August 17, 2020

An Introduction to Statistical Energy Analysis

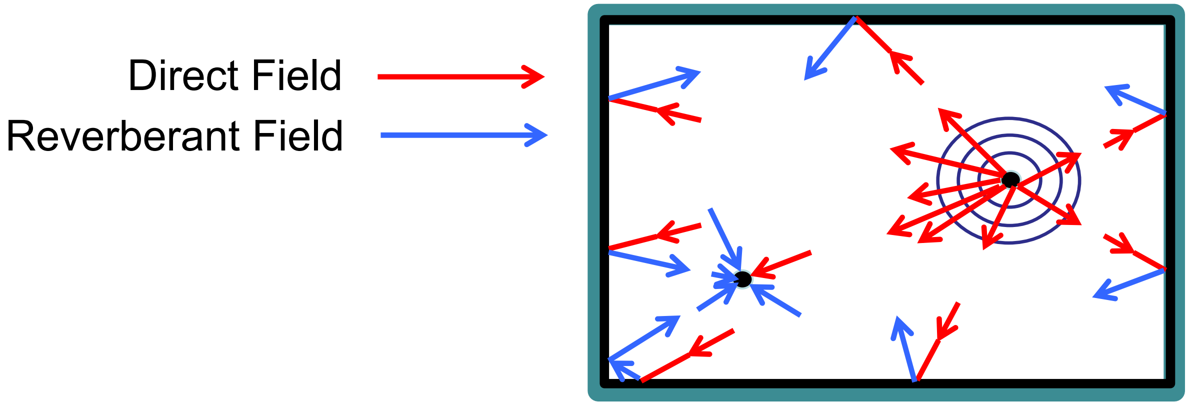
David Herrin
University of Kentucky

Vibro-Acoustics Consortium

Overview

- Basic Theory
- Fundamentals
- Measuring SEA Parameters
- Examples

Room Acoustics Theory



$$L_p = L_W + 10\log\left(\underbrace{\frac{\Gamma}{4\pi r^2}}_{\text{Direct Field}} + \underbrace{\frac{4}{R_r}}_{\text{Reverberant Field}}\right)$$

$$p_{ref} = 20 \times 10^{-6}\ \text{Pa}$$

$$W_{ref} = 1.0 \times 10^{-12}\ \text{W}$$

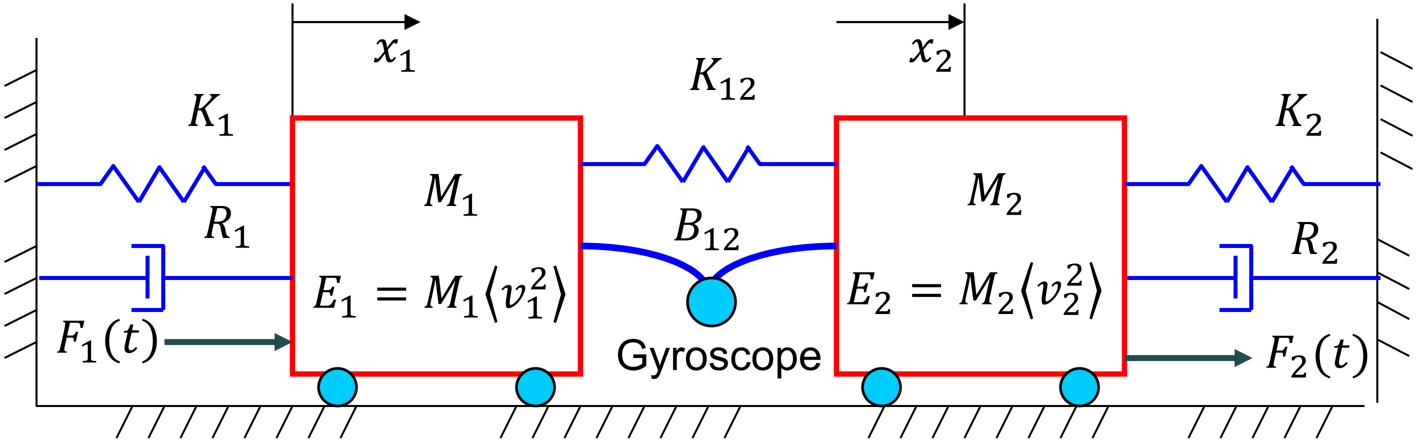
In the Reverberant Field

$$L_p = L_W + 10\log_{10}\left(\frac{4}{R_r}\right) \qquad L_p = 10\log_{10}\left(\frac{p^2}{p_{ref}^2}\right)$$

$$10\log_{10}\left(\frac{W}{W_{ref}}\right) = 10\log_{10}\left(\left(\frac{R_r}{4}\right)\left(\frac{p^2}{p_{ref}^2}\right)\right)$$

$$\underset{\substack{\uparrow\\ \text{Input Power}}}{\frac{W}{W_{ref}}} = \underset{\substack{\uparrow\\ \text{“Loss Factor”}}}{\left(\frac{R_r}{4}\right)}\underset{\substack{\uparrow\\ \text{Acoustic Energy}}}{\left(\frac{p^2}{p_{ref}^2}\right)} \quad\Longrightarrow\quad W_{in} = \eta_i \omega E_i$$

Theory Coupled Simple Oscillator

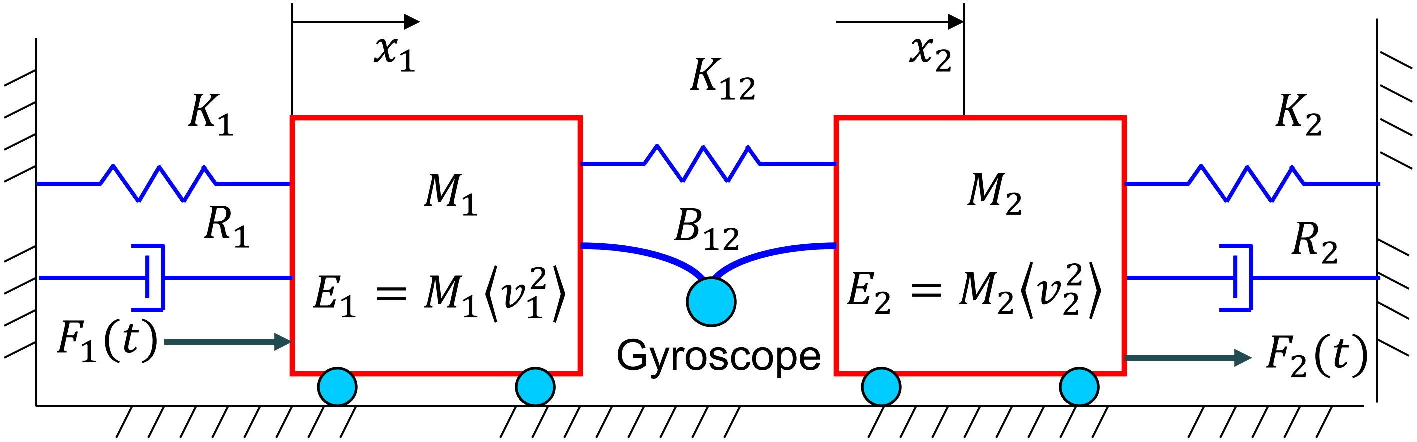


$$W_{12} = \beta(E_1 - E_2)$$

W_{12}	Power transferred from subsystem 1 to 2 (for a frequency band)
β	Constant which is a function of the oscillator properties
E_1, E_2	Energy in subsystems 1 and 2 (for a frequency band)

Assumption: Forces are assumed to be broadband and incoherent.

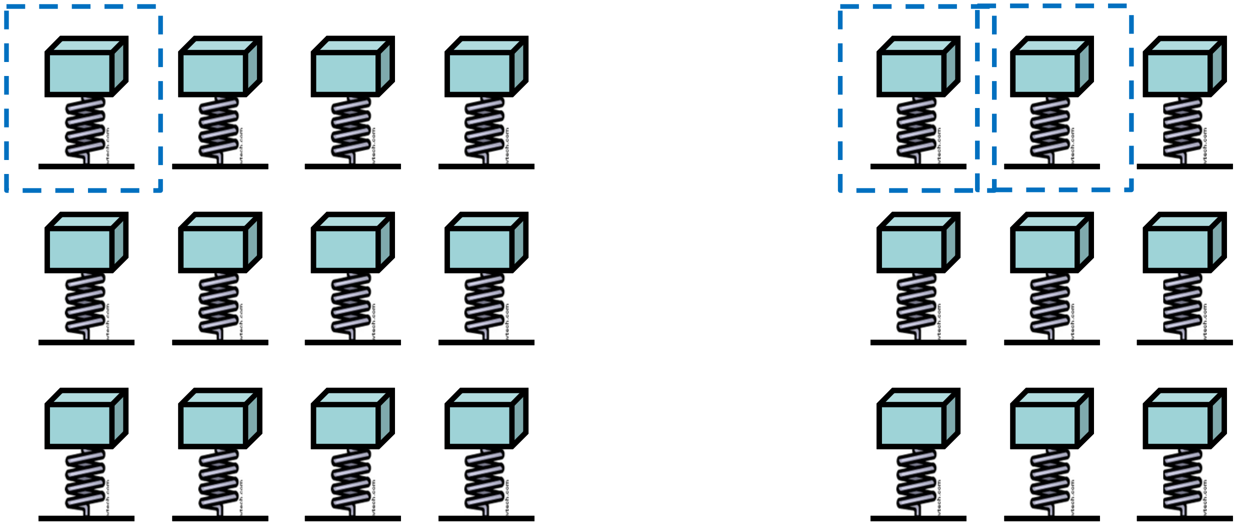
Theory Coupled Simple Oscillator



$$W_{12} = \beta(E_1 - E_2)$$

- Power flow is proportional to the differences in the (modal) energies.
- The constant relating the power flow to energy difference is a function of the coupling parameters and oscillator properties.
- Power flows from the oscillator with higher (modal) energy to the oscillator with lower (modal) energy.

Theory Coupled Multi-Resonant Case



N_1 modes or oscillators

N_2 modes or oscillators

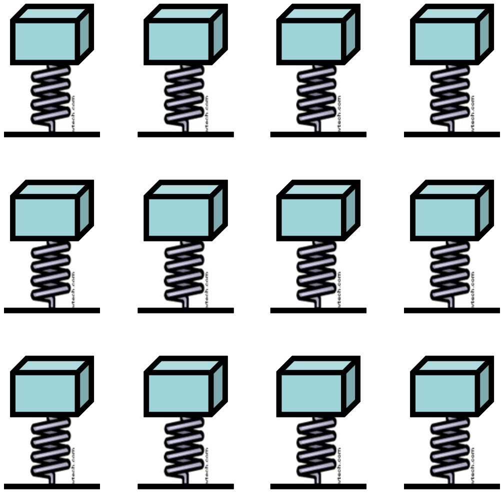
$$\bar{E}_1 = \frac{E_1}{N_1}$$

$$\bar{E}_2 = \frac{E_2}{N_2}$$

$$W_{12} = \langle \beta \rangle_{N_1 N_2} N_1 N_2 (\bar{E}_1 - \bar{E}_2)$$

Theory Coupled Multi-Resonant Case

Assumption More modes implies greater energy storage potential.



$$W_{12} = \langle\beta\rangle_{N_1N_2}N_1N_2(\bar{E}_1 - \bar{E}_2)$$

$$W_{12} = \omega(\eta_{12}E_1 - \eta_{21}E_2)$$

Coupling Loss Factors

$$\eta_{12} = \frac{\langle\beta\rangle_{N_1N_2}N_2}{\omega} \qquad \eta_{21} = \frac{\langle\beta\rangle_{N_1N_2}N_1}{\omega}$$

$$\eta_{21} = \frac{N_1\eta_{12}}{N_2}$$

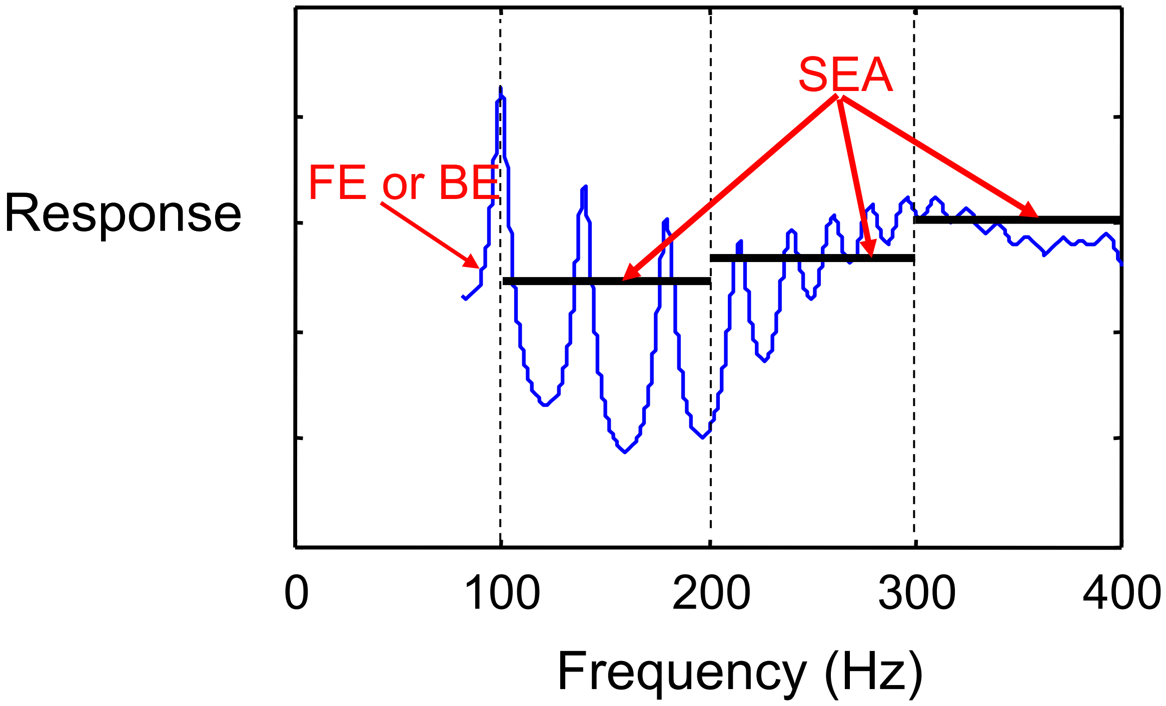
Overview

- Basic Theory
- Fundamentals
- Measuring SEA Parameters
- Examples

What is SEA?

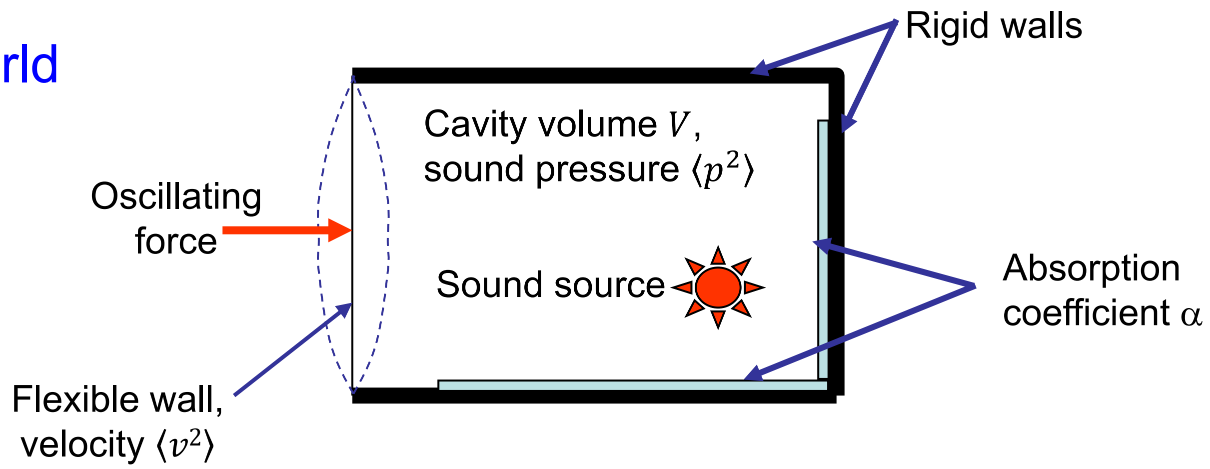
- SEA is a lumped parameter approach for vibro-acoustic analysis that accounts for the flow of energy throughout the system based on statistical coupling of the system modes.
- Lumped parameter: each component or subsystem is a single entity having its own energy, either acoustic or vibrational.
- Energy flow: steady-state averages are determined from energy balances for each component or subsystem.
- Statistical averages: sound pressure and vibration velocity are values averaged over a band of frequencies and over space.

Frequency Averaging



What is SEA? A Simple Example

Physical World



Rigid walls

Cavity volume V, sound pressure $\langle p^2\rangle$

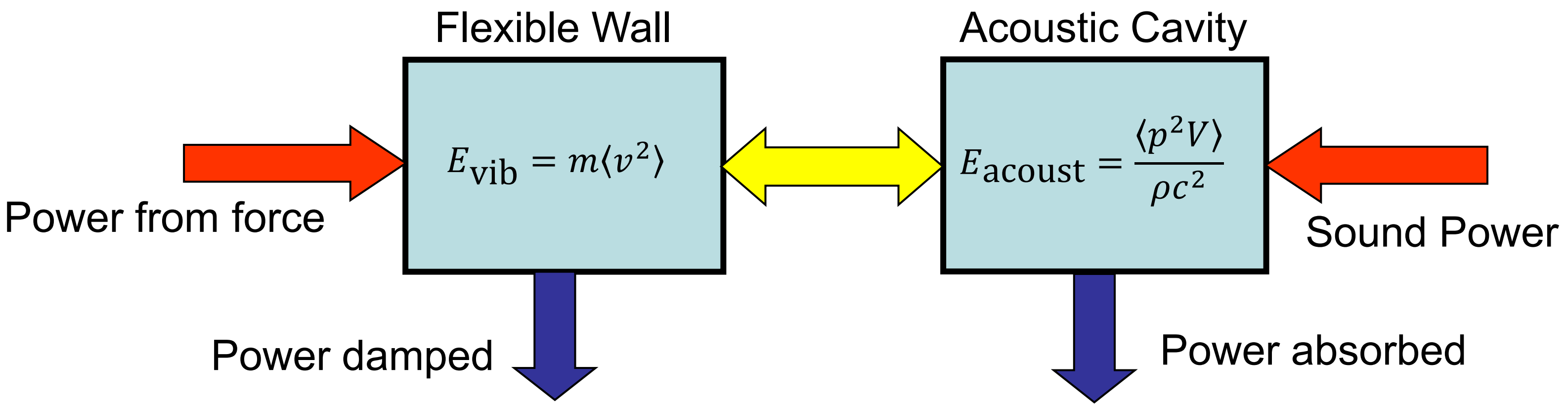
Oscillating force

Sound source

Absorption coefficient α

Flexible wall, velocity $\langle v^2\rangle$

SEA World



Flexible Wall

Acoustic Cavity

$E_{\text{vib}} = m\langle v^2\rangle$

$E_{\text{acoust}} = \dfrac{\langle p^2 V\rangle}{\rho c^2}$

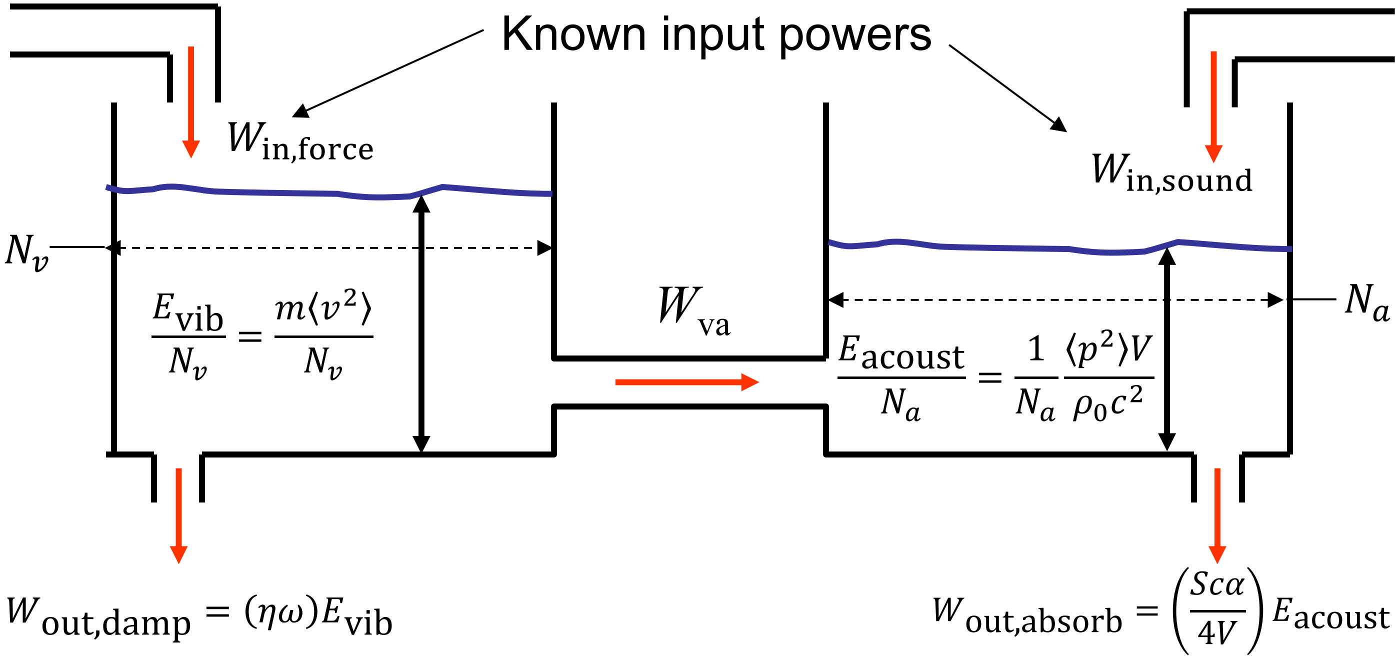
Power from force

Sound Power

Power damped

Power absorbed

Water Tank Analogy



$$\frac{E_{\text{vib}}}{N_v} = \frac{m\langle v^2\rangle}{N_v}$$

$$\frac{E_{\text{acoust}}}{N_a} = \frac{1}{N_a}\frac{\langle p^2\rangle V}{\rho_0 c^2}$$

$$W_{\text{out,damp}} = (\eta\omega)E_{\text{vib}}$$

$$W_{\text{out,absorb}} = \left(\frac{Sc\alpha}{4V}\right)E_{\text{acoust}}$$

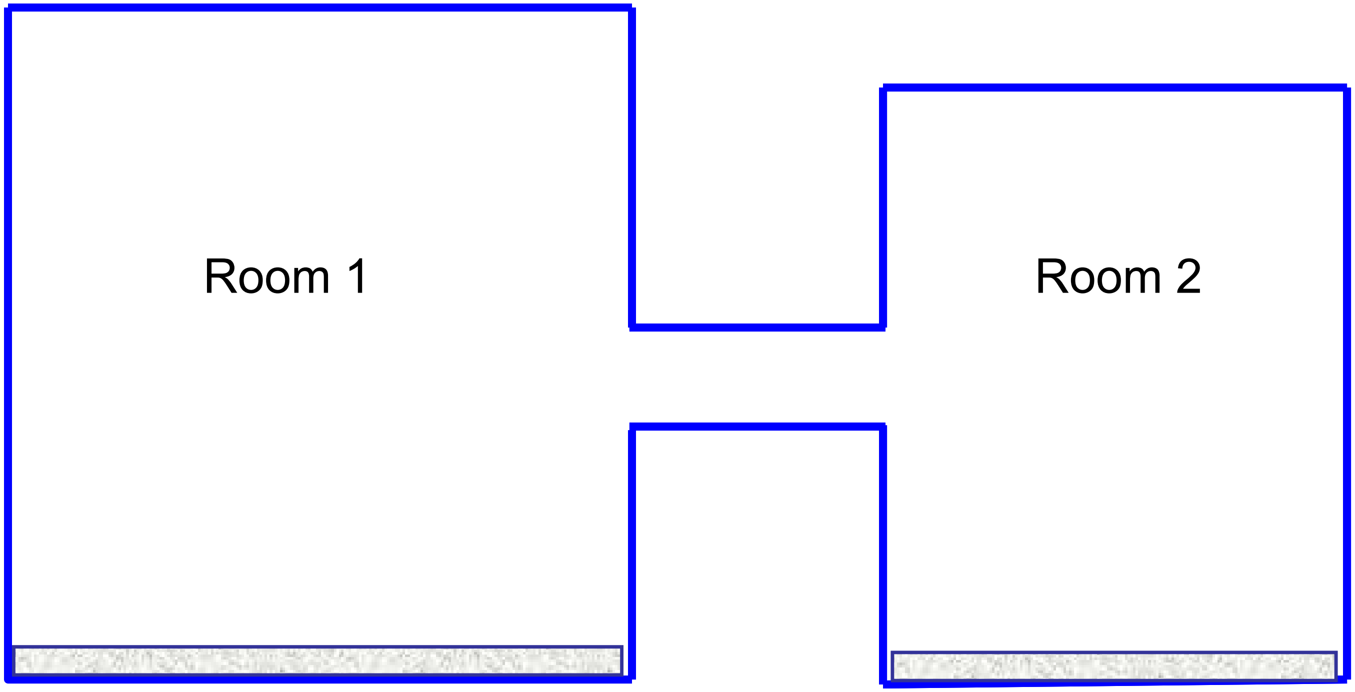
$$\boxed{W_{\text{in,force}} = W_{\text{out,damp}} + W_{\text{va}}}$$

$$\boxed{W_{\text{in,sound}} = W_{\text{out,absorb}} - W_{\text{va}}}$$

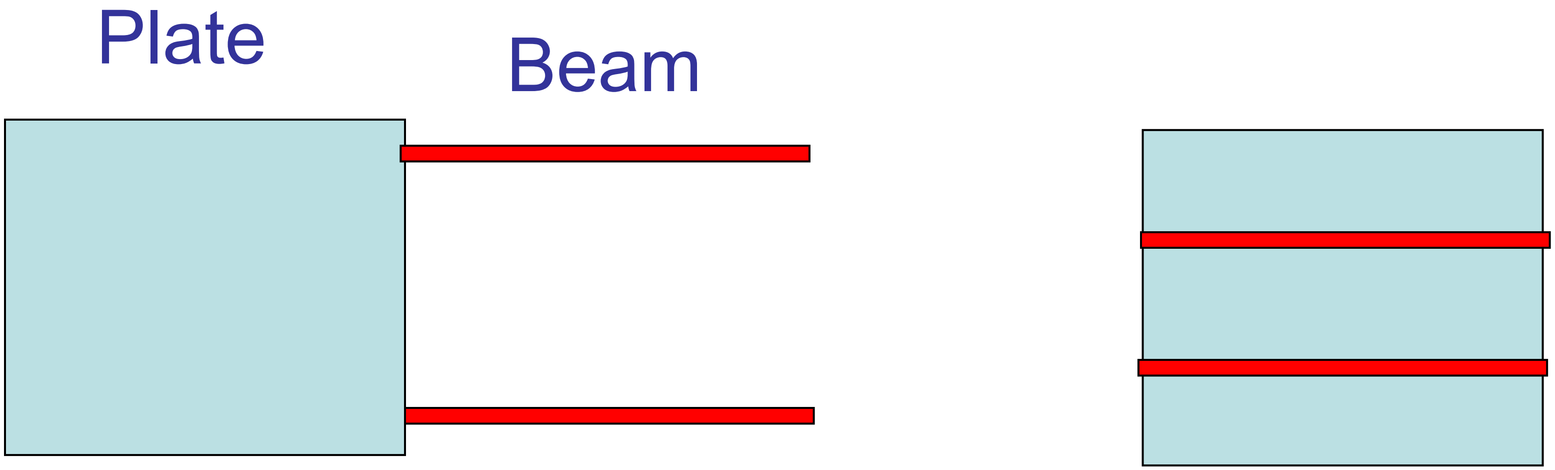
(Usually, E_{vib} and E_{acoust} are our unknowns)

Assumptions Weak Coupling

- Energy is uniform everywhere within a subsystem.
- More energy is dissipated in a subsystem than is transmitted to other subsystems.



Assumptions Weak Coupling

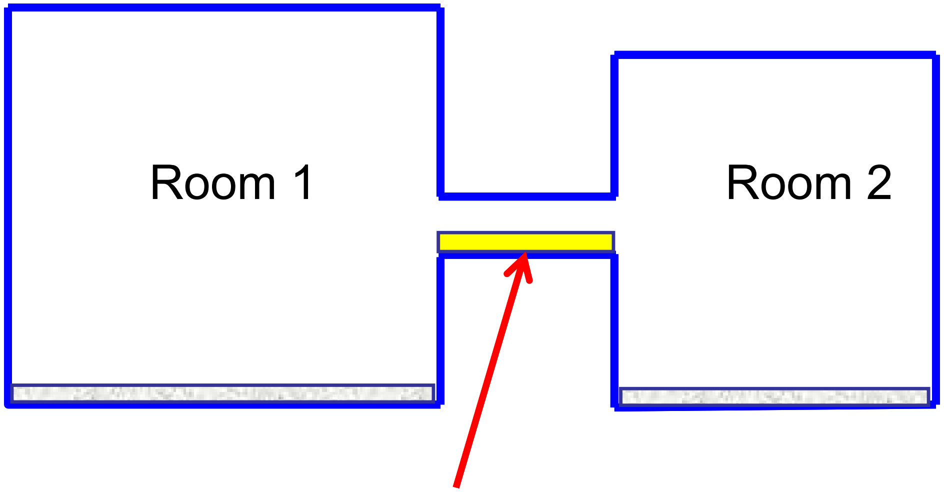


YES

No

Understanding Loss Factors

Damping Loss Factor – Energy lost by structural damping and acoustic radiation damping.
Coupling Loss Factor – Energy lost by transmission across a junction.



Energy absorbed at junction is included in the damping loss factors (not the coupling loss factor). Some damping at junctions is beneficial to SEA since it promotes weak coupling.

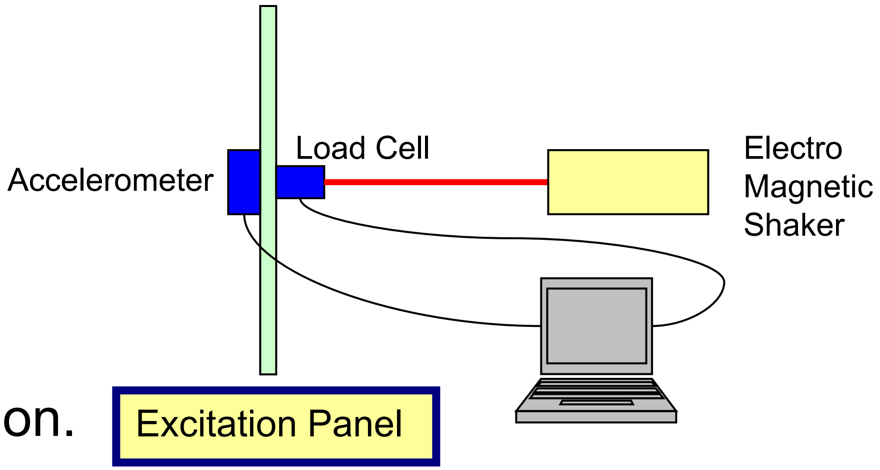
Overview

- Basic Theory
- Fundamentals
- Measuring SEA Parameters
- Examples

Measuring Input Power

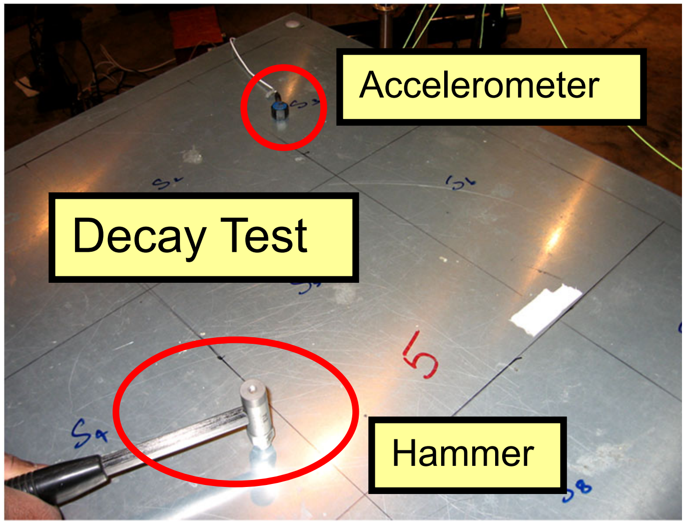
$$W_i = \mathrm{Re}(F_i v_i^*) = F_i^2\,\mathrm{Re}[Y_i]$$

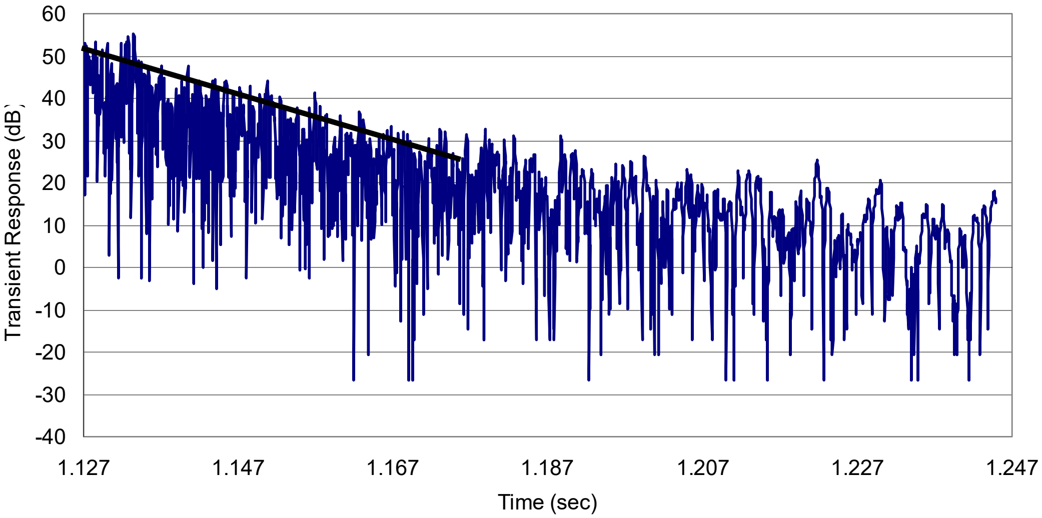
Y_i is the mobility at the input location.



Lyon and Dejong (1995)

Measuring Damping Loss Factors



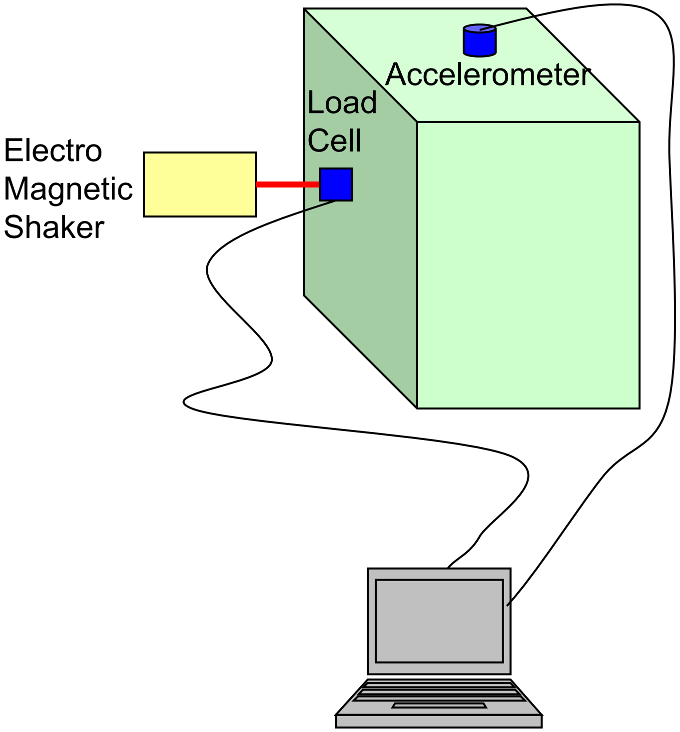


$$\eta_i = \frac{\gamma_i}{27.3f}$$ where γ_i is the initial slope of the transient response.

Measuring Energy

Structural Subsystems

$$E_i = M_i^{eq} \langle V_i^2 \rangle_{sp}$$

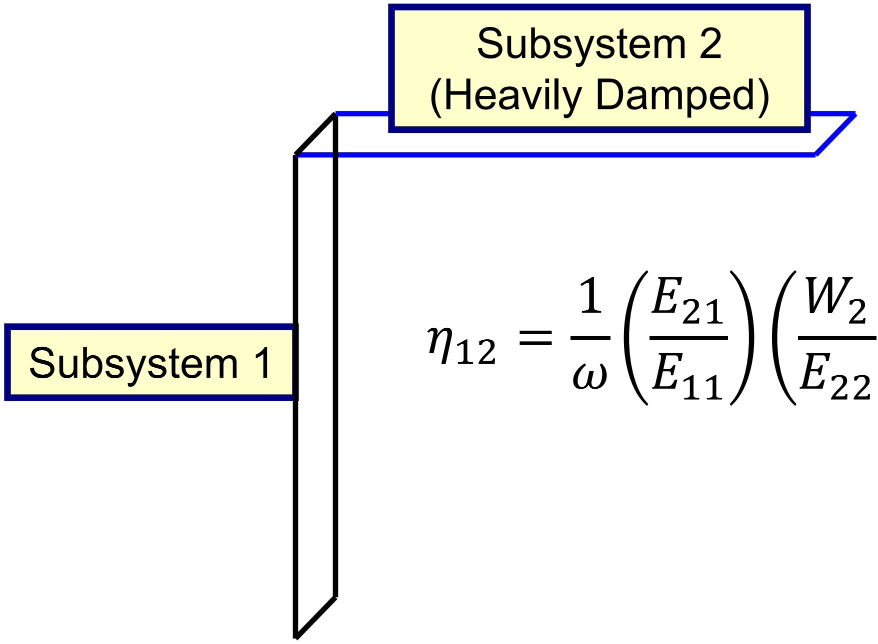


Acoustical Subsystems

$$E_i = \frac{V}{\rho c^2} \langle p_i \rangle_{sp}^2$$

$\langle \quad \rangle_{sp}$ indicates spatial averaging.

Measuring Coupling Loss Factors

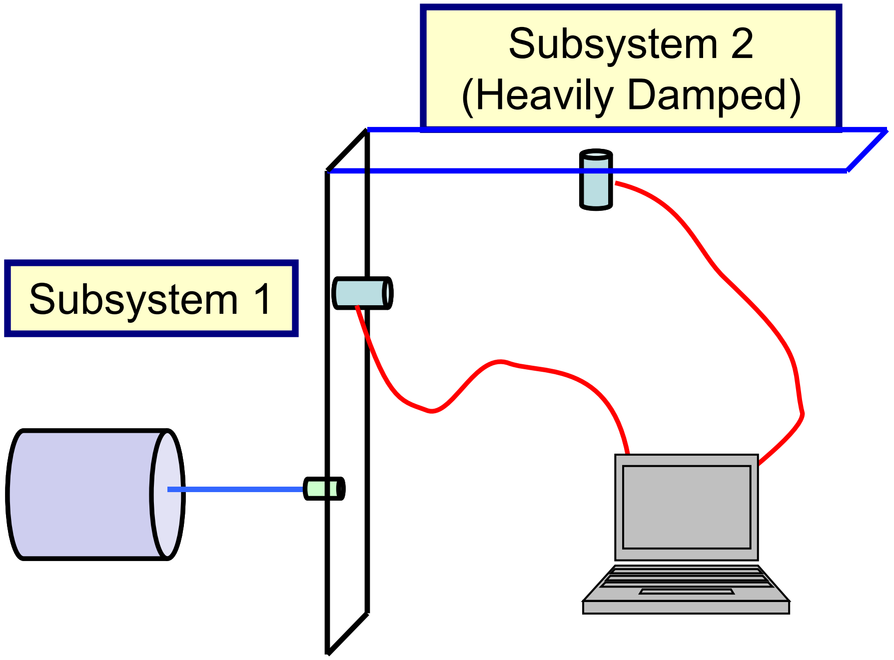


$$\eta_{12} = \frac{1}{\omega}\left(\frac{E_{21}}{E_{11}}\right)\left(\frac{W_2}{E_{22}}\right)$$

E_{ji} – Energy of Response Panel j with respect to Excitation Panel i

W_i – Input power for subsystem i

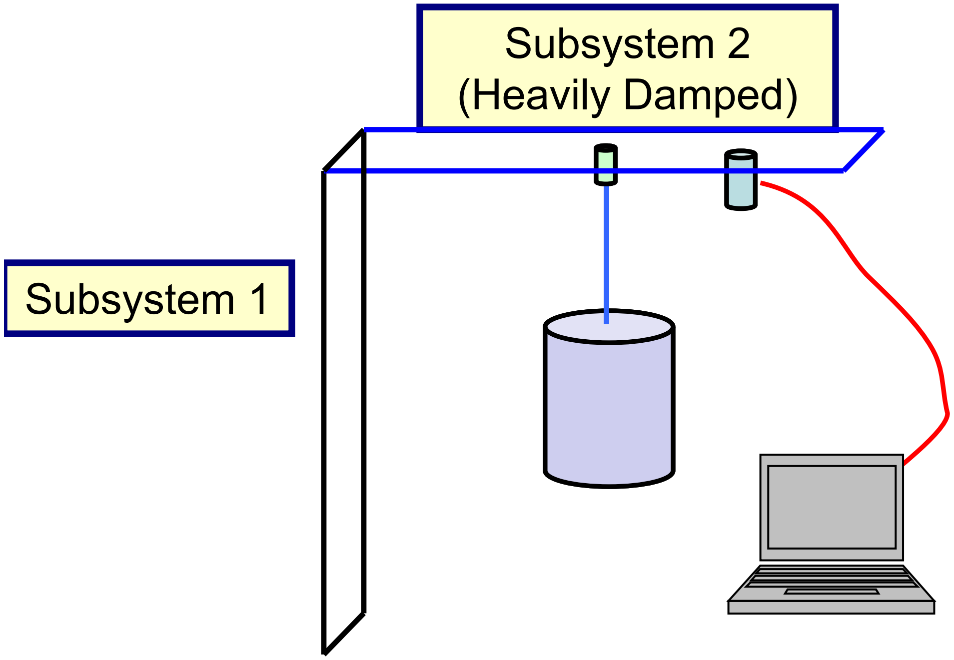
Step 1 Excite Subsystem 1



Measure accelerations (spatially average over several points) to find E_{11} and E_{21}.

$$\eta_{12} = \frac{1}{\omega}\left(\frac{E_{21}}{E_{11}}\right)\left(\frac{W_2}{E_{22}}\right)$$

Step 2 Excite Subsystem 2



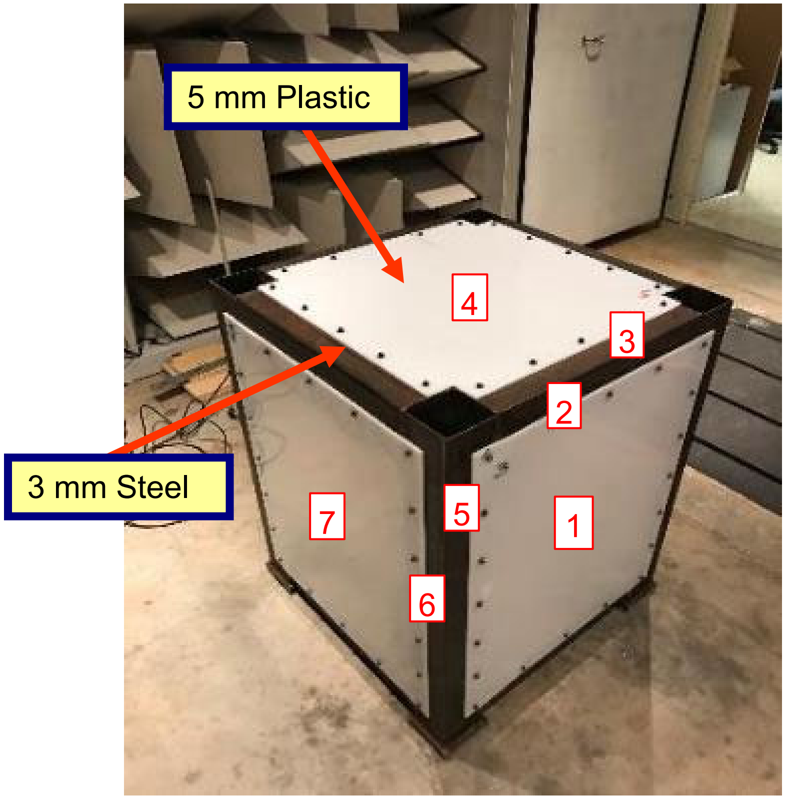
Measure W_2 using impedance head and E_{22} accelerometer.

$$\eta_{12} = \frac{1}{\omega}\left(\frac{E_{21}}{E_{11}}\right)\left(\frac{W_2}{E_{22}}\right)$$

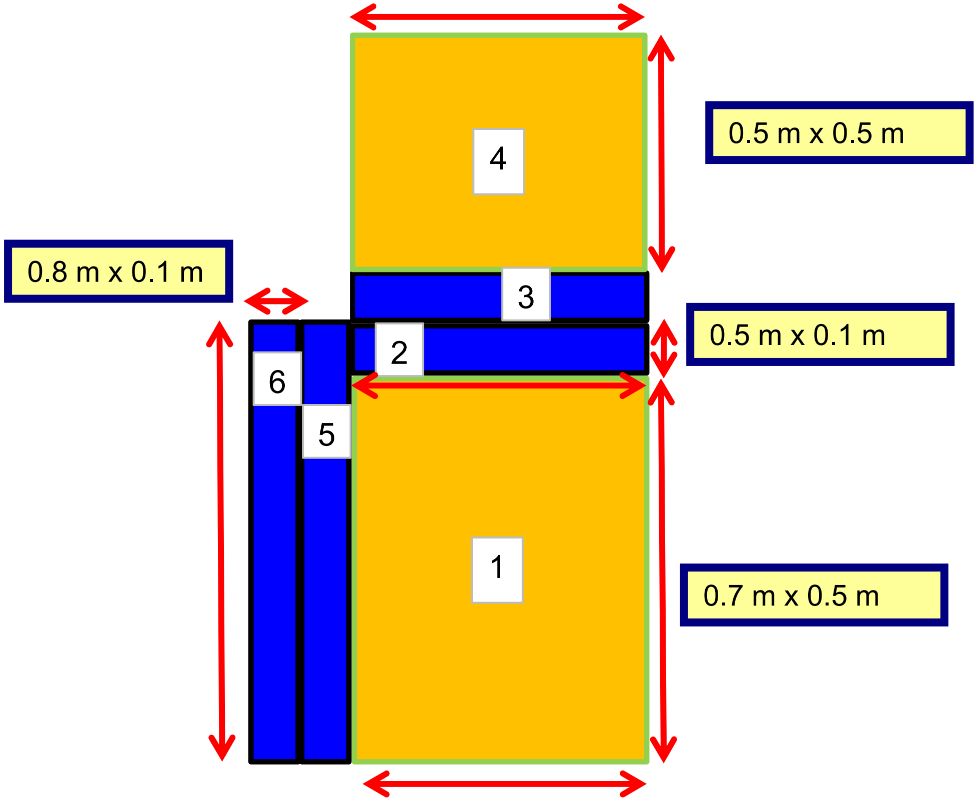
Overview

- Basic Theory
- Fundamentals
- Measuring SEA Parameters
- Examples

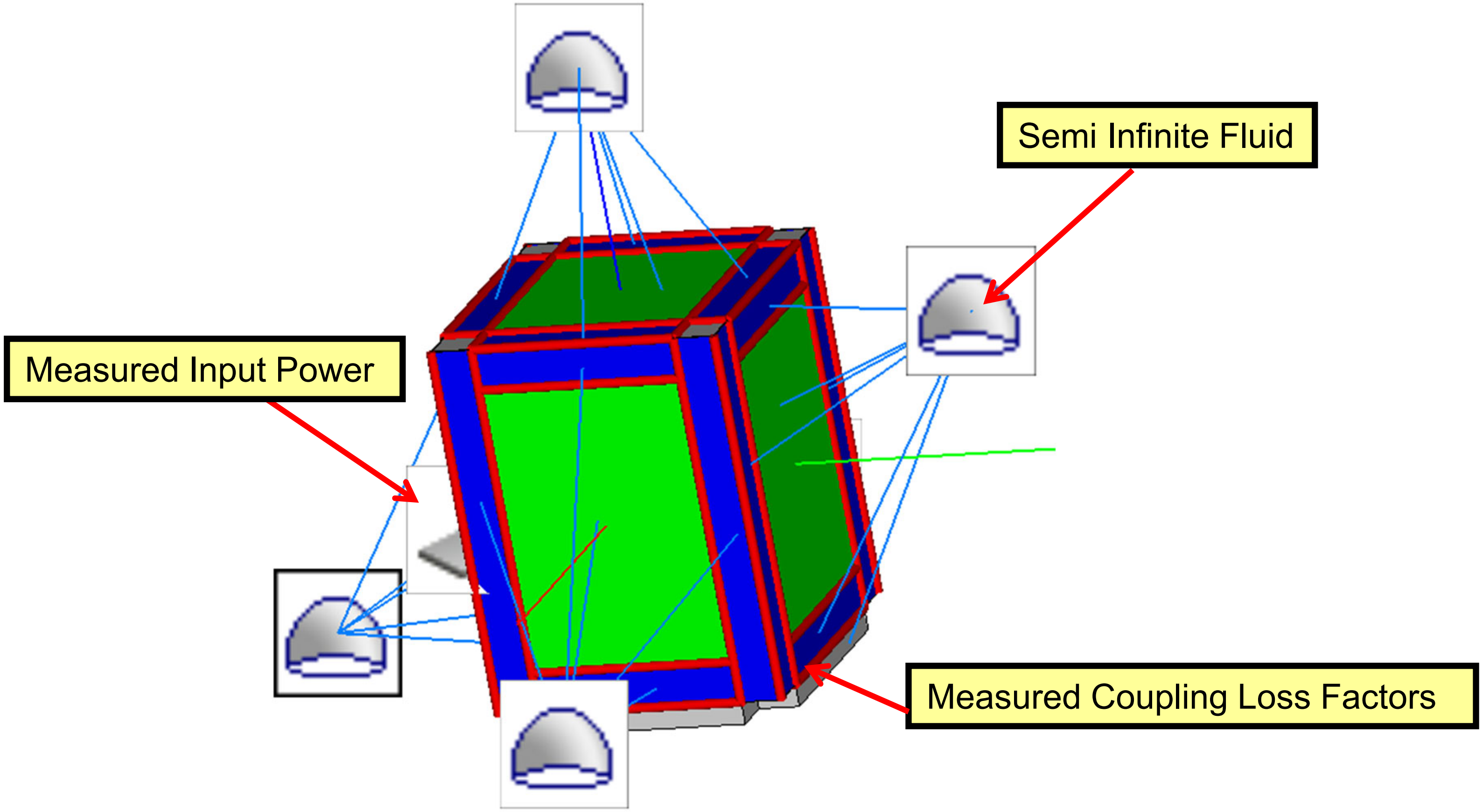
Test Article



Subsystems used for experimental SEA



VA-One Model

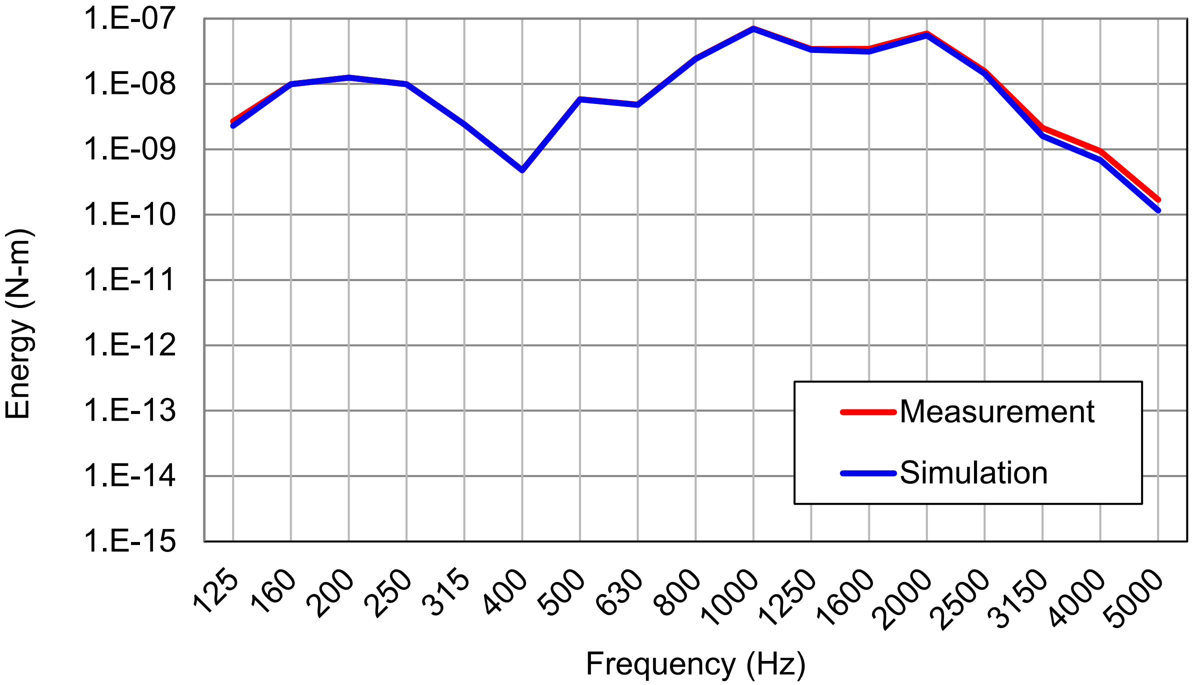


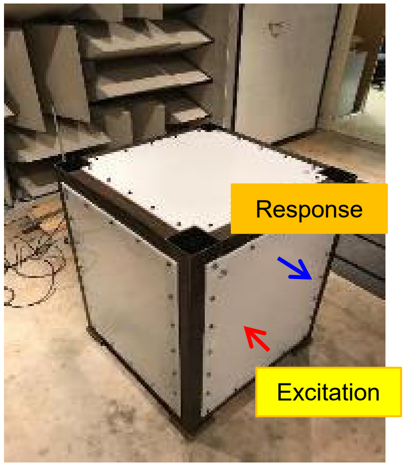
Semi Infinite Fluid

Measured Input Power

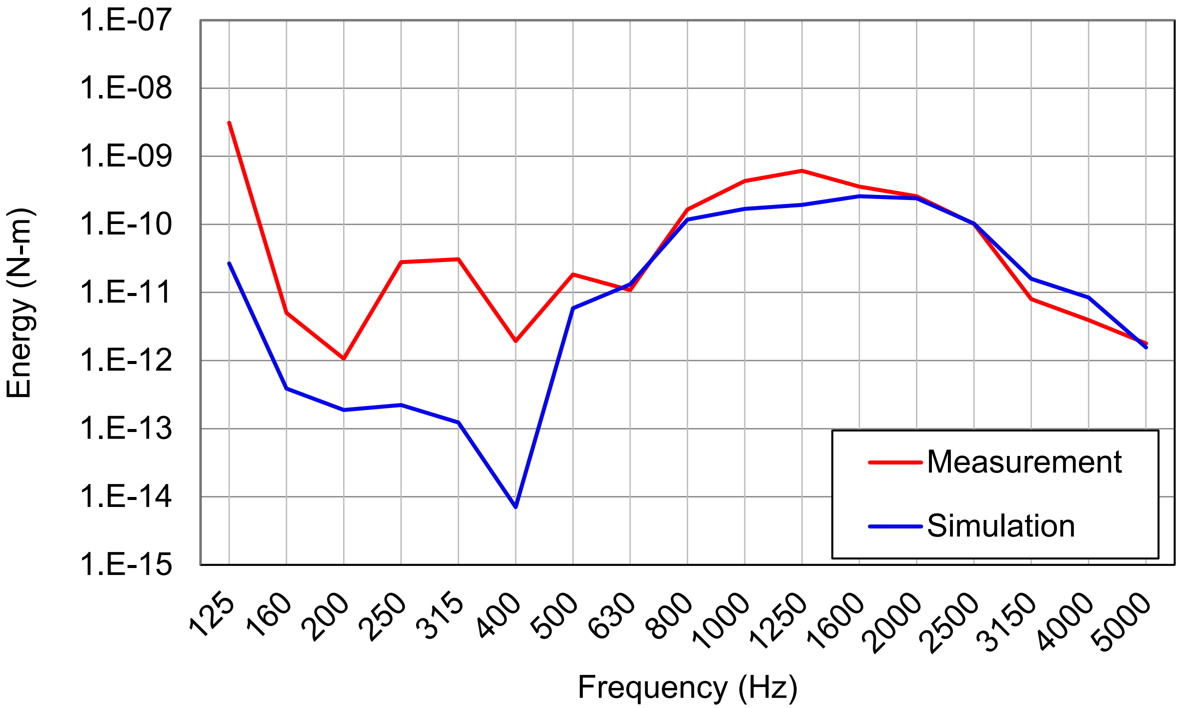
Measured Coupling Loss Factors

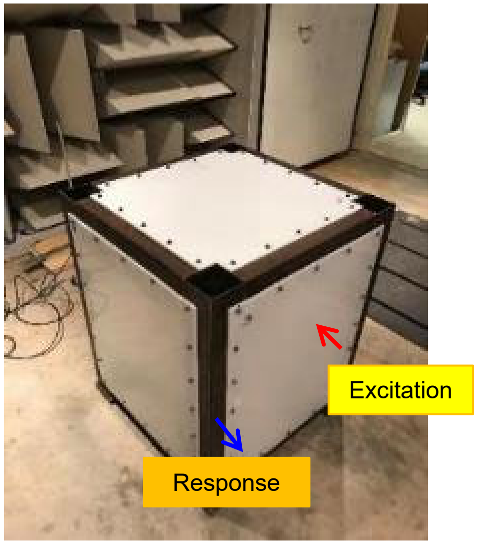
Single Input Flexural Energy



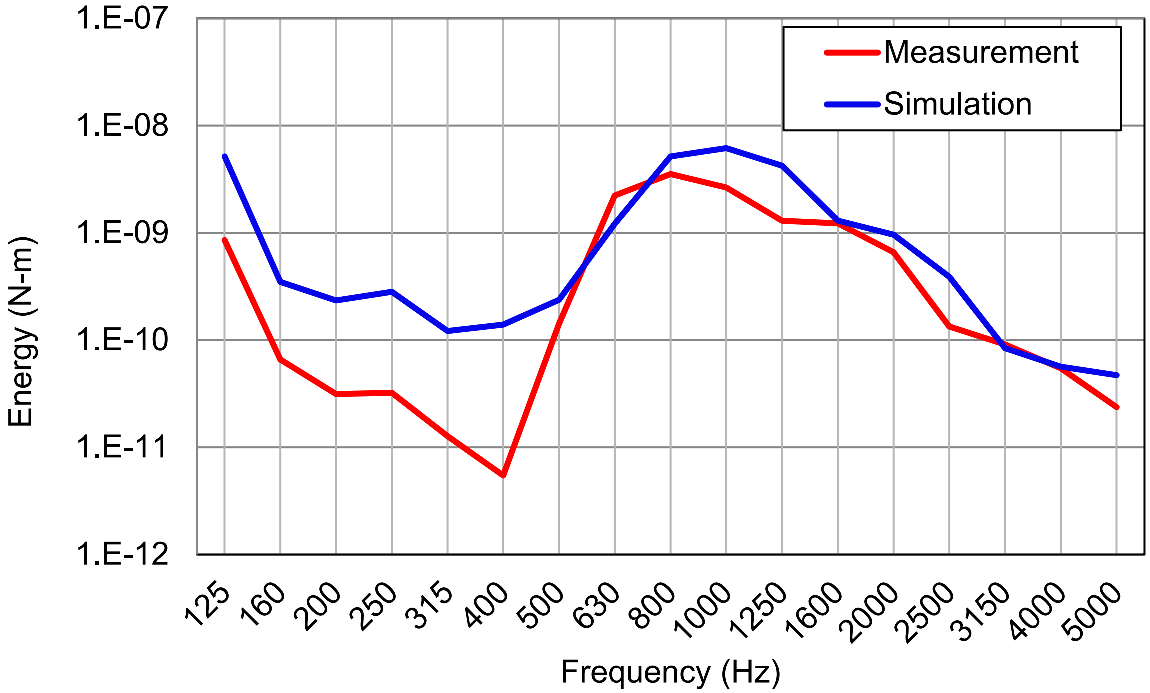


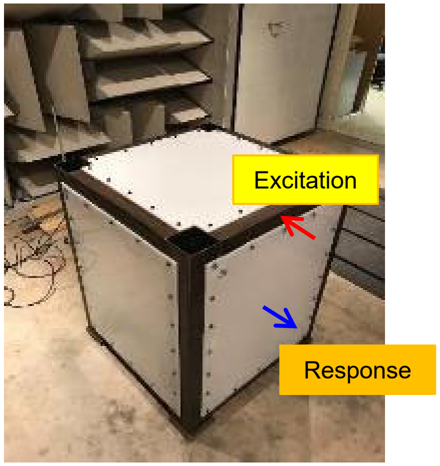
Single Input Flexural Energy



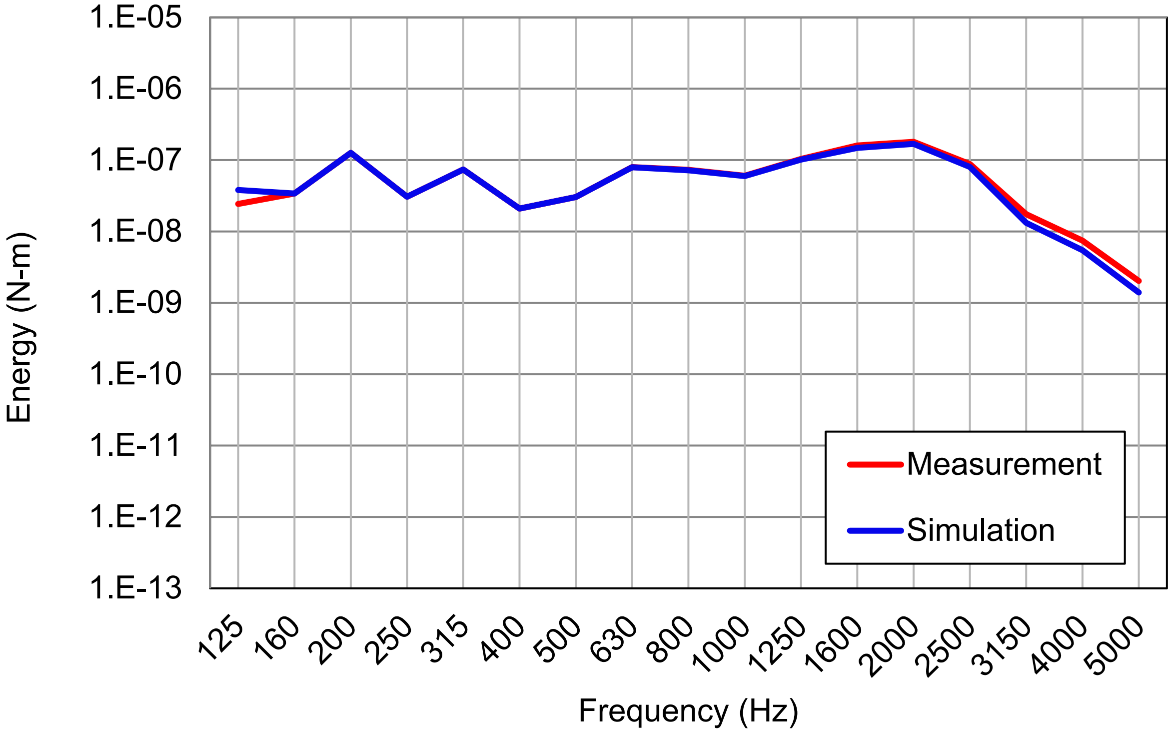


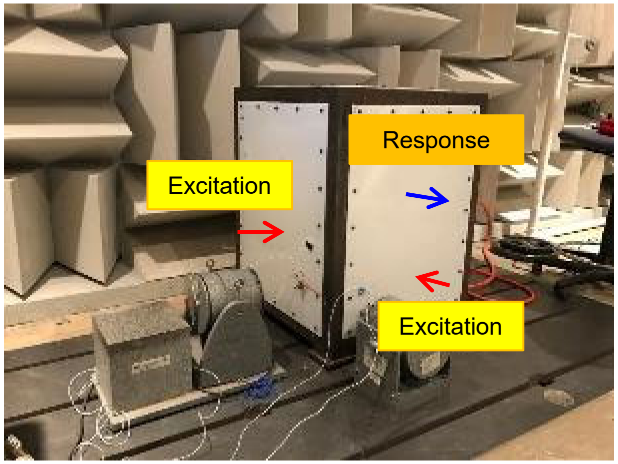
Single Input Flexural Energy



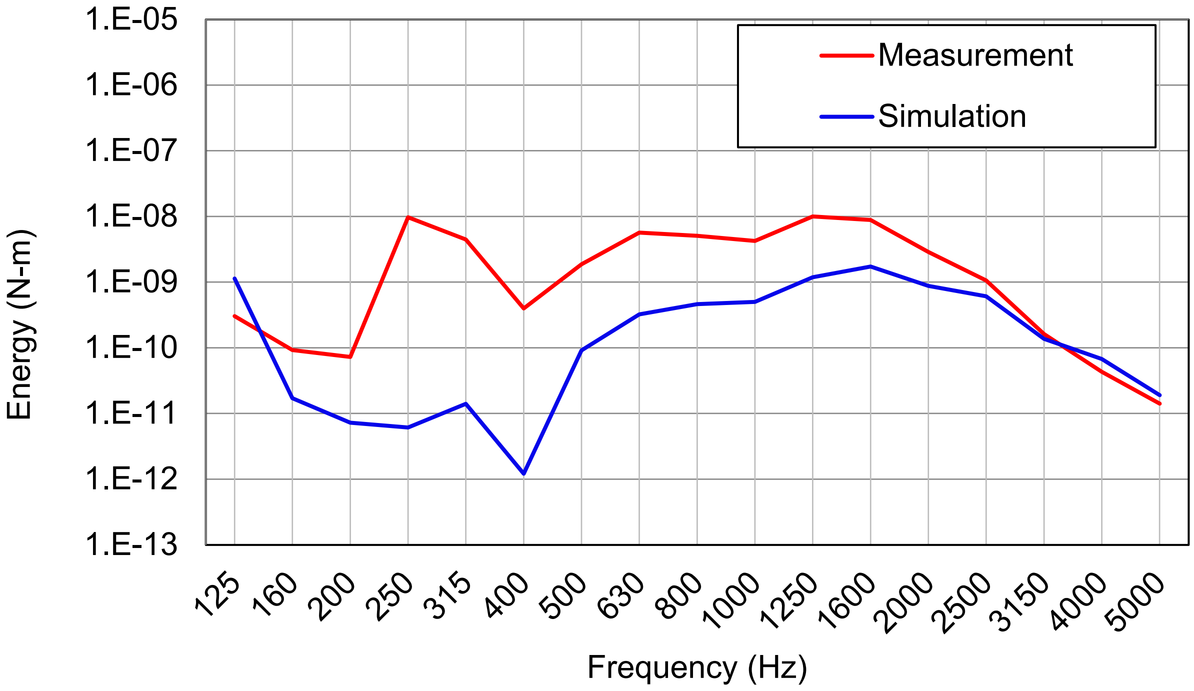


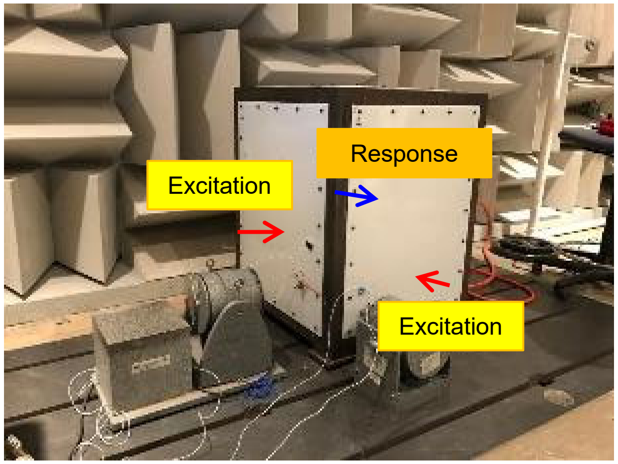
Two Inputs Flexural Energy





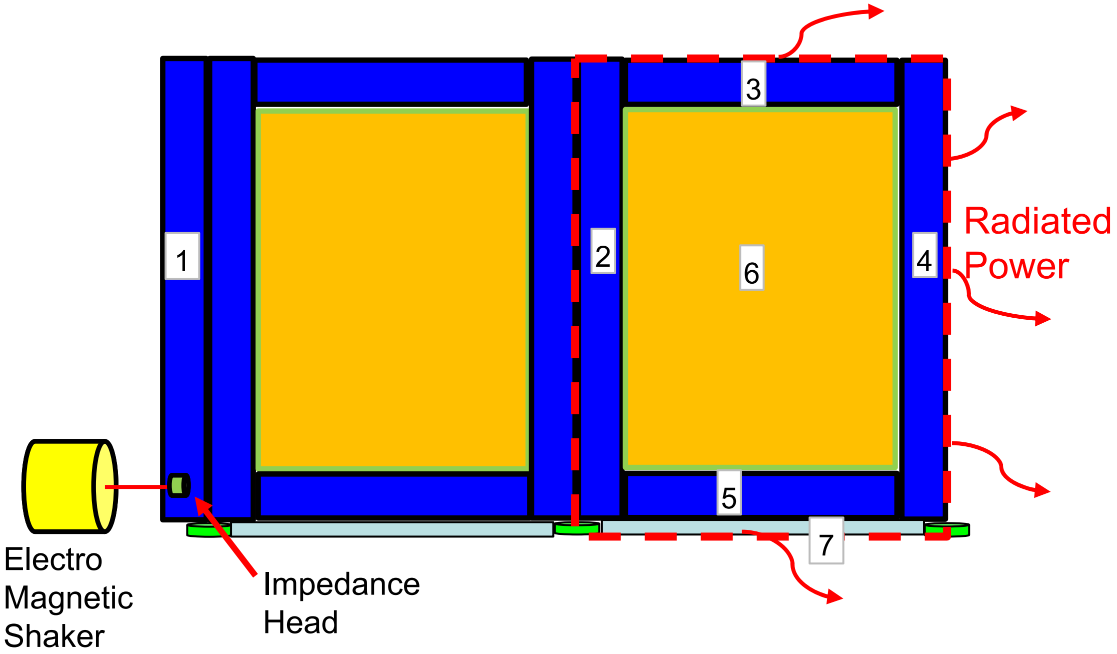
Two Inputs Flexural Energy



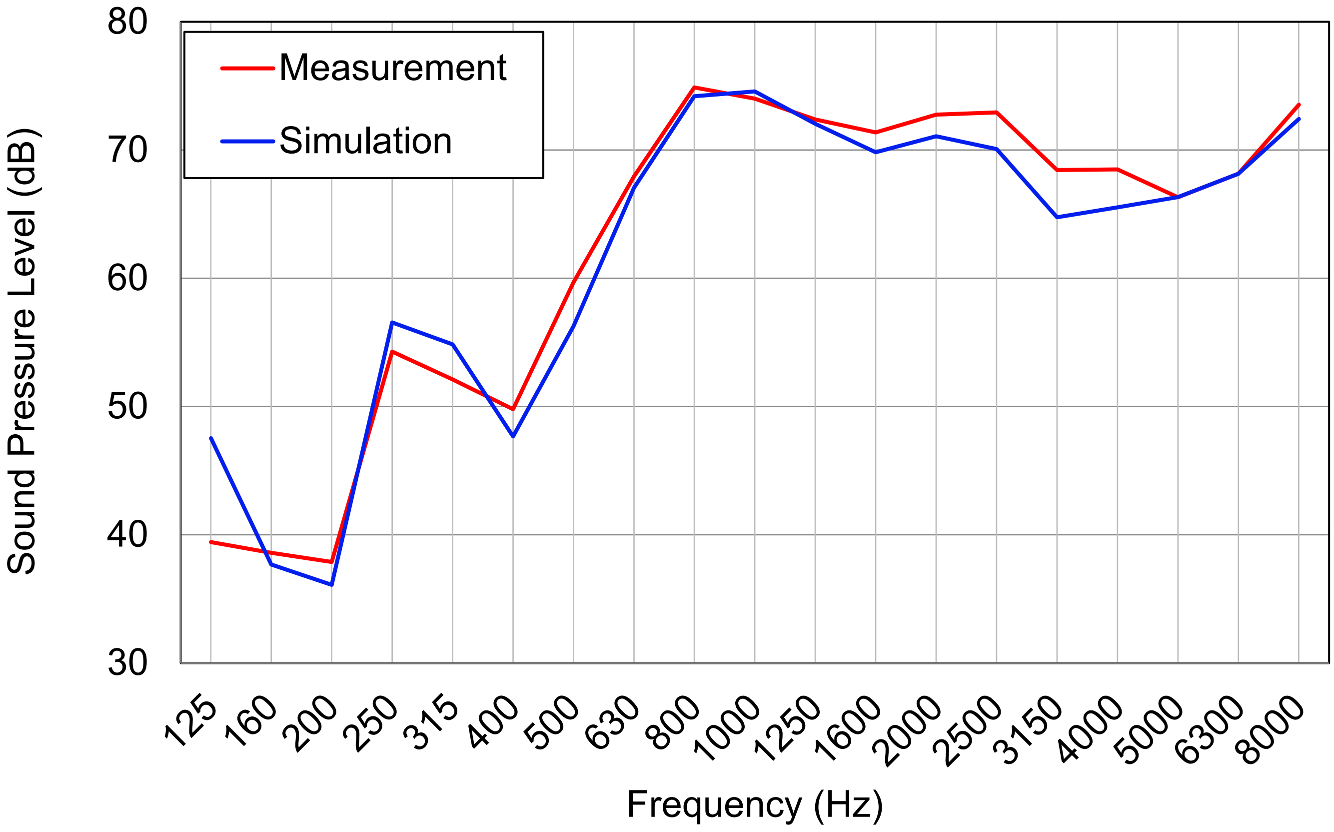


Experiment Setup

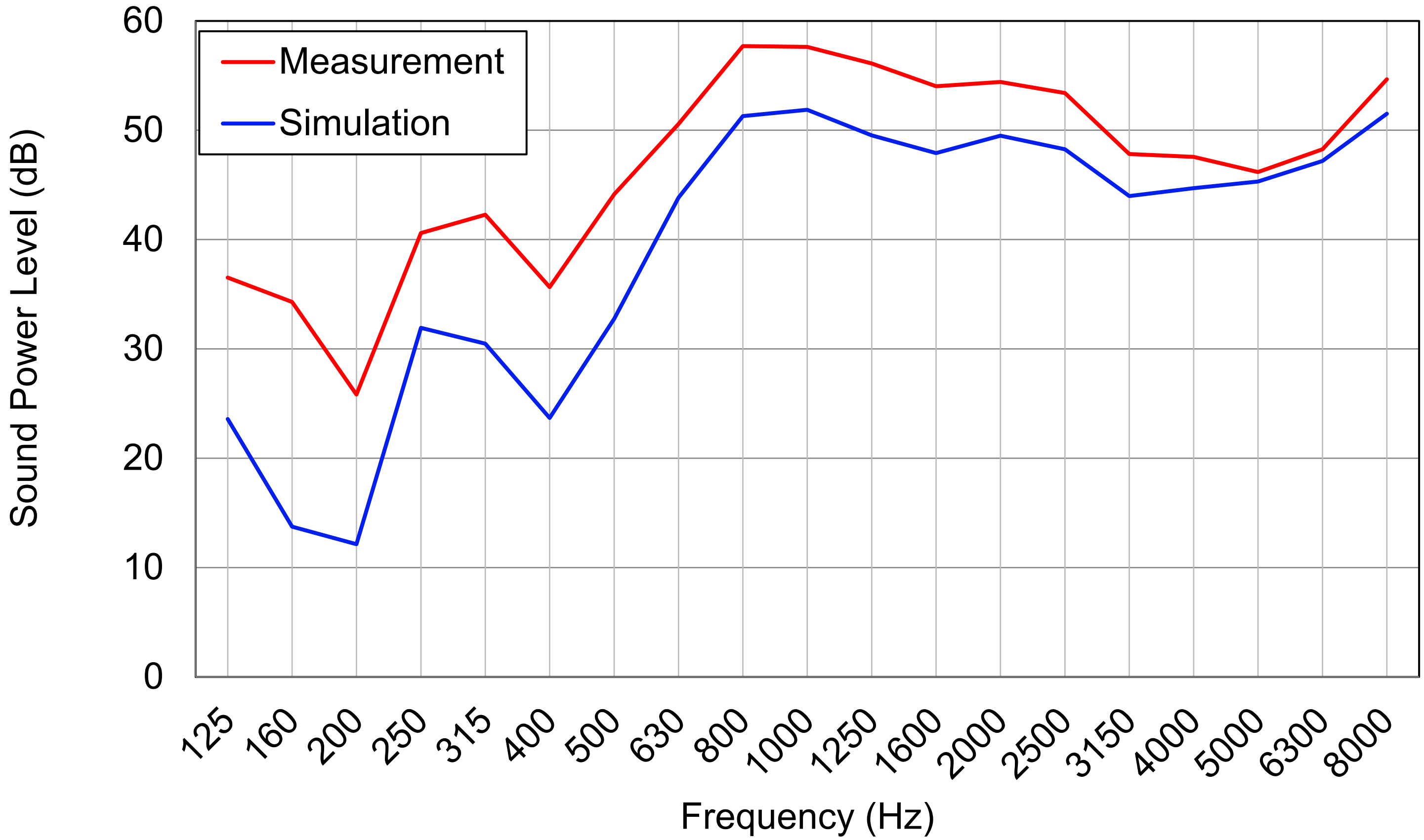
- Electromagnetic shaker is attached on steel panel as input source
- Average sound pressure level inside the acoustic cavity is measured
- Radiated sound power from one side of the enclosure is measured



Cavity Sound Pressure Level Comparison

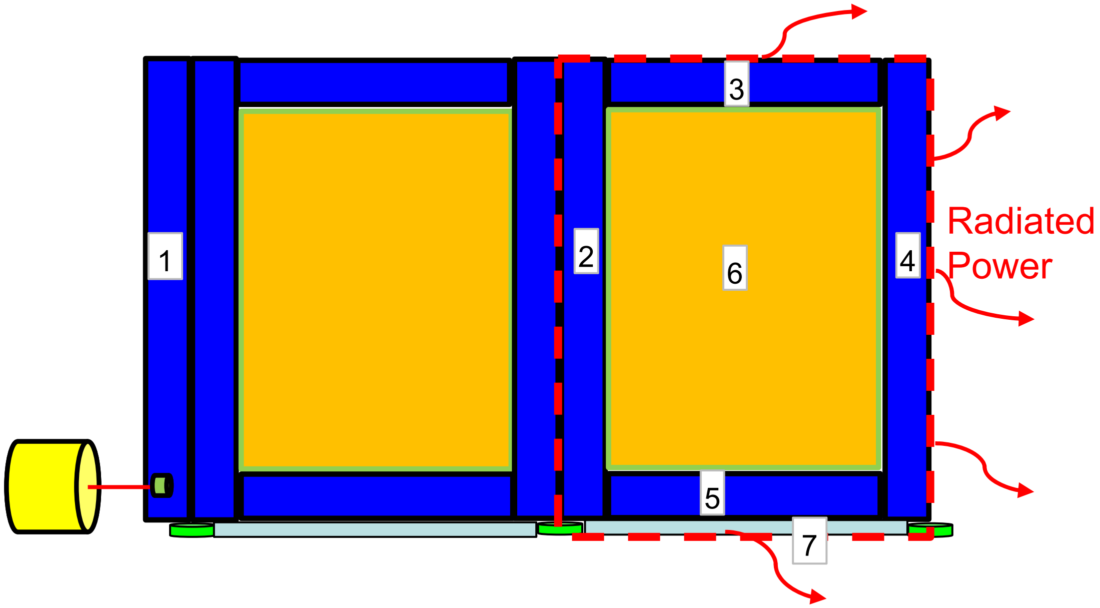


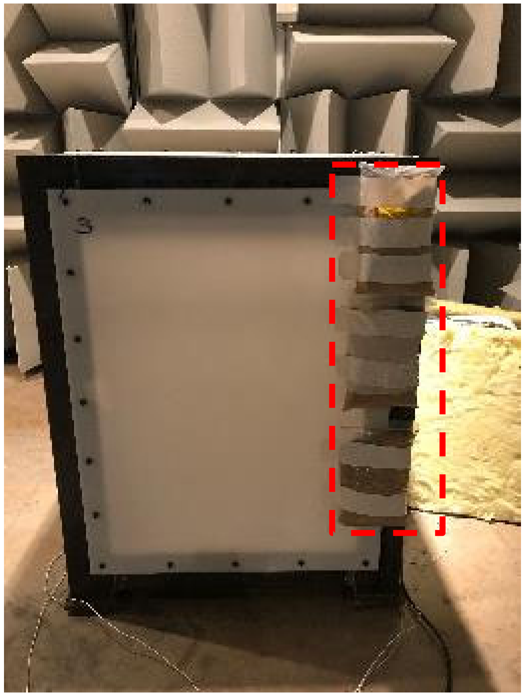
Radiated Sound Power Level Comparison



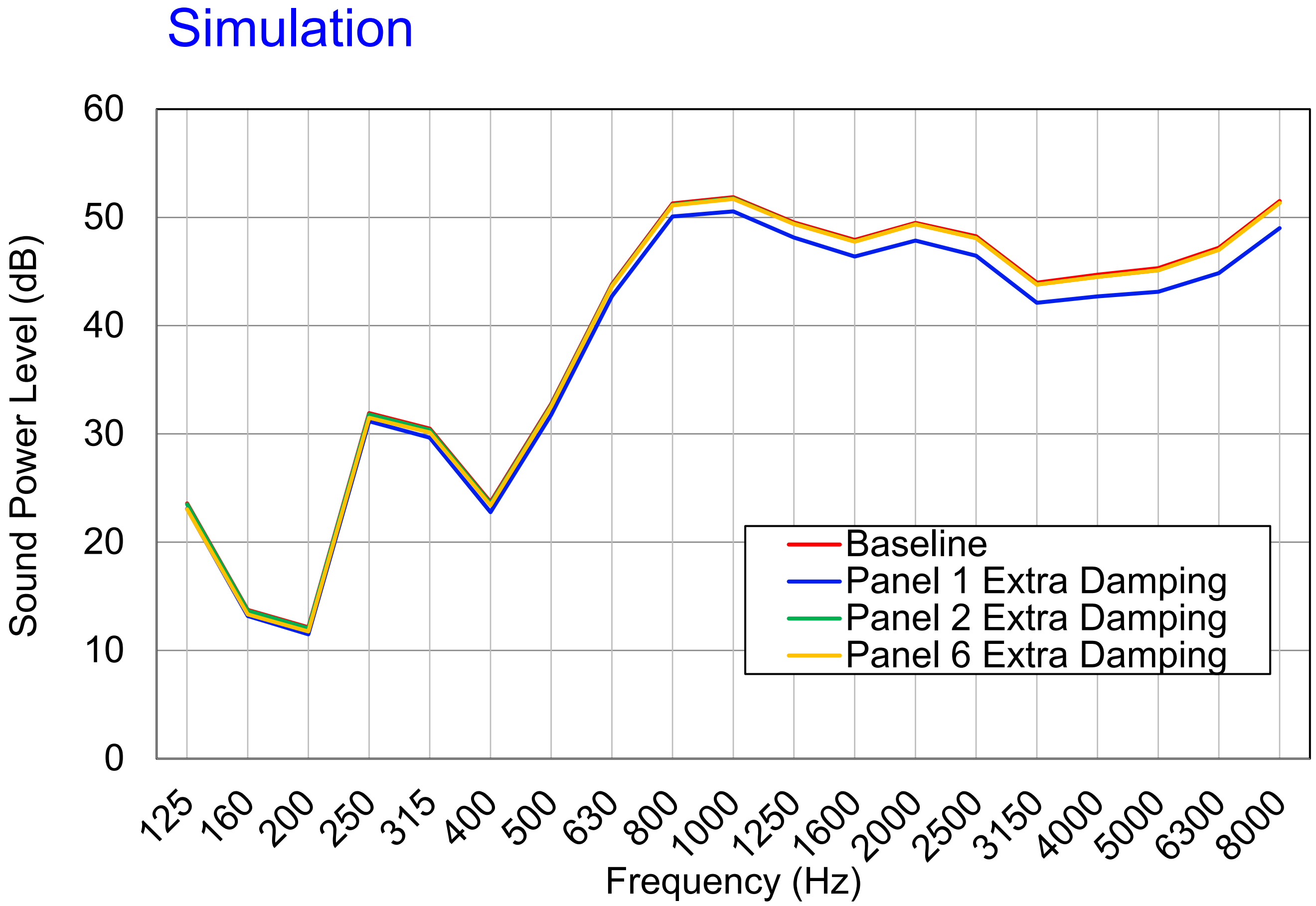
Treatment 1 Structural Damping

- Damping applied on panels 1, 2, and 6.



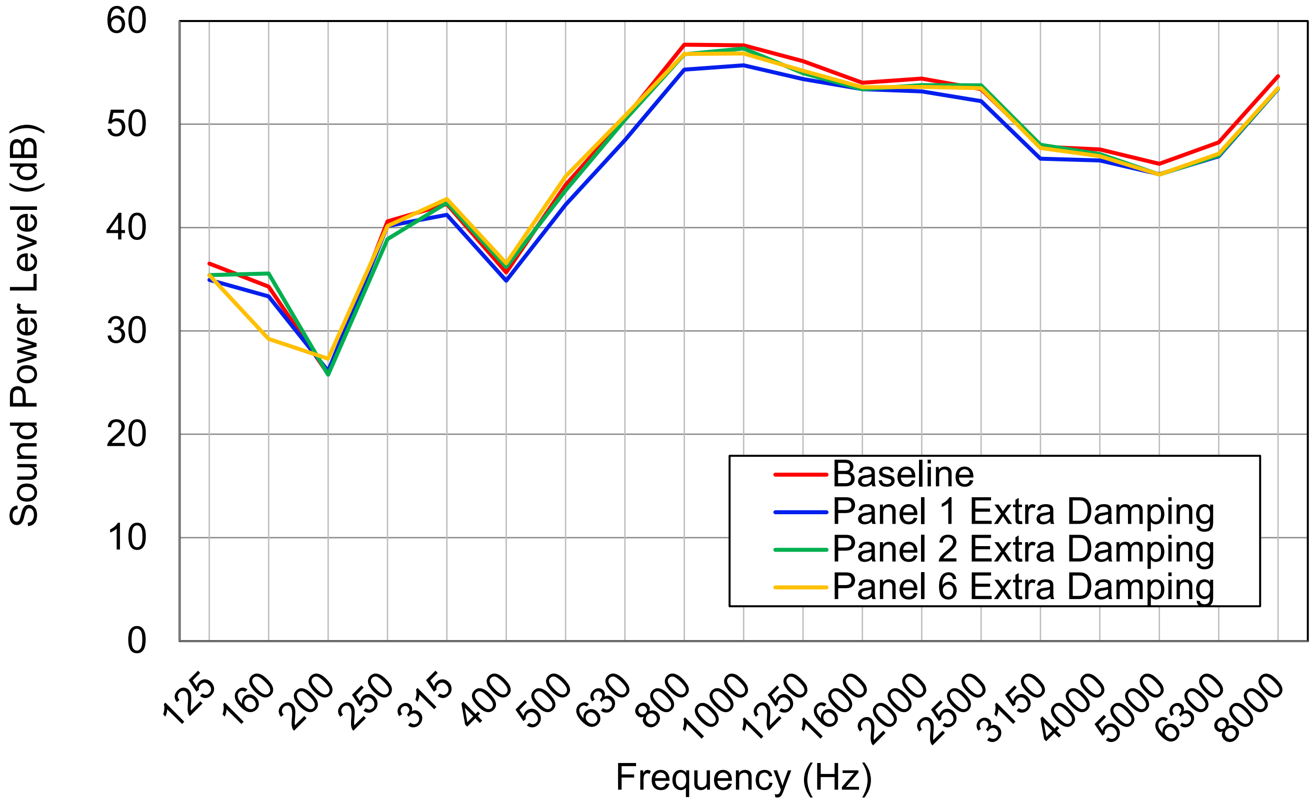


Treatment 1 Structural Damping

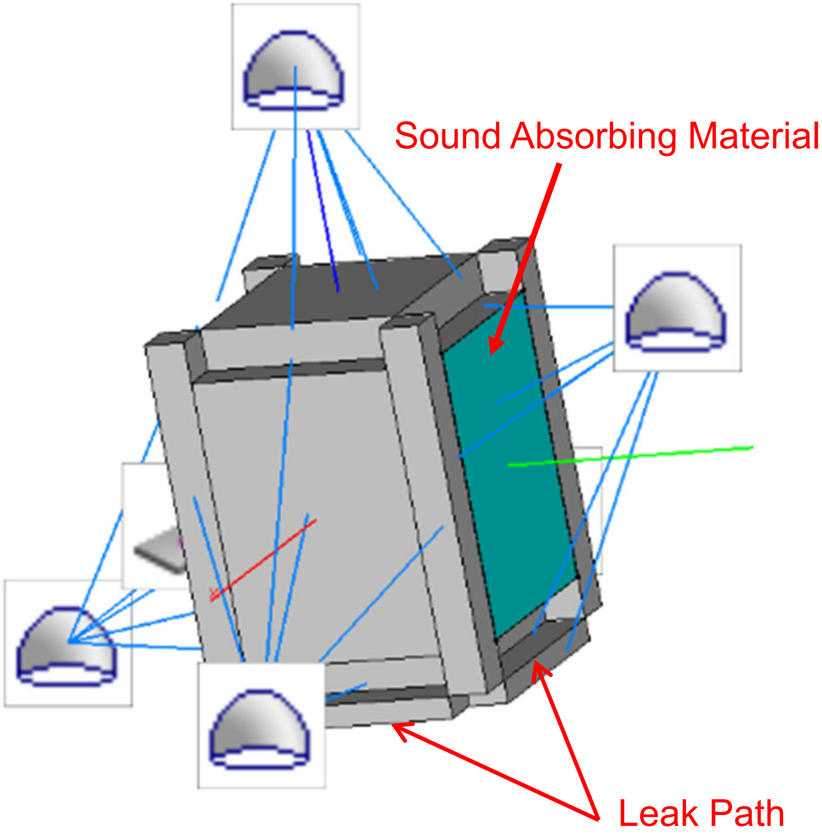


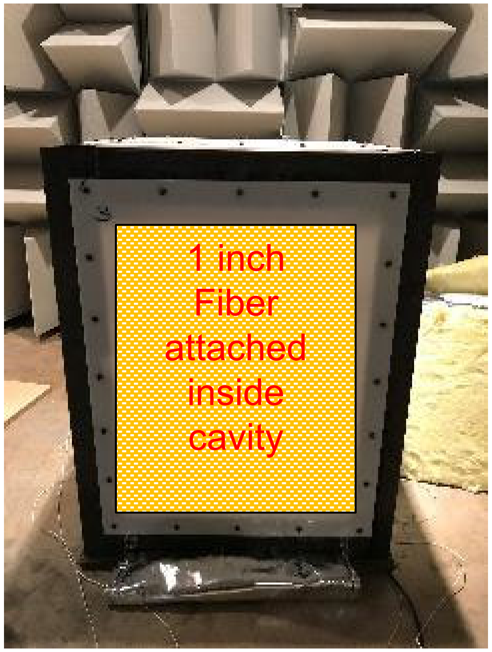
Treatment 1 Structural Damping

Measurement

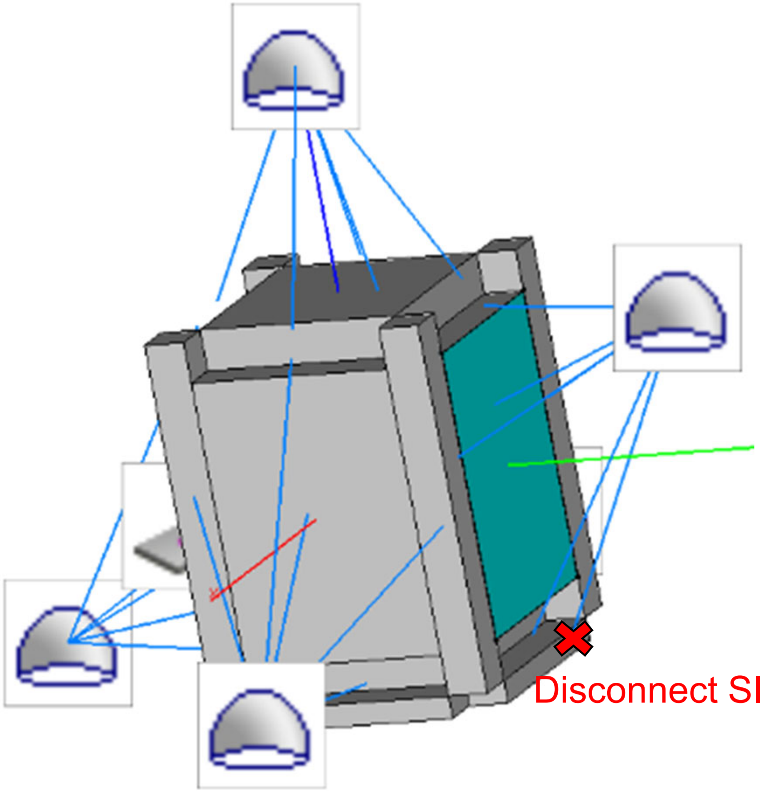


Treatment 2 Sound Absorption





Treatment 3 Barrier in Leak

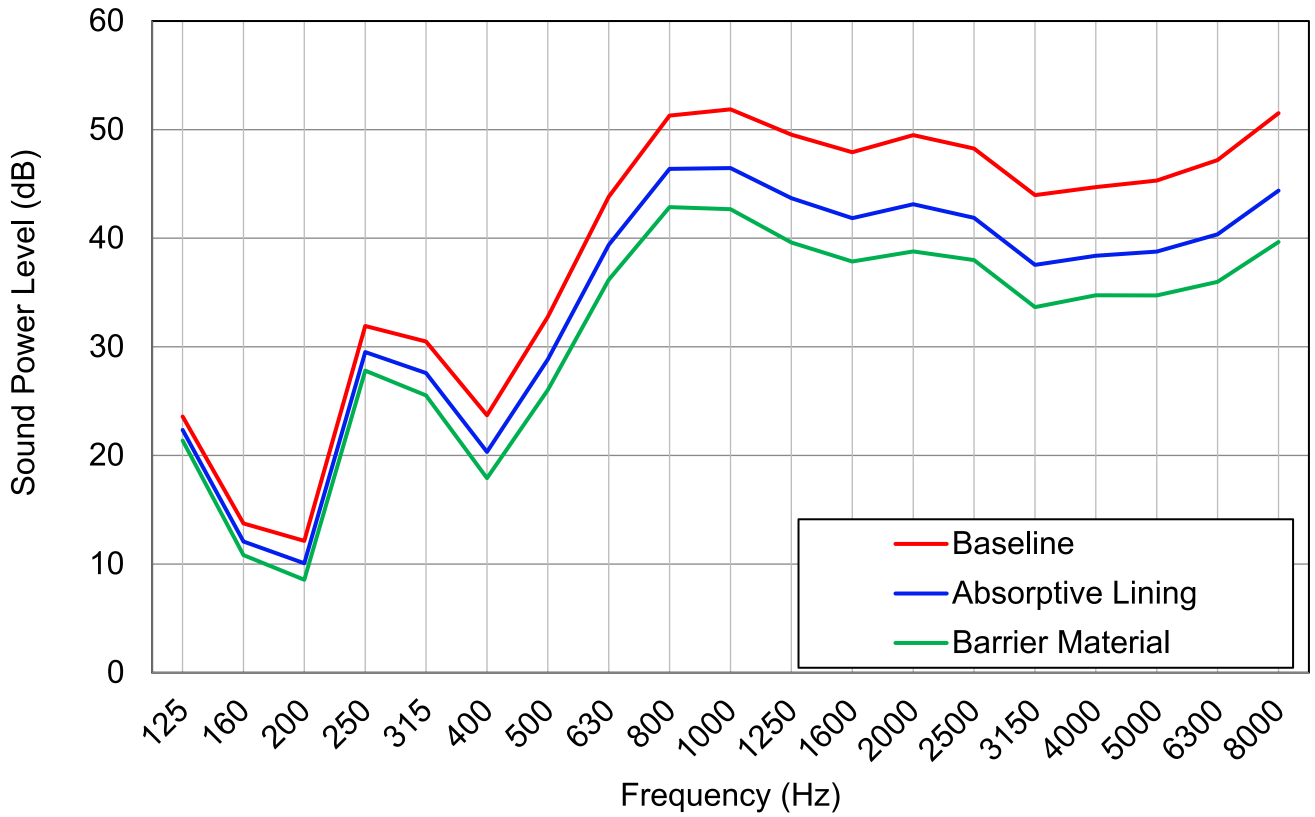


Disconnect SIF from cavity

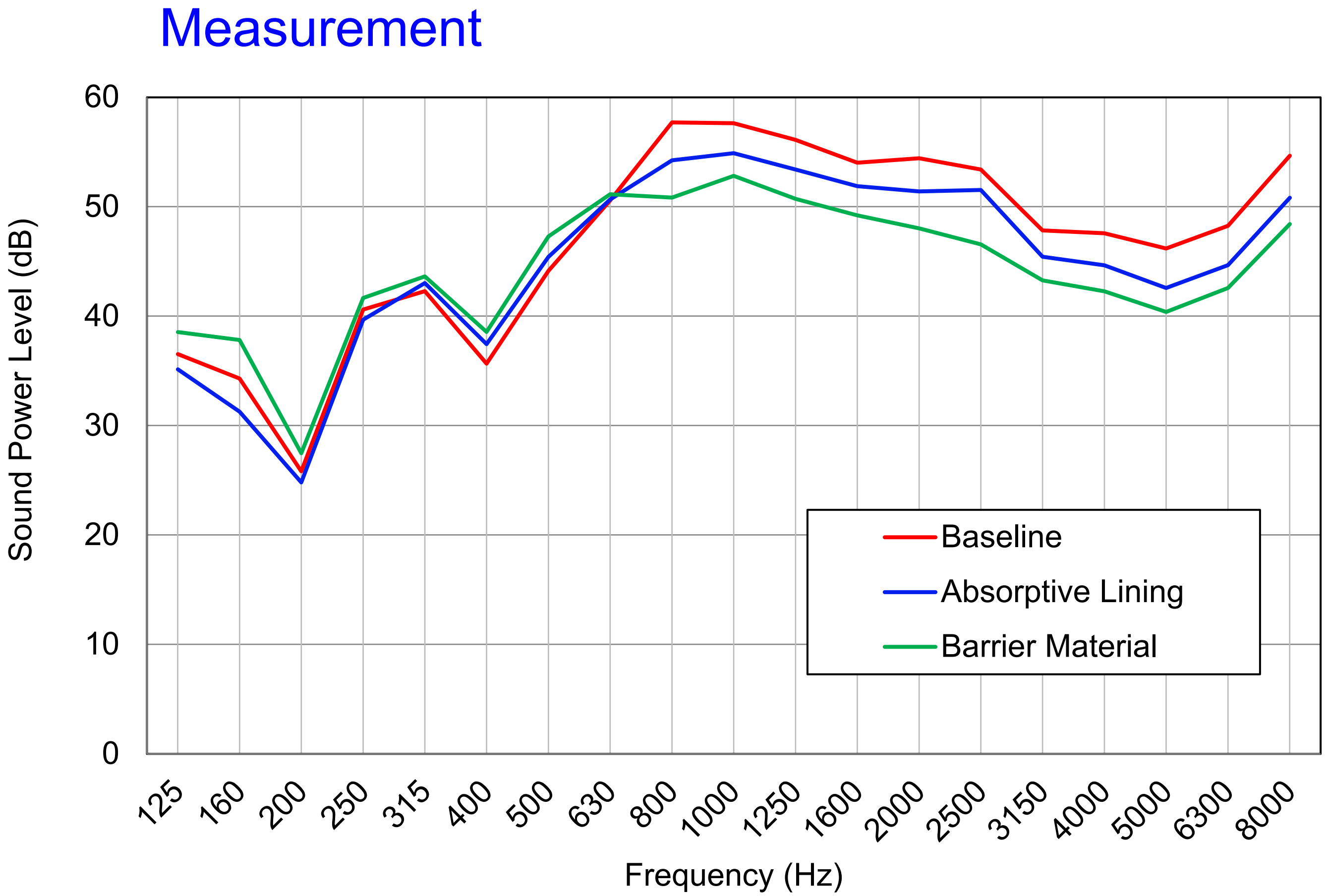
Cavity gap is blocked by lagging material

Effect of Acoustical Treatments

Simulation



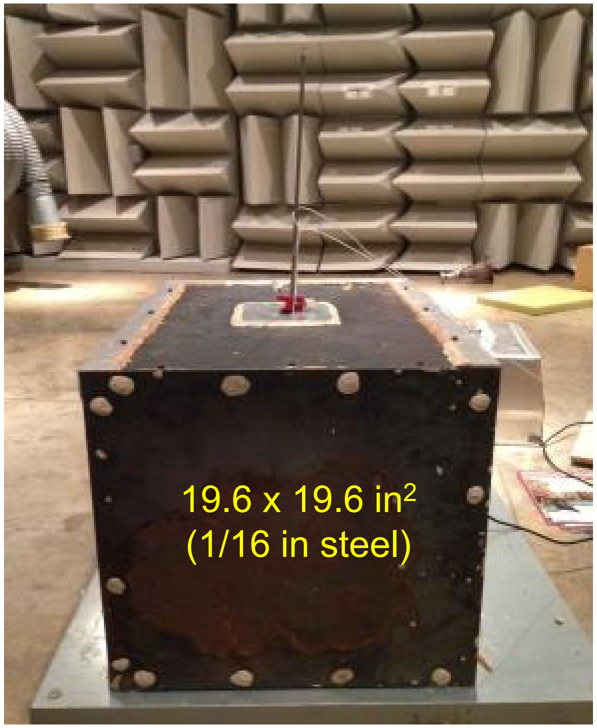
Effect of Acoustical Treatments



Experiment Sample Boxes

36 x 36 x 36 in^3 wood box
(1/2 inch thick)

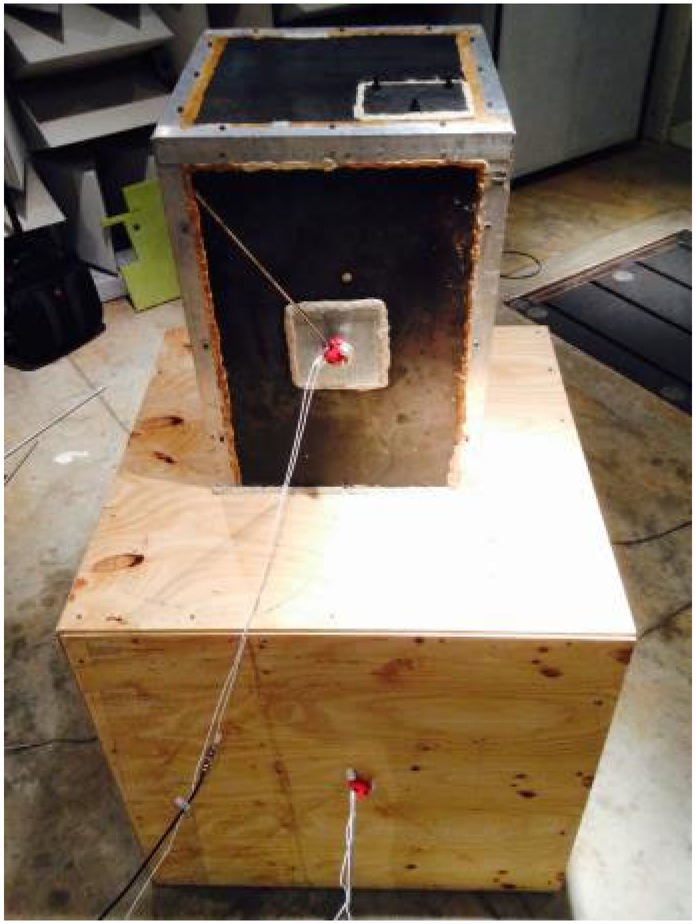
26.4 x 19.6 x 19.6 in^3 steel box



19.6 x 19.6 in^2
(1/16 in steel)

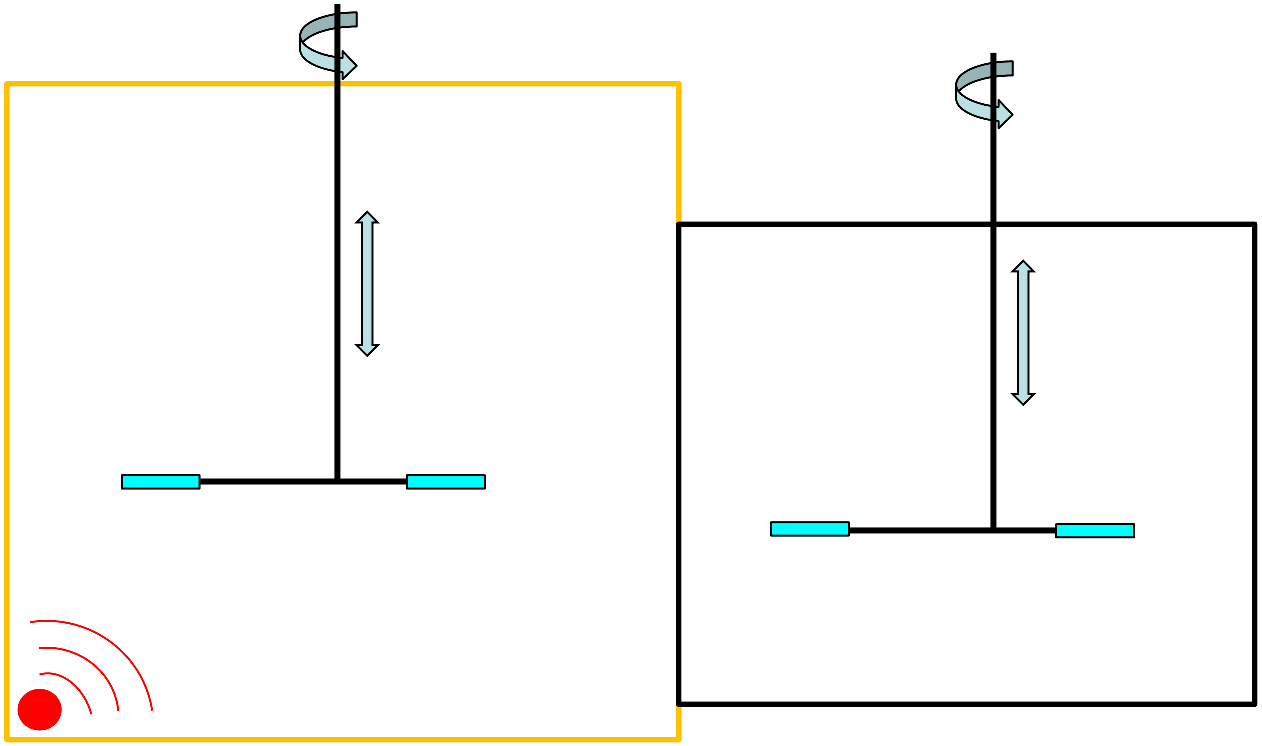
Experiment Approach

- Upper box rests on lower box.
- Source is placed in the lower box.
- Average sound pressure level (SPL) of two boxes measured.

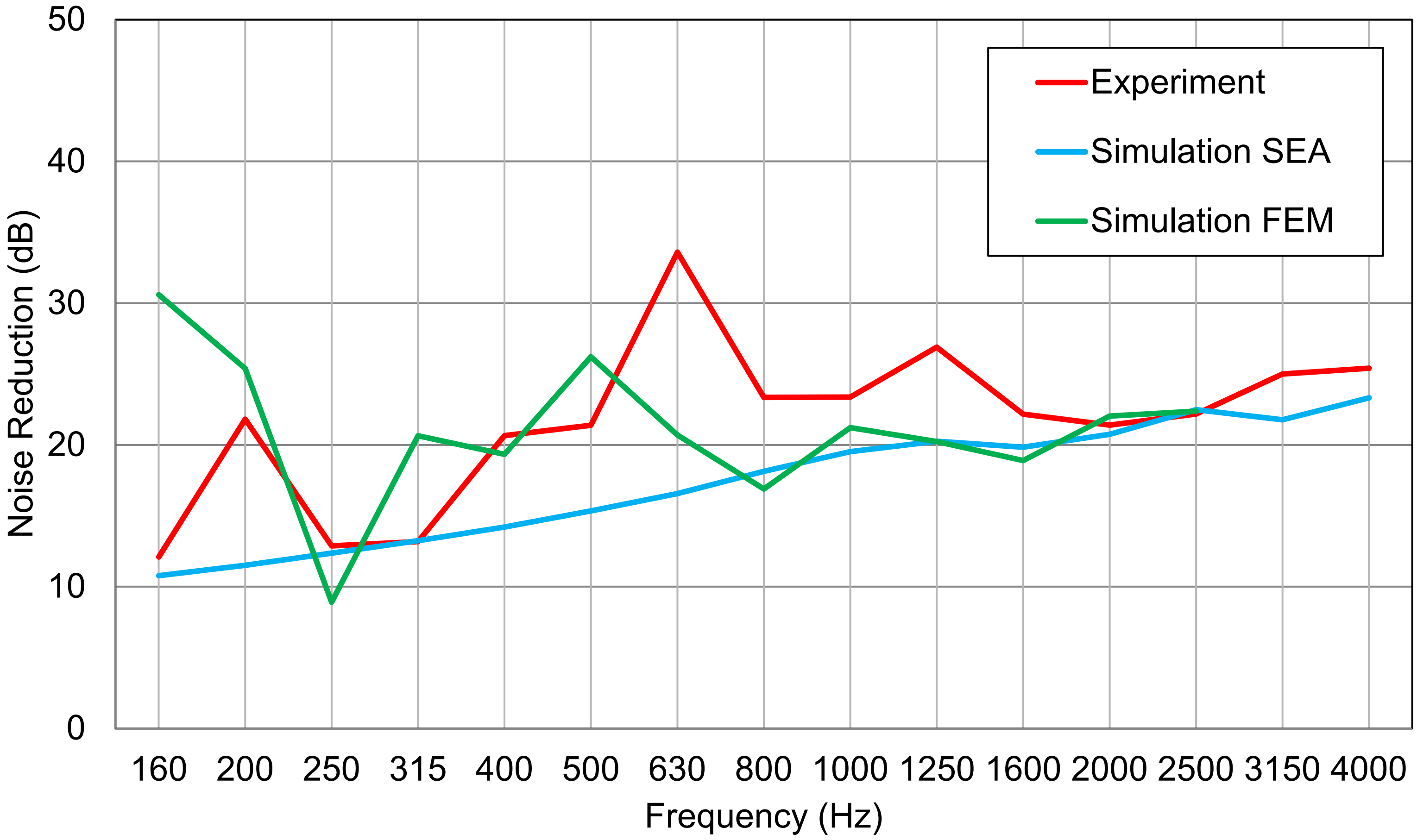


Experiment Approach

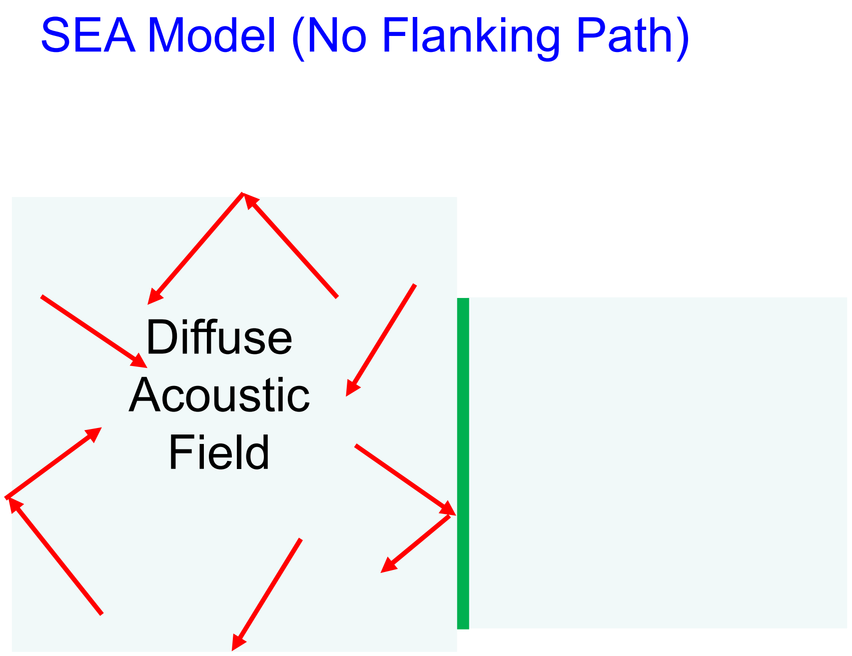
- Loudspeaker source placed in larger box.
- Two microphones roved inside of boxes to identify spatially averaged SPL.

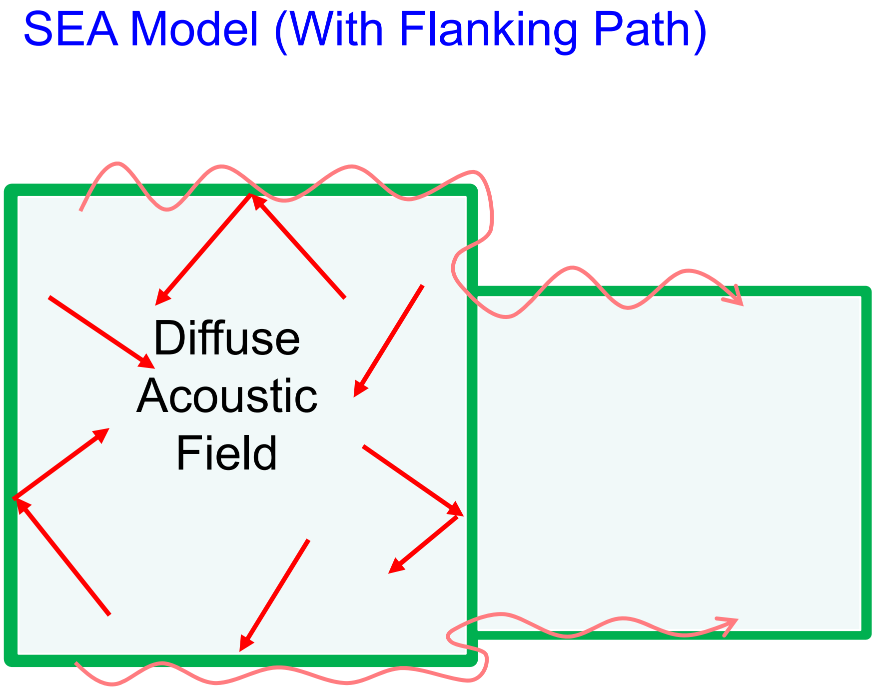


Noise Reduction Comparison

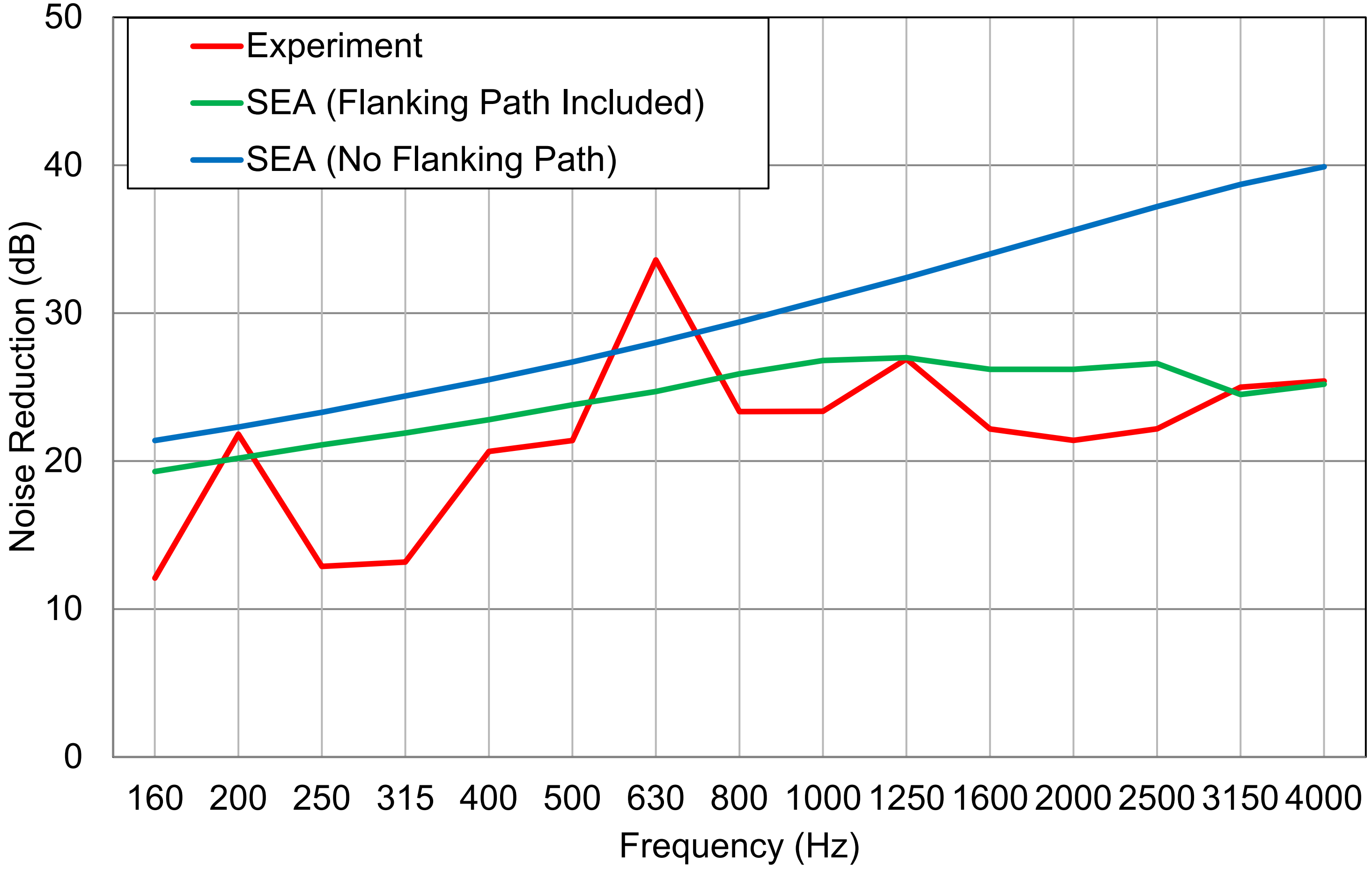


Simulation SEA VA One





Noise Reduction Comparison



Overview

- Basic Theory
- Fundamentals
- Measuring SEA Parameters
- Examples

References

- C. B. Burroughs, R. W. Fischer, and F. R. Kern, “An Introduction to Statistical Energy Analysis,” *J. Acoust. Soc. Amer.*, Vol. 101, No. 4, pp. 1779-1789 (1997).
- F. J. Fahy, “Statistical Energy Analysis: A Critical Review,” *Phil. Trans. R. Soc. Lond. A*, Vol. 346, pp. 431-447 (1994).
- R. H. Lyon and R. G. DeJong, *Theory and Application of Statistical Energy Analysis*, RH Lyon Corp. (1998).