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Introduction to Wavelets

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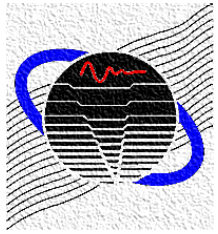
University of Kentucky

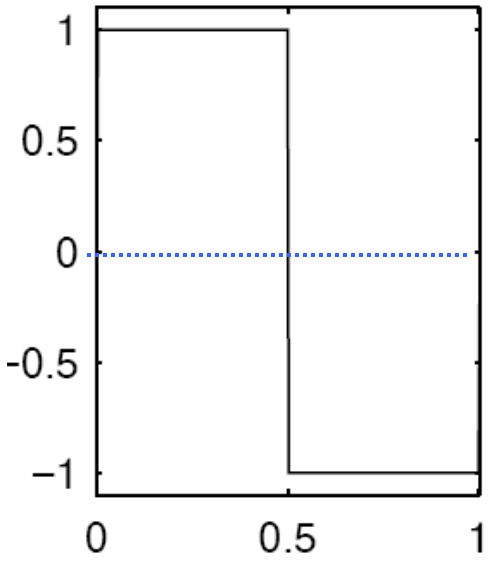


Vibro-Acoustics Consortium

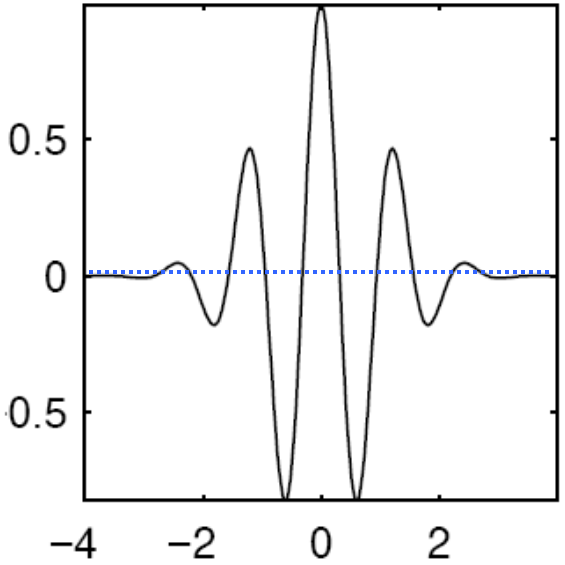
What is a Wavelet?

A waveform of limited duration with an average value of zero (i.e. a small wave).

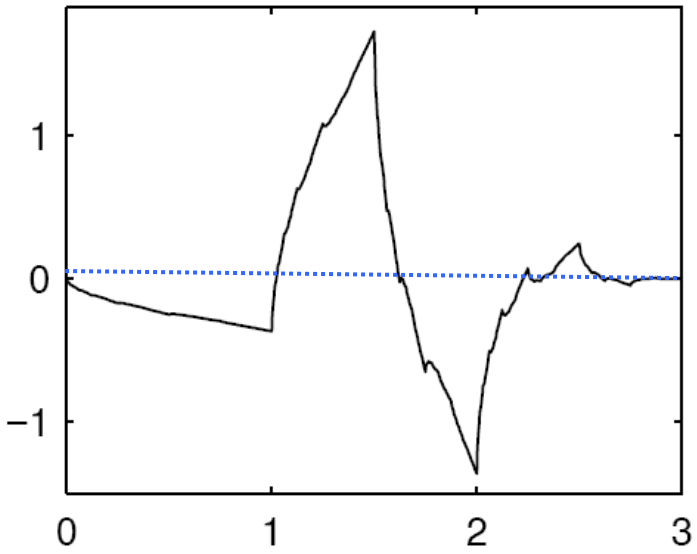
Haar Wavelet



Morlet Wavelet

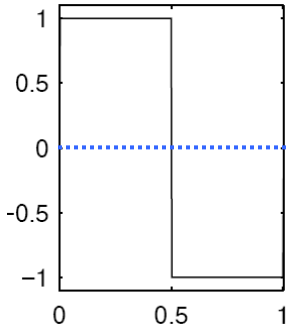


Daubechies 2 (db2)



What is a Wavelet?

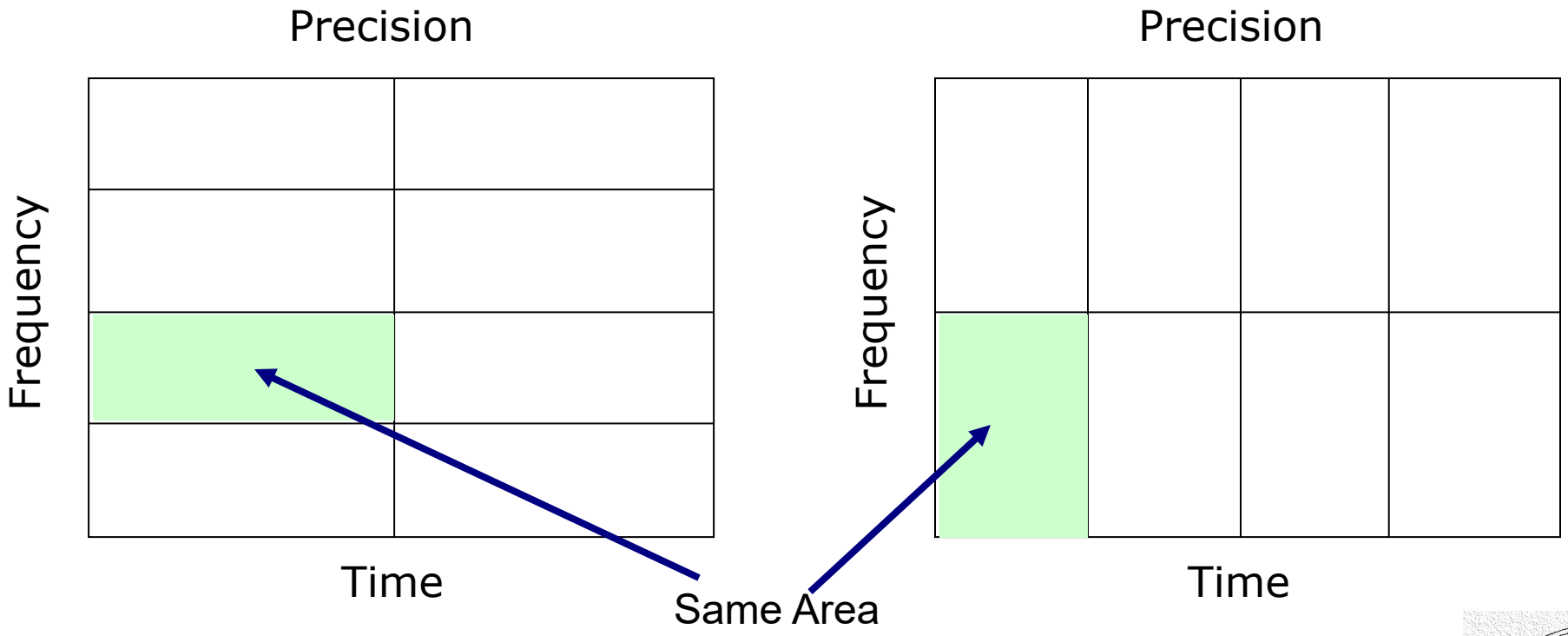
Stretching the Haar Wavelet



+1 -1
+1 +1 -1 -1
+1 +1 +1 +1 -1 -1 -1 -1
+1 +1 +1 +1 +1 +1 +1 +1 -1 -1 -1 -1 -1 -1 -1 -1

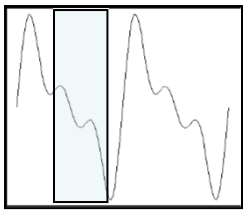
What is a Wavelet?

Heisenberg Uncertainty Principle – It is impossible to know exact frequency content at an exact time.

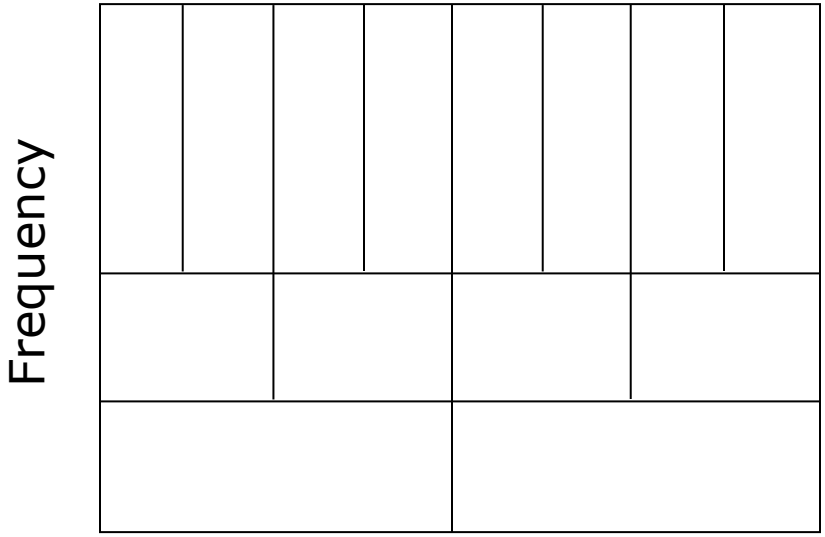


Wavelet Transform

Intuitively

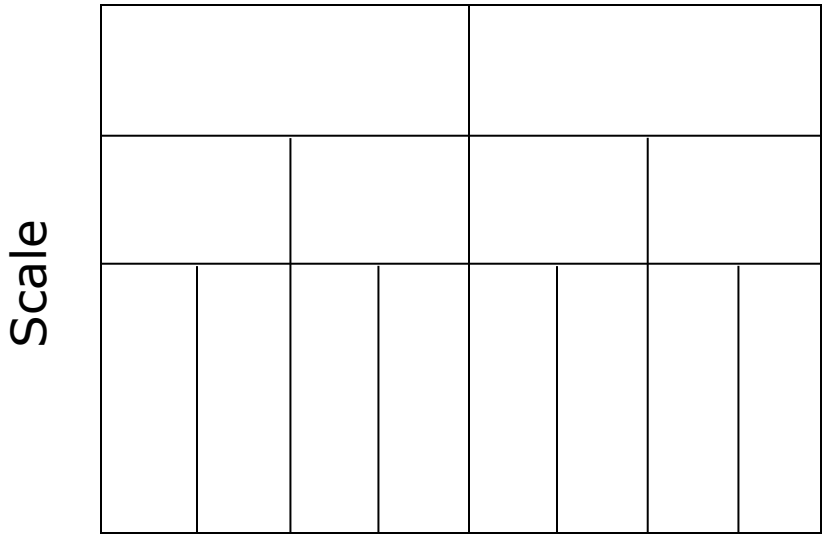


Precision



Wavelet

Precision



Music Analogy (i.e. Scale)

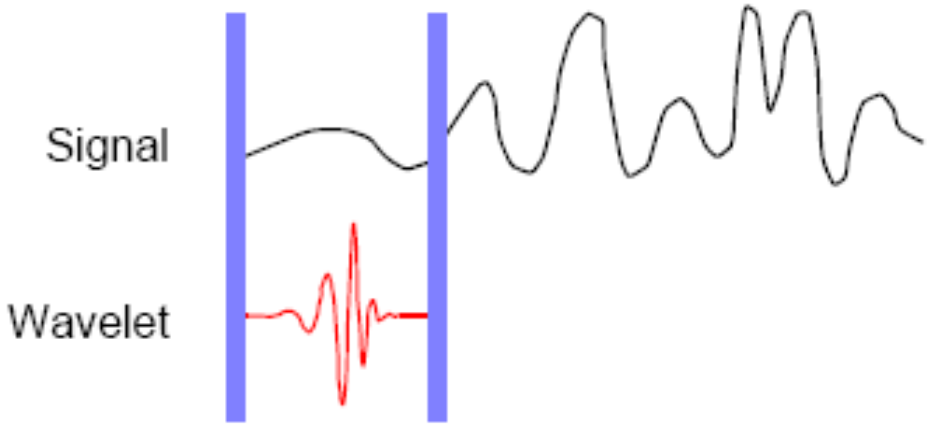
Low notes (low frequency) need longer to be correctly generated while high notes (high frequency) can be generated quickly.

Wavelet Transforms

- Continuous Wavelet Transform
- Discrete Wavelet Transform

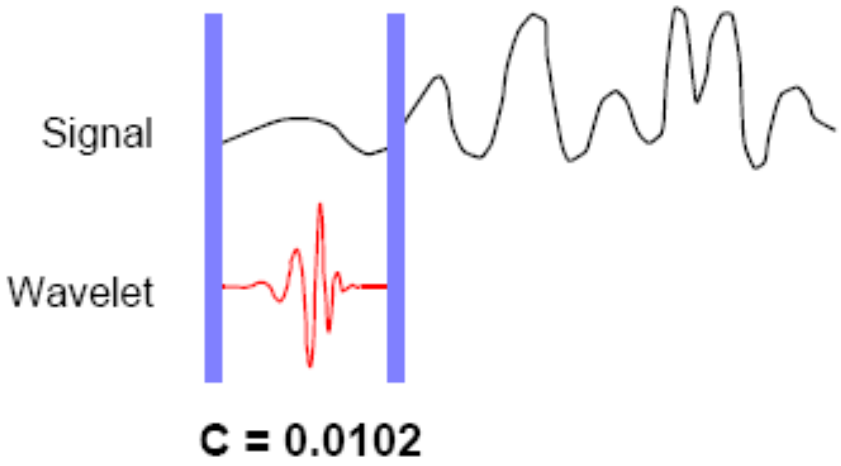
Continuous Wavelet Transform Step 1

Take a wavelet and compare it to a section at the start of the original signal.



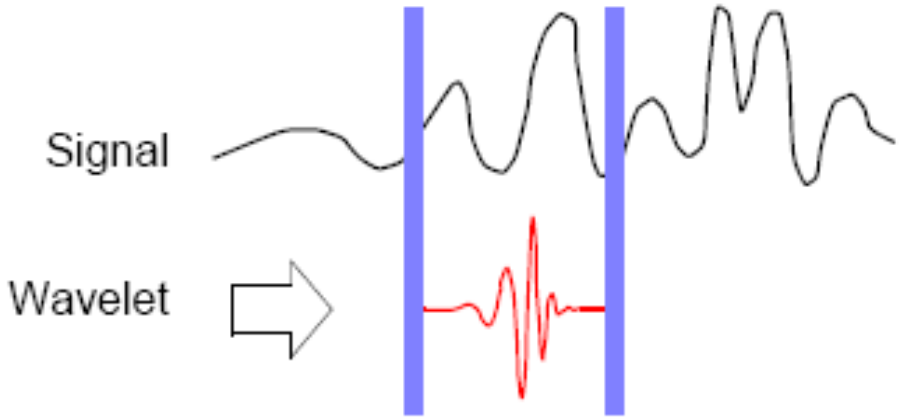
Continuous Wavelet Transform Step 2

Calculate a number C that indicates how closely correlated the wavelet is with this section of the signal. The higher C is the more similarity. Correlation depends partly on the wavelet selected.



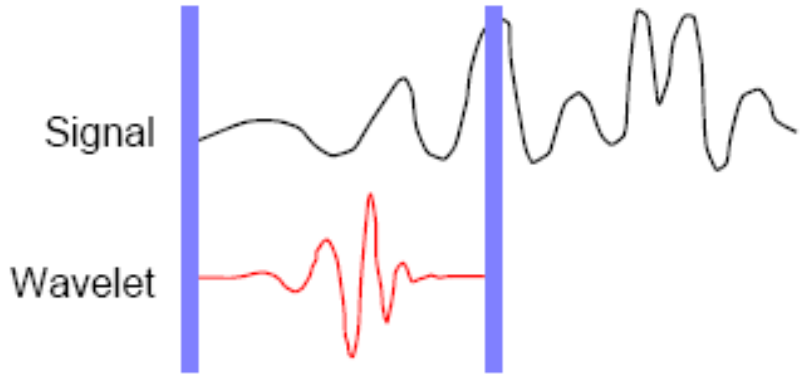
Continuous Wavelet Transform Step 3

Shift wavelet slightly to the right and repeat steps 1 and 2.



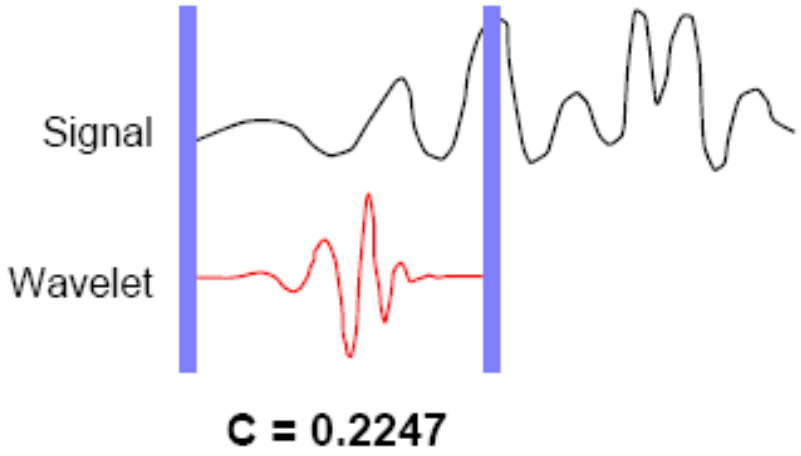
Continuous Wavelet Transform Step 4

Scale (stretch) the wavelet slightly and repeat again.

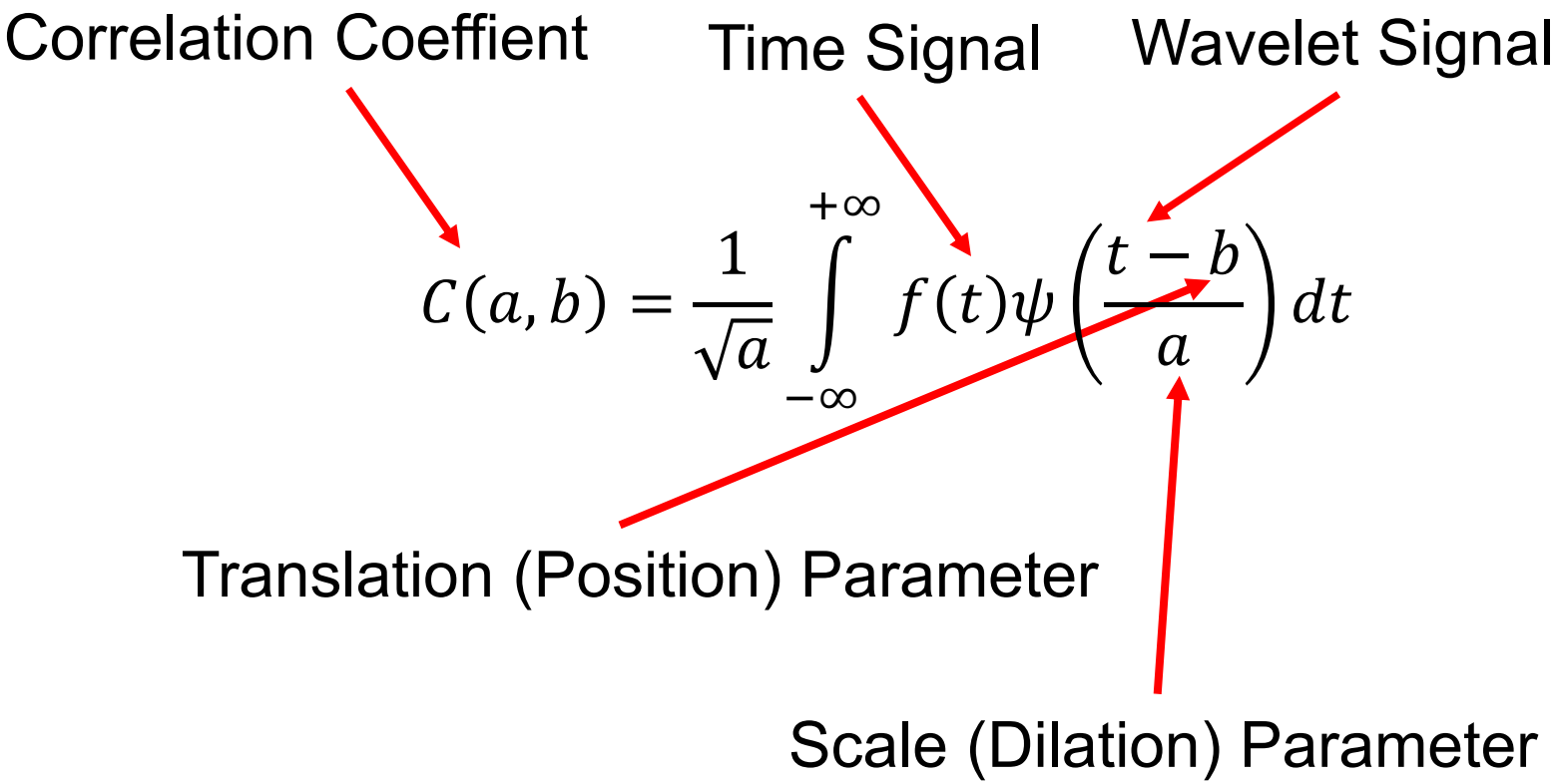


Continuous Wavelet Transform Step 5

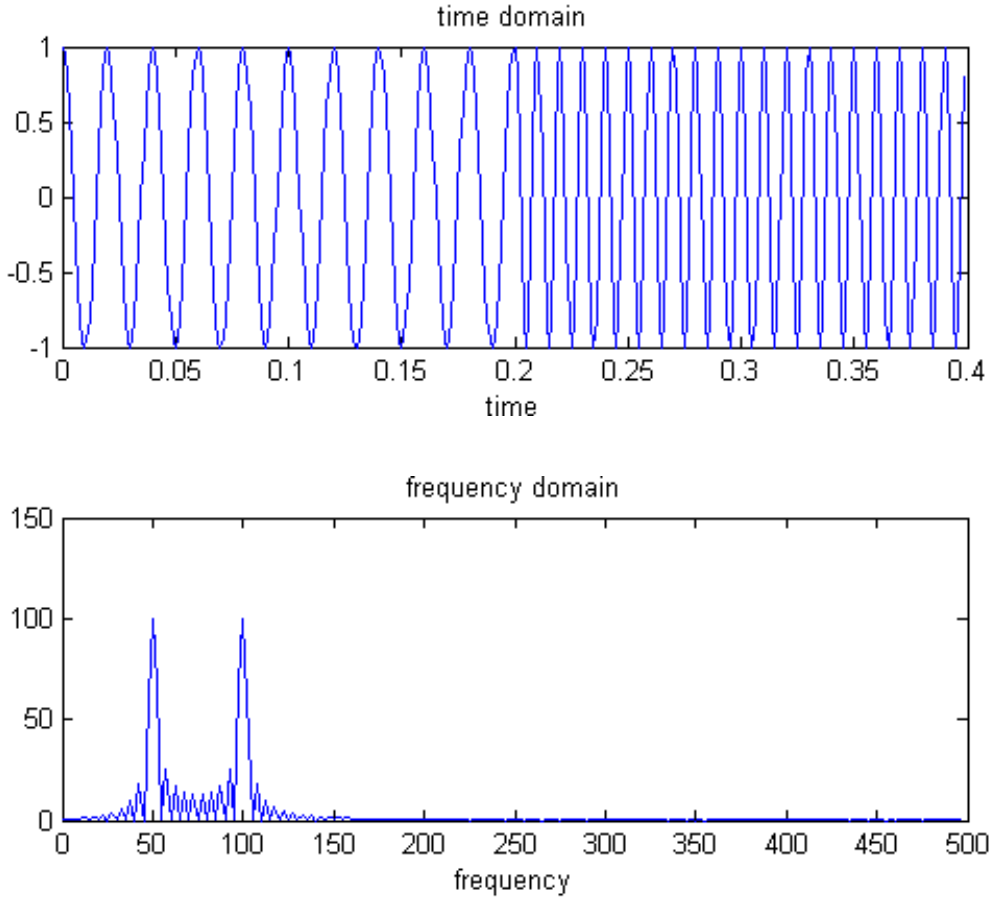
Repeat the process for all scales. If we do this at 10 scales and for 400 time steps, we will have a 400 by 10 set of correlation coefficients. A wavelet that resembles what you are looking for will give a strong correlation at some stretch and shift.



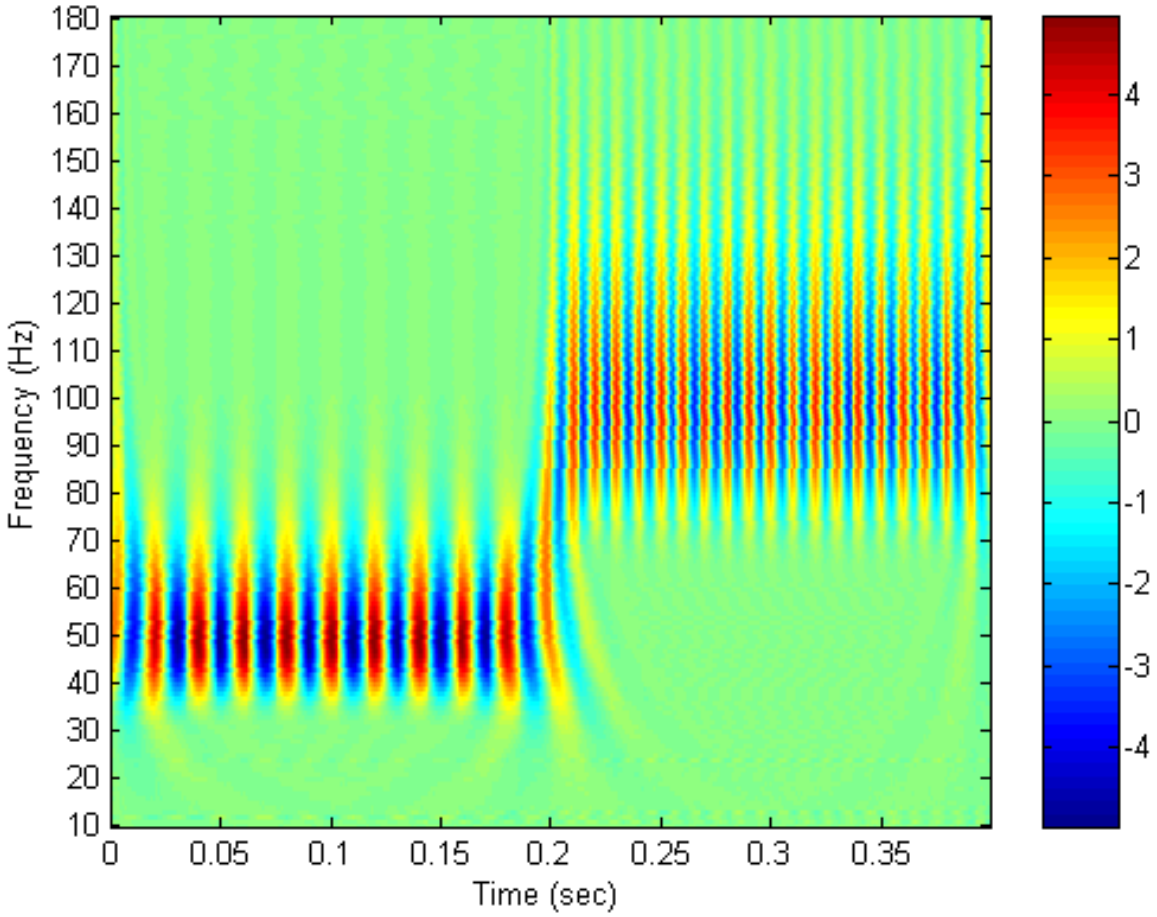
Continuous Wavelet Transform



Example Non-Stationary Signal

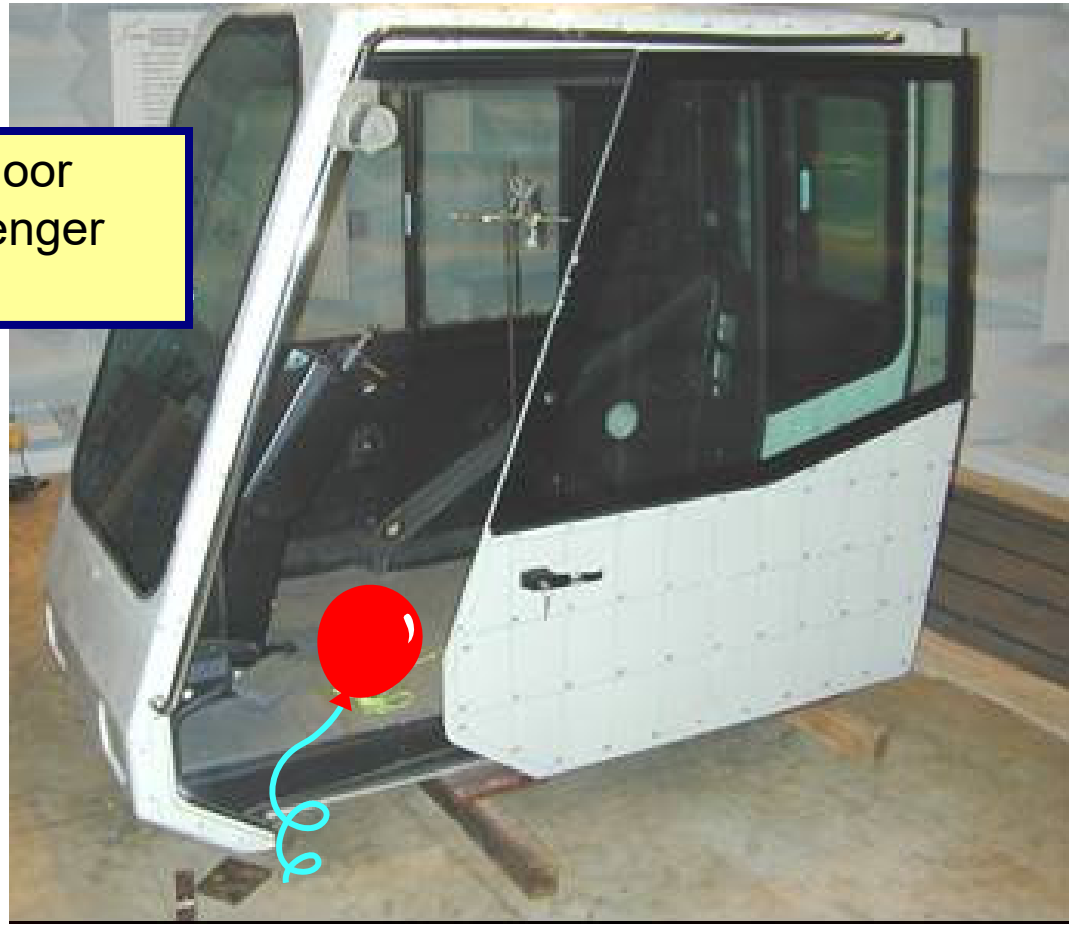


Example Non-Stationary Signal

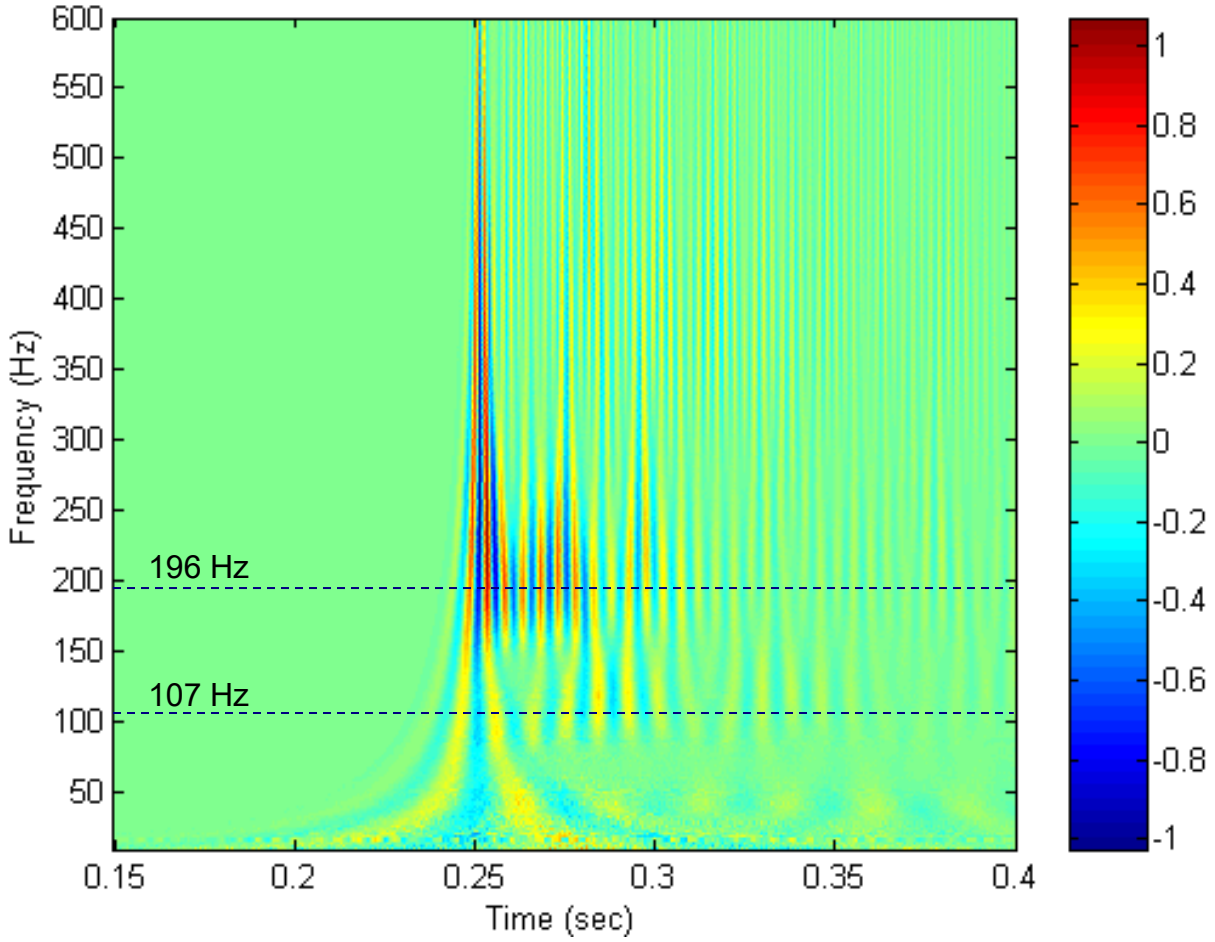


Example Cab Reverberation

Balloon located close to door towards front of the passenger compartment.

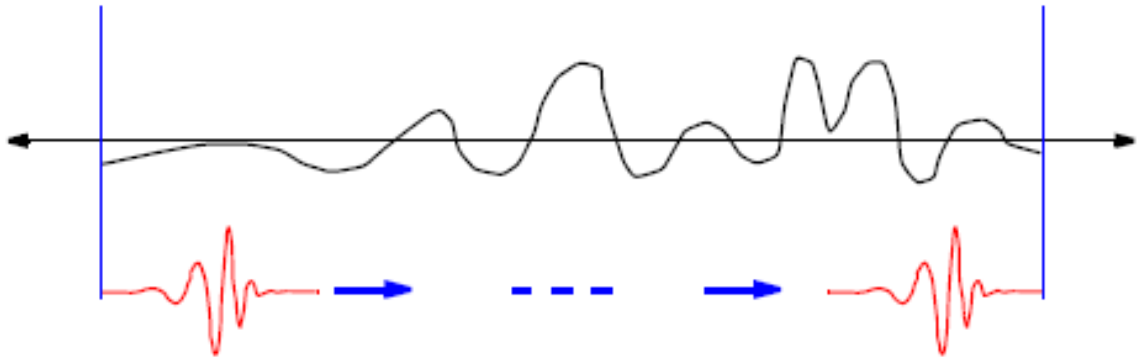


Example Cab Reverberation



Continuous Wavelet Transform

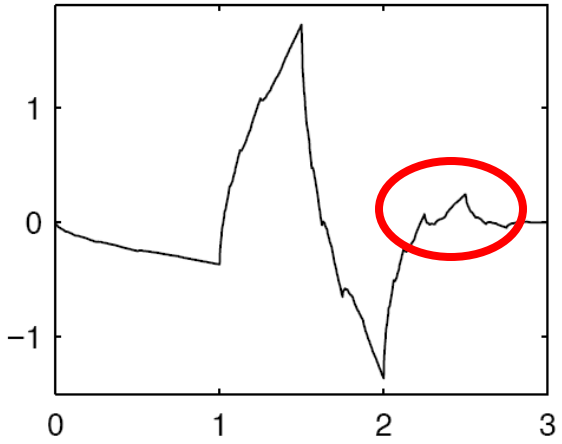
- ✓ Smooth shifting and scaling
- ✓ Use any wavelet you want
- ✓ Inverse CWT is difficult



Discrete Wavelet Transform

- ✓ Signals can be reconstructed after being DWT
- ✓ Scales are powers of 2
- ✓ Wavelets must be constructed from digital filters

Daubechies 2 (db2)



Wavelet Filters

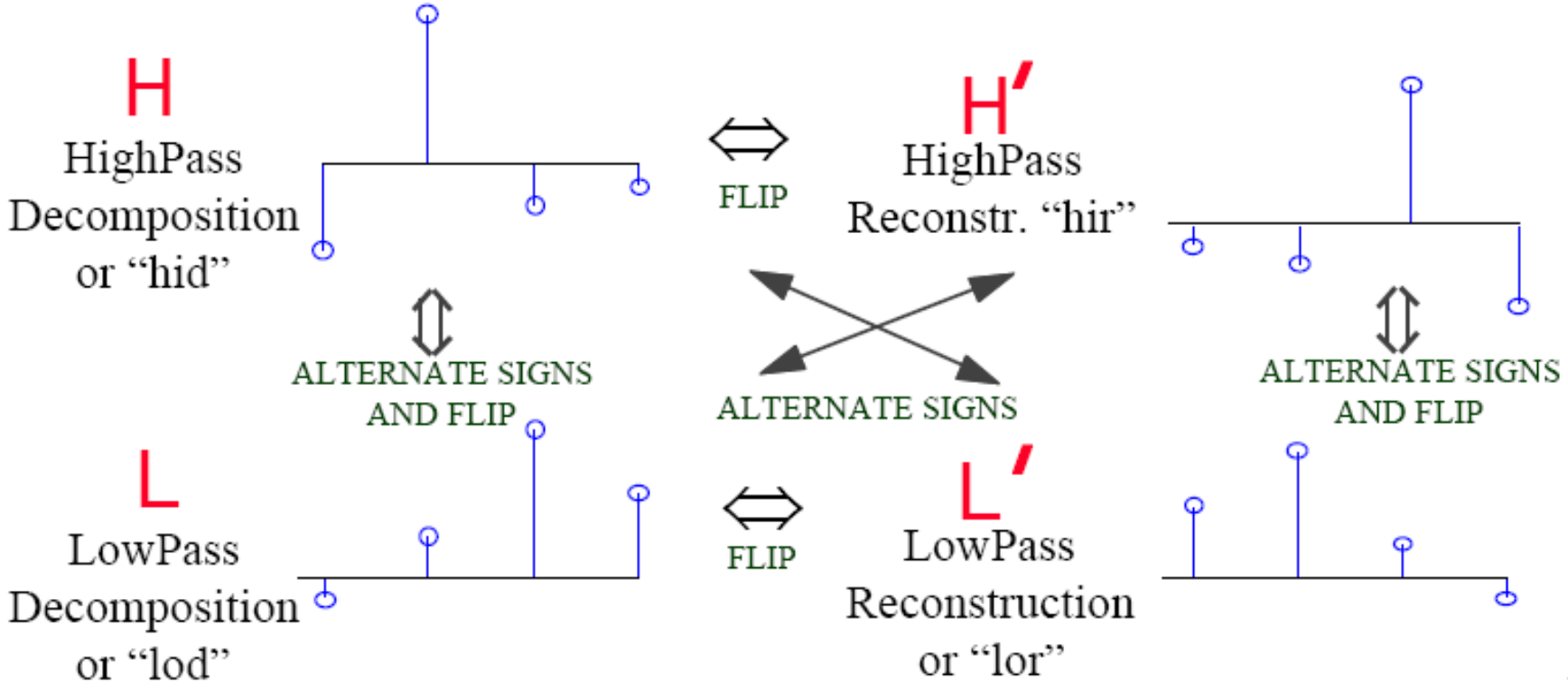
Low Pass Filter

Low Pass Reconstruction Filter

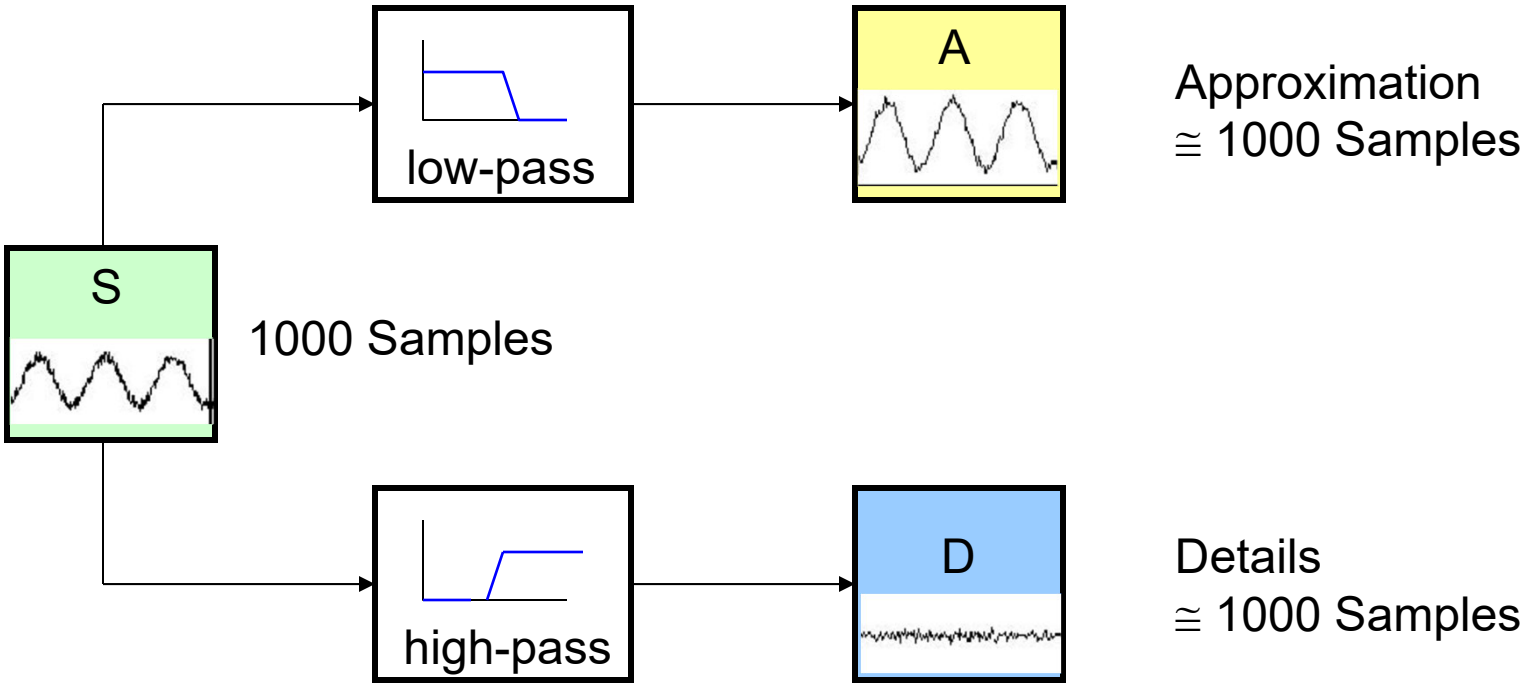
High Pass Filter

High Pass Reconstruction Filter

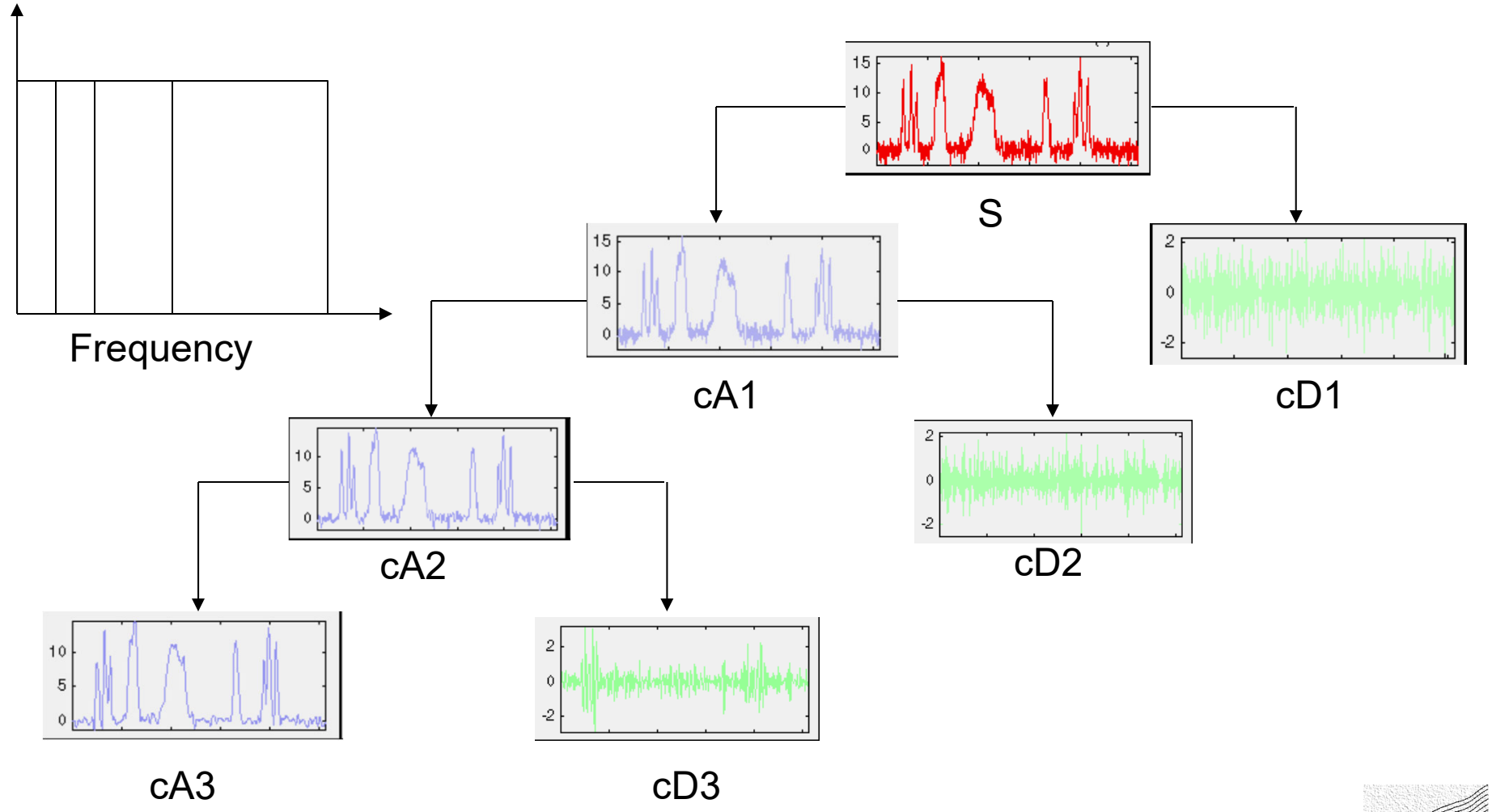
Wavelet Filters



Approximation and Details



Multiple Level Decomposition



Downsampling and Upsampling

Original Signal

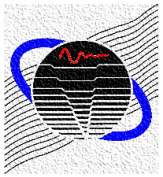
0.12 0.15 0.18 0.15 0.12 0.09 0.06 0.03 0.00 0.03 0.05 0.09 0.12

Downsampled Signal (remove every other data point)

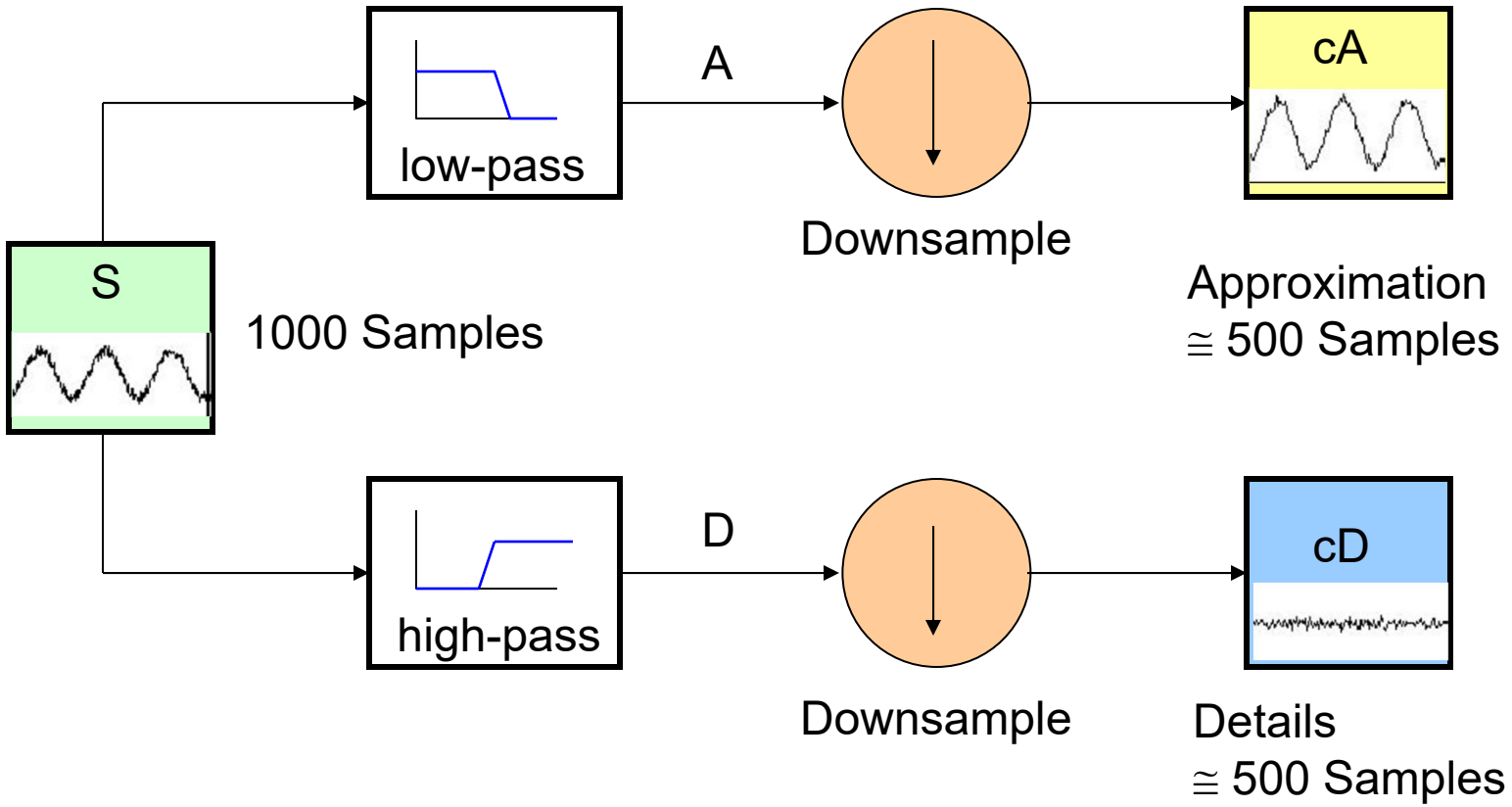
0.12 0.18 0.12 0.06 0.00 0.05 0.12

Upsampled Signal (insert 0's between every other data point)

0.12 0.00 0.18 0.00 0.12 0.00 0.06 0.00 0.00 0.00 0.05 0.00 0.12

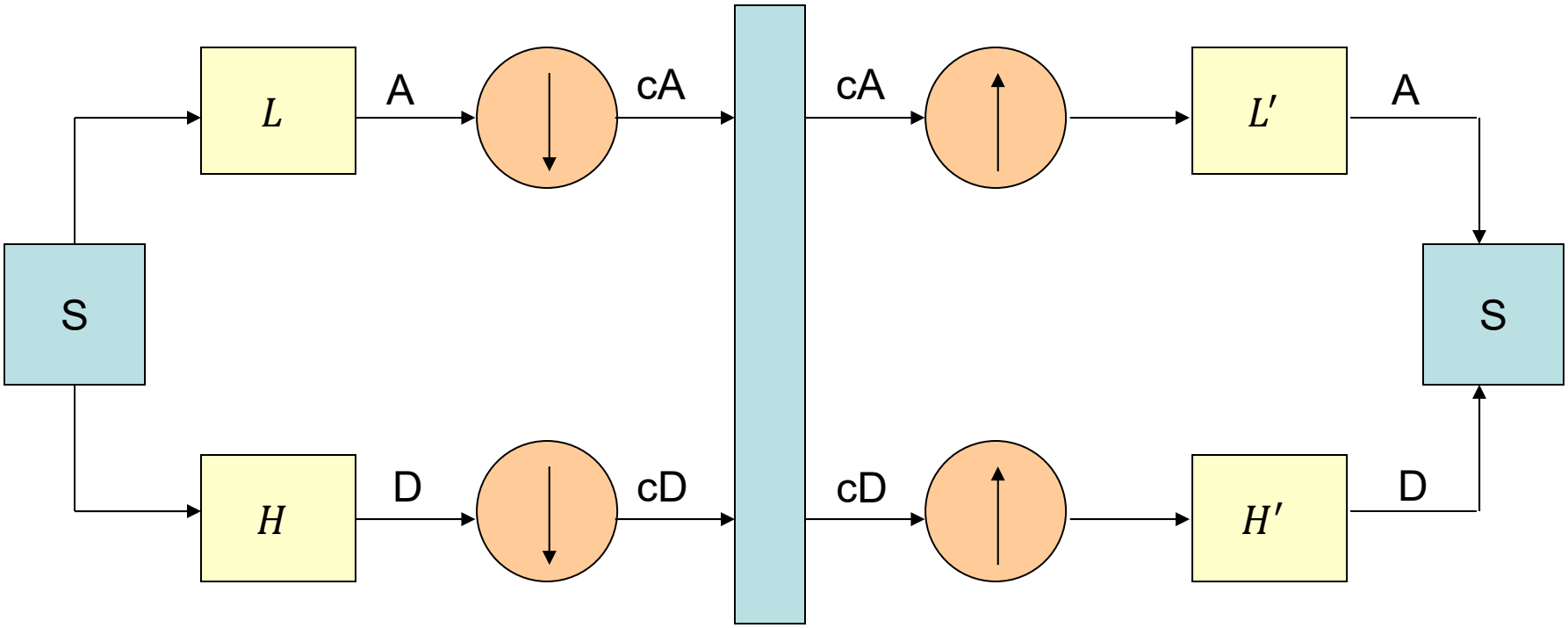


With Wavelet Filters



Downsample – Throw away every other term in the data

Discrete Wavelet Transform



Wavelet Transforms

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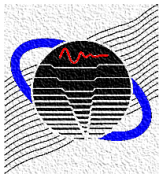
- Discrete Wavelet Transform



Wavelets Reference

D. L. Fugal, Conceptual Wavelets in Digital Signal Processing: an In-Depth, Practical Approach for the Non-Mathematician, Space and Signals Technical Publishing, San Diego (2009).