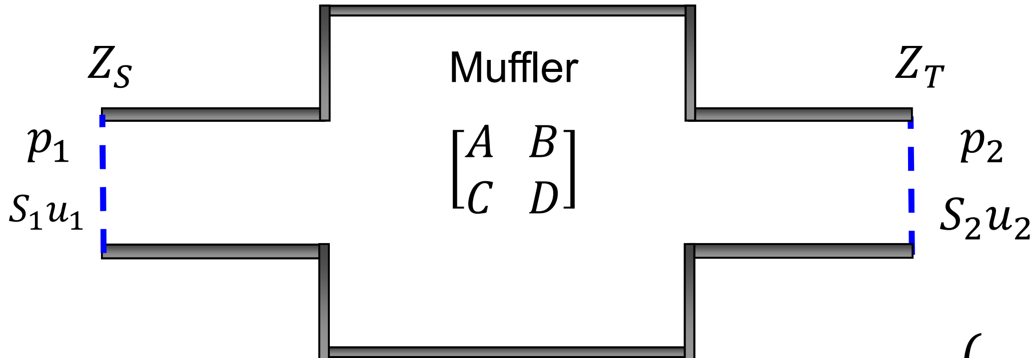
Test Case 4 Parallel Baffle Silencer



References

Ruan, K., T. W. Wu, and Herrin, D. W., “Correlation between Boundary and Finite Element Determination of Large Silencer Transmission Loss,” Noise Control Engineering Journal (Accepted).

Appendix *Equivalent IL* Method Derivation



$$\begin{Bmatrix} p_1 \\ S_1 u_1 \end{Bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{Bmatrix} p_2 \\ S_2 u_2 \end{Bmatrix} \Rightarrow \begin{cases} p_1 = Ap_2 + B(S_2 u_2) \\ S_1 u_1 = Cp_2 + D(S_2 u_2) \end{cases} \Rightarrow \begin{cases} p_1 = Ap_2 + B\left(\dfrac{p_2}{Z_T}\right) \\ \dfrac{p_S - p_1}{Z_S} = Cp_2 + D\left(\dfrac{p_2}{Z_T}\right) \end{cases}$$

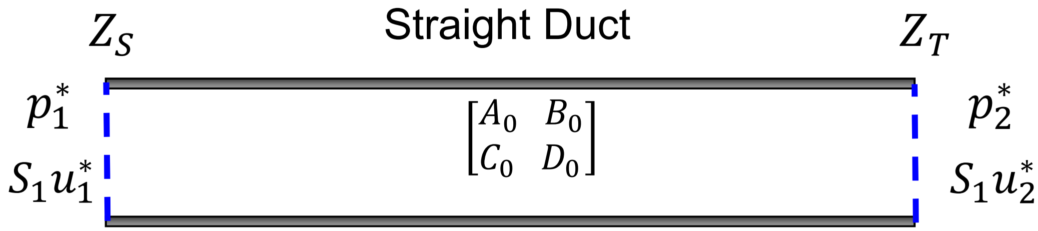
$$Z_S = \frac{p_S - p_1}{S_1 u_1} \Rightarrow S_1 u_1 = \frac{p_S - p_1}{Z_S}$$

$$Z_T = \frac{p_2}{S_2 u_2} \Rightarrow S_2 u_2 = \frac{p_2}{Z_T}$$

$$p_2 = \frac{1}{A + \dfrac{1}{Z_T} B + Z_S C + \dfrac{Z_S}{Z_T} D} \cdot p_S$$

Appendix *Equivalent IL* Method Derivation

Similarly,



$$p_2^* = \frac{1}{A_0 + \frac{1}{Z_T} B_0 + Z_S C_0 + \frac{Z_S}{Z_T} D_0} \cdot p_S$$

$$IL = 10 \cdot \log_{10} \left\{ \frac{|p_2^*|^2 S_1/(2\rho_0 c)}{|p_2|^2 S_2/(2\rho_0 c)} \right\} = 20 \cdot \log_{10} \left\{ \left| \frac{A + \frac{1}{Z_T} B + Z_S C + \frac{Z_S}{Z_T} D}{A_0 + \frac{1}{Z_T} B_0 + Z_S C_0 + \frac{Z_S}{Z_T} D_0} \right| \right\} + 10 \cdot \log_{10} \frac{S_1}{S_2}$$

Appendix *Equivalent IL* Method Derivation

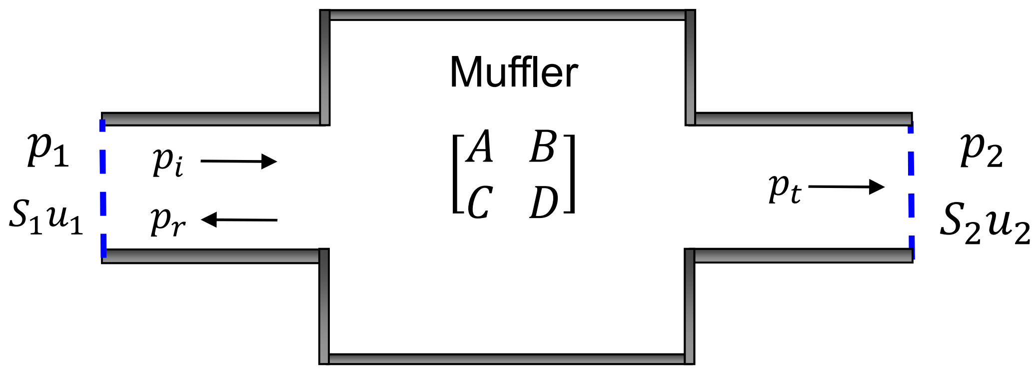
Given an anechoic source and termination

For muffler $Z_S = \frac{\rho_0 c}{S_1} \quad Z_T = \frac{\rho_0 c}{S_2}$

For straight duct $Z_S = Z_T = \frac{\rho_0 c}{S_1} \quad \begin{bmatrix} A_0 & B_0 \\ C_0 & D_0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$IL = 20 \cdot \log_{10}\left\{\frac{1}{2}\left|A + \frac{S_2}{\rho c_0}B + \frac{\rho c_0}{S_1}C + \frac{S_2}{S_1}D\right|\right\} + 10 \cdot \log_{10}\frac{S_1}{S_2}$$

Appendix *Equivalent IL* Method Derivation



$$\begin{Bmatrix} p_1 \\ S_1 u_1 \end{Bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{Bmatrix} p_2 \\ S_2 u_2 \end{Bmatrix}$$

$$p_1 = p_i + p_r \ , \ S_1 u_1 = S_1 \left(\frac{p_i - p_r}{\rho_0 c} \right)$$

$$p_2 = p_t \ , S_2 u_2 = S_2 \frac{p_t}{\rho_0 c}$$

$$\Rightarrow \begin{cases} p_i + p_r = A p_t + B S_2 \left(\dfrac{p_t}{\rho_0 c} \right) \\ S_1 \left(\dfrac{p_i - p_r}{\rho_0 c} \right) = C p_t + D S_2 \left(\dfrac{p_t}{\rho_0 c} \right) \end{cases}$$

$$\Rightarrow \frac{p_i}{p_t} = \frac{1}{2} \left(A + \frac{B S_2}{\rho_0 c} + \frac{\rho_0 c}{S_1} C + \frac{S_2}{S_1} D \right)$$

$$\Rightarrow TL = 10 \cdot \log_{10} \frac{|p_i|^2 S_1 / (2 \rho_0 c)}{|p_t|^2 S_1 / (2 \rho_0 c)} = 20 \cdot \log_{10} \left| \frac{1}{2} \left(A + \frac{B S_2}{\rho_0 c} + \frac{\rho_0 c}{S_1} C + \frac{S_2}{S_1} D \right) \right| + 10 \cdot \log_{10} \frac{S_1}{S_2}$$