Overview of Prasad's 1987 paper on "A Four Load Method for Evaluation of Acoustical Source Impedance in a Duct"

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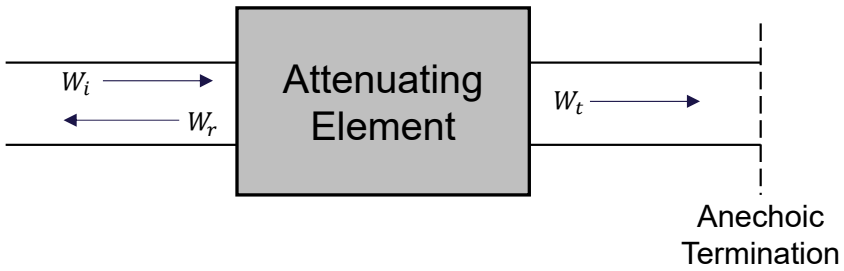
Background
Transmission Loss and Insertion Loss

Transmission Loss (TL)

$$TL = 20\log_{10}\left(\frac{W_i}{W_t}\right)$$

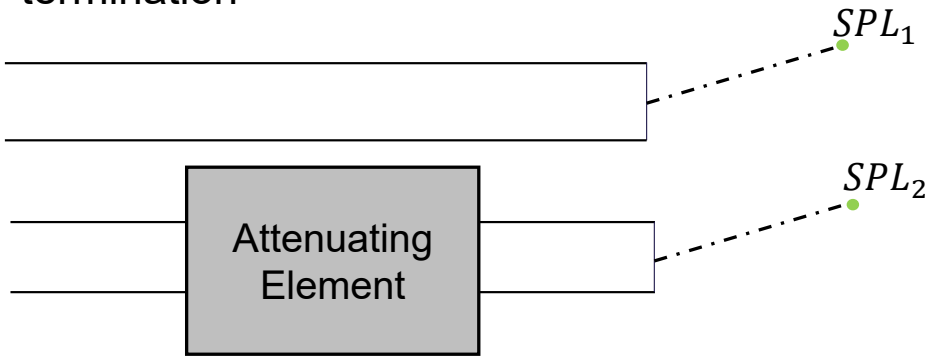
- Property of the attenuating element
- May not reflect the attenuating element's performance in an actual system
- Easily predicted using plane wave, FE or BE analysis



Insertion Loss (IL)

$$IL = SPL_1 - SPL_2$$

- Property of the system
- Typically measured
- Preferred method for defining performance
- Prediction requires knowledge of the characteristics of the source and termination

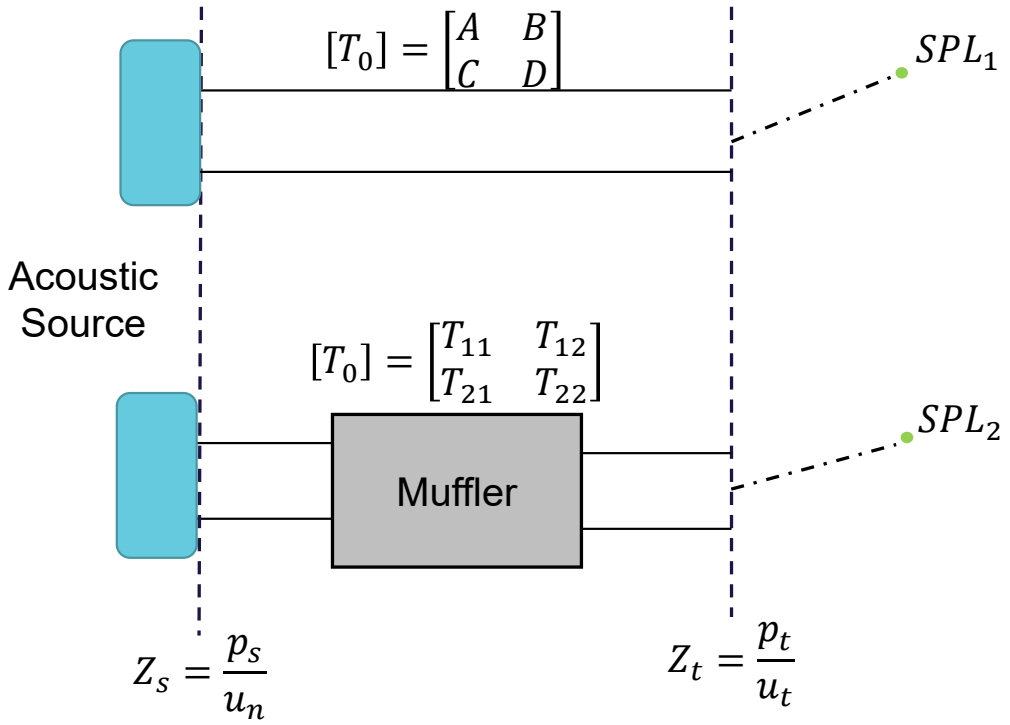


Background
Prediction of Insertion Loss

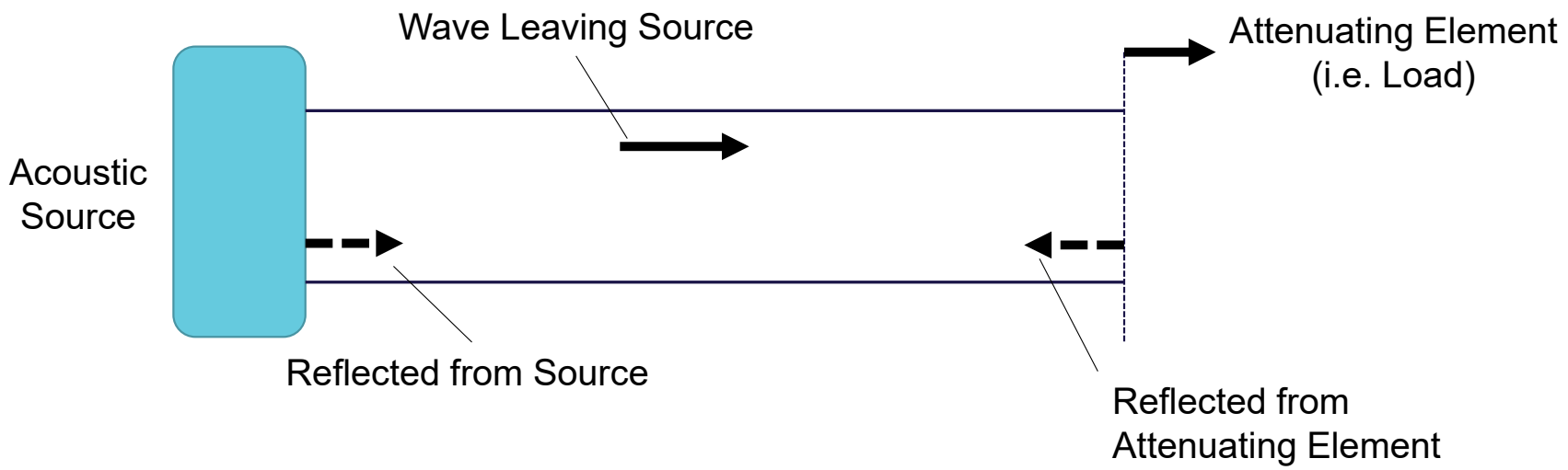
- Analytical analysis is possible, but the source and terminations must be accurately simulated.
- For plane wave analysis (Prasad and Crocker, 1981)

$$IL = 20\log_{10}\left|\frac{T_{11}Z_t + T_{12} + T_{21}Z_tZ_s + T_{22}Z_s}{A_{11}Z_t + A_{12} + A_{21}Z_tZ_s + A_{22}Z_s}\right|$$

- Termination (Z_t) impedance can be estimated using handbook equations.
- Source impedance (Z_s) has no mathematical solution



Background
Source Impedance



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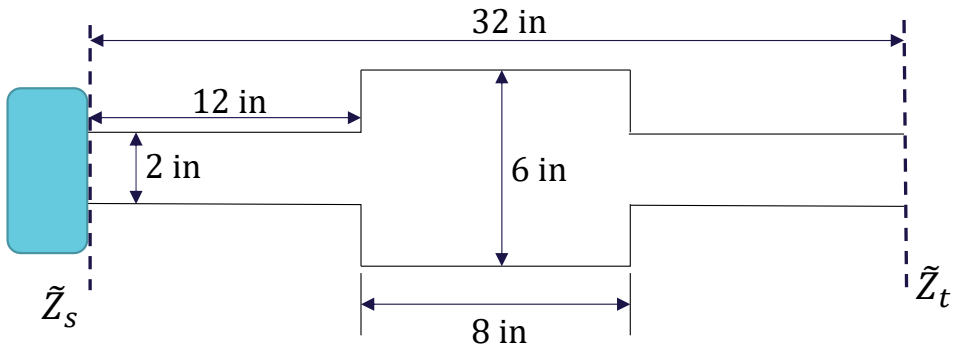
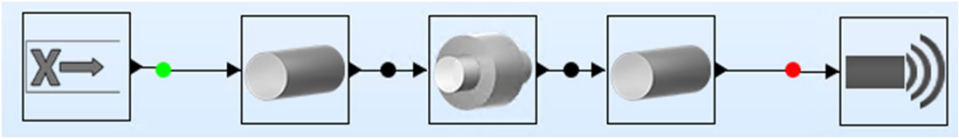
Source Impedance



Case 1: Anechoic Source

$$Z_s = \frac{\rho c}{S}$$

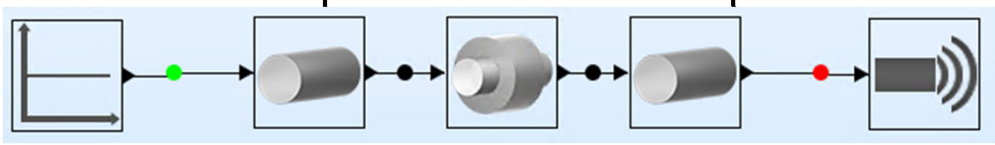
$$Z_t = \left[\frac{1}{4}(ka)^2 + j0.6133ka\right]\frac{\rho c}{S}$$



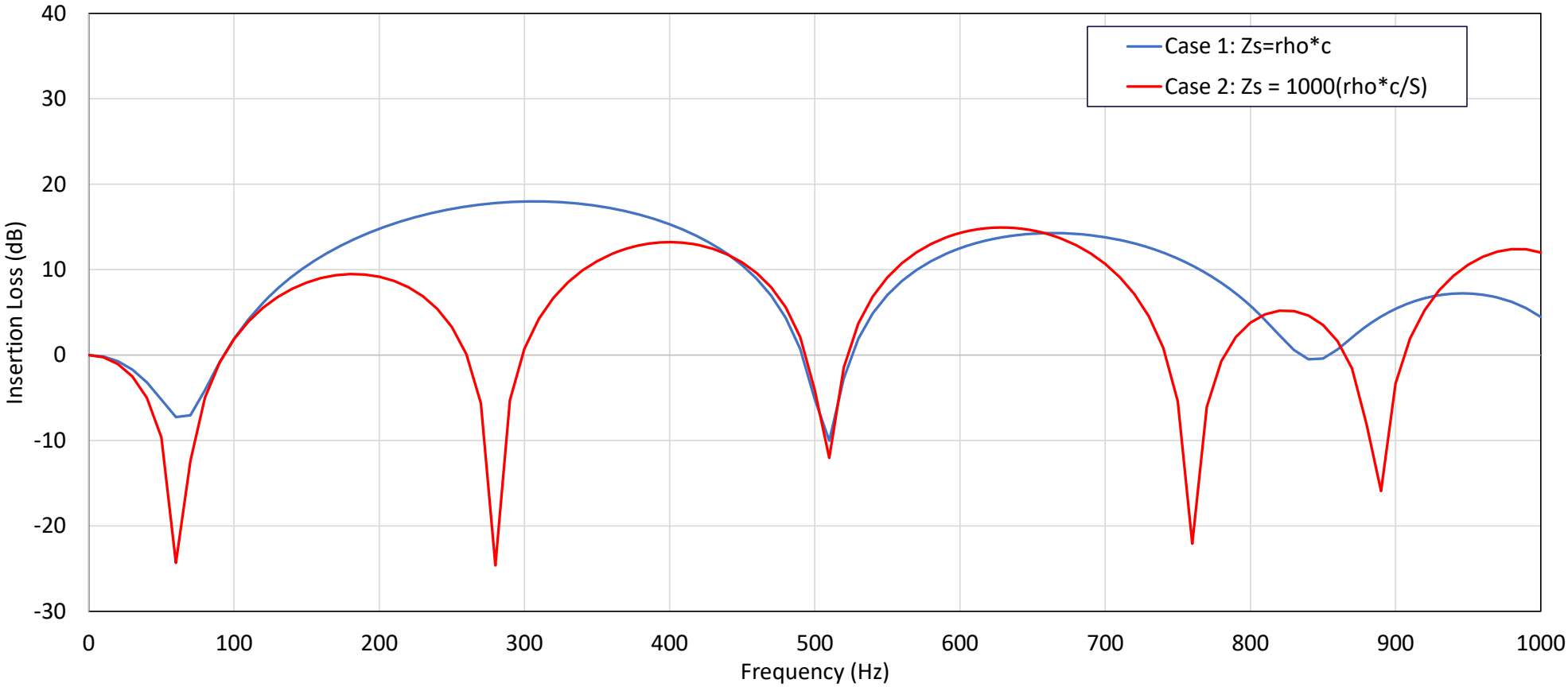
Case 2: Large Source Impedance

$$Z_s = 1000\left(\frac{\rho c}{s}\right)$$

$$Z_t = \left[\frac{1}{4}(ka)^2 + j0.6133ka\right]\frac{\rho c}{S}$$



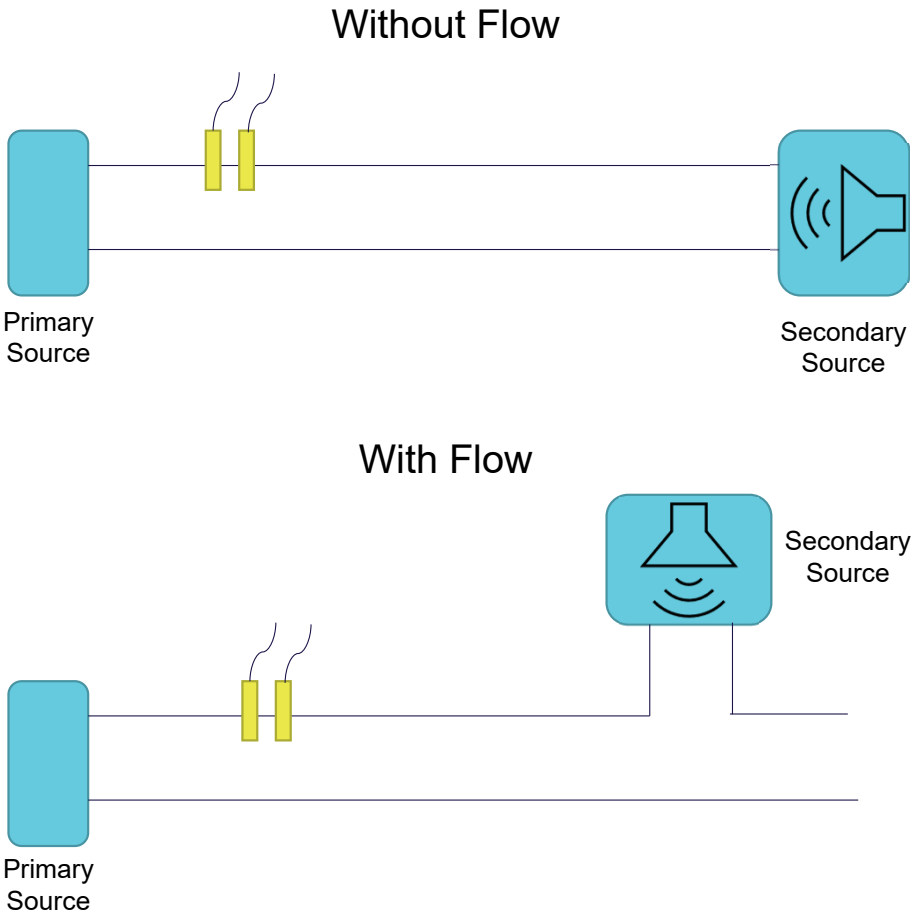
Background
Source Impedance



Background
Direct Measurement of Source Impedance

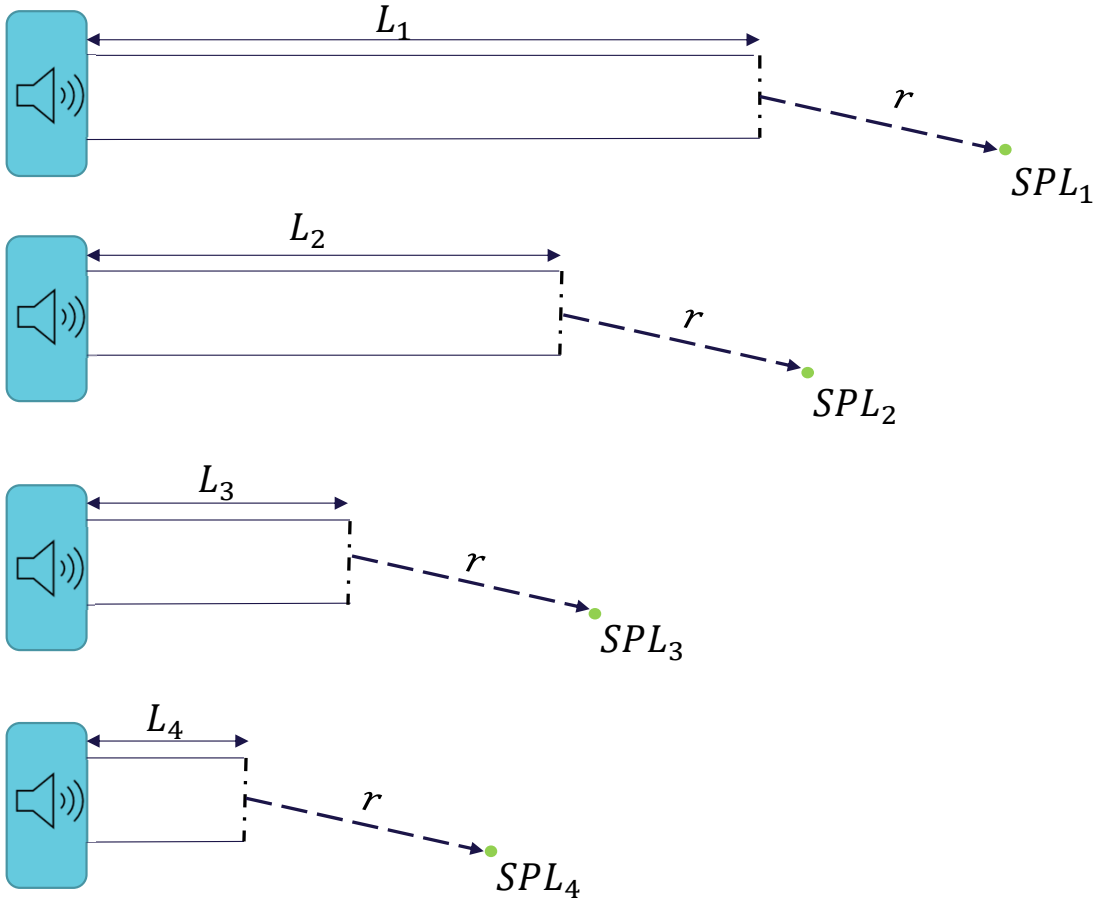
Two-microphone method (Prasad and Crocker, 1983)

- Described in ASTM E-1050 and ISO 10534-2
- Measurements are made inside the duct
- A second source is necessary
- Challenges
  - Microphones must be placed in the flow path
  - A secondary source must be attached to the system and be sufficiently powerful.



Four Load Method of Measuring Source Impedance

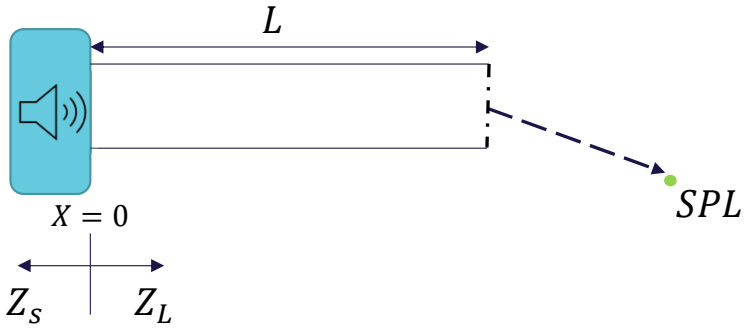
- Measurements are made outside of the duct
- Does not require a secondary source
- Four different straight ducts are used as 4 different loads on the source
- Using measured sound pressure levels outside each duct the source impedance is calculated

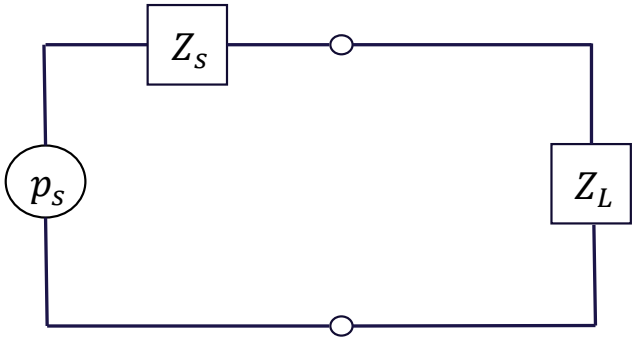


Develop the Four Load Method

- Analogous circuit for a duct of length L connected to a source
  - Source characterized by
    i. Source Pressure (p_s)
    ii. Source Impedance (Z_s)
  - Load characterized by the load impedance (Z_L)
- The pressure at the inlet of the duct is

$$p_L = \left\{ \frac{Z_L}{Z_s + Z_L} \right\} p_s$$



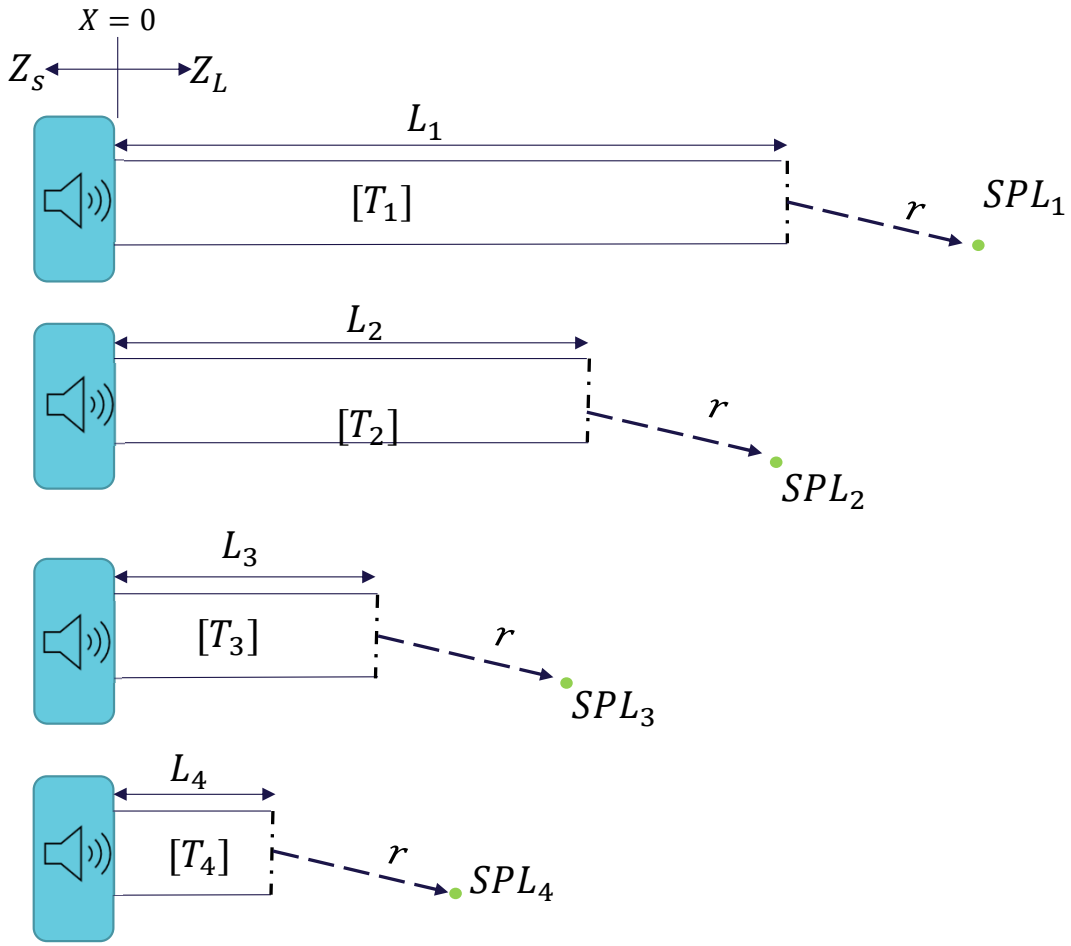


Develop the Four Load Method

- Measure sound pressure levels at a single point outside the duct
- Z_L is defined by the four pole parameters termination impedance (Z_t)
- Assume that Z_s and p_s do not change for the different lengths
- Four loads → Four load pressures → Four equations for Z_s and p_s
- Four unknowns

$$Z_s = R_s + jX_s$$
$$p_s = p_s^+ + jp_s^-$$

$$Z_s = f([T_1], [T_2], [T_3], [T_4], Z_t, \mathrm{SPL}_1, \mathrm{SPL}_2, \mathrm{SPL}_3, \mathrm{SPL}_4)$$



Develop the Four Load Method

Real Part of Source Impedance

$$R_s = \left[\frac{(a_2c_1-a_1c_2)(a_3d_2-a_2d_3)-(a_3c_2-a_2c_3)(a_2d_1-a_1d_2)}{(a_2b_1-a_1b_2)(a_3d_2-a_2d_3)-(a_3b_2-a_2b_3)(a_2d_1-a_1d_2)}\right]$$

Imaginary Part of Source Impedance

$$X_s = \left[\frac{(a_2c_1-a_1c_2)(a_3b_2-a_2b_3)-(a_3c_2-a_2c_3)(a_2b_1-a_1b_2)}{(a_2d_1-a_1d_2)(a_3b_2-a_2b_3)-(a_3d_2-a_2d_3)(a_2b_1-a_1b_2)}\right]$$

$$\alpha_m = \left\{\frac{\left|C_{L_m}Z_t + D_{L_m}\right|}{\left|C_{L_{m+1}}Z_t + D_{L_{m+1}}\right|}\right\} 10^{SPL_m - SPL_{m+1}/20}$$

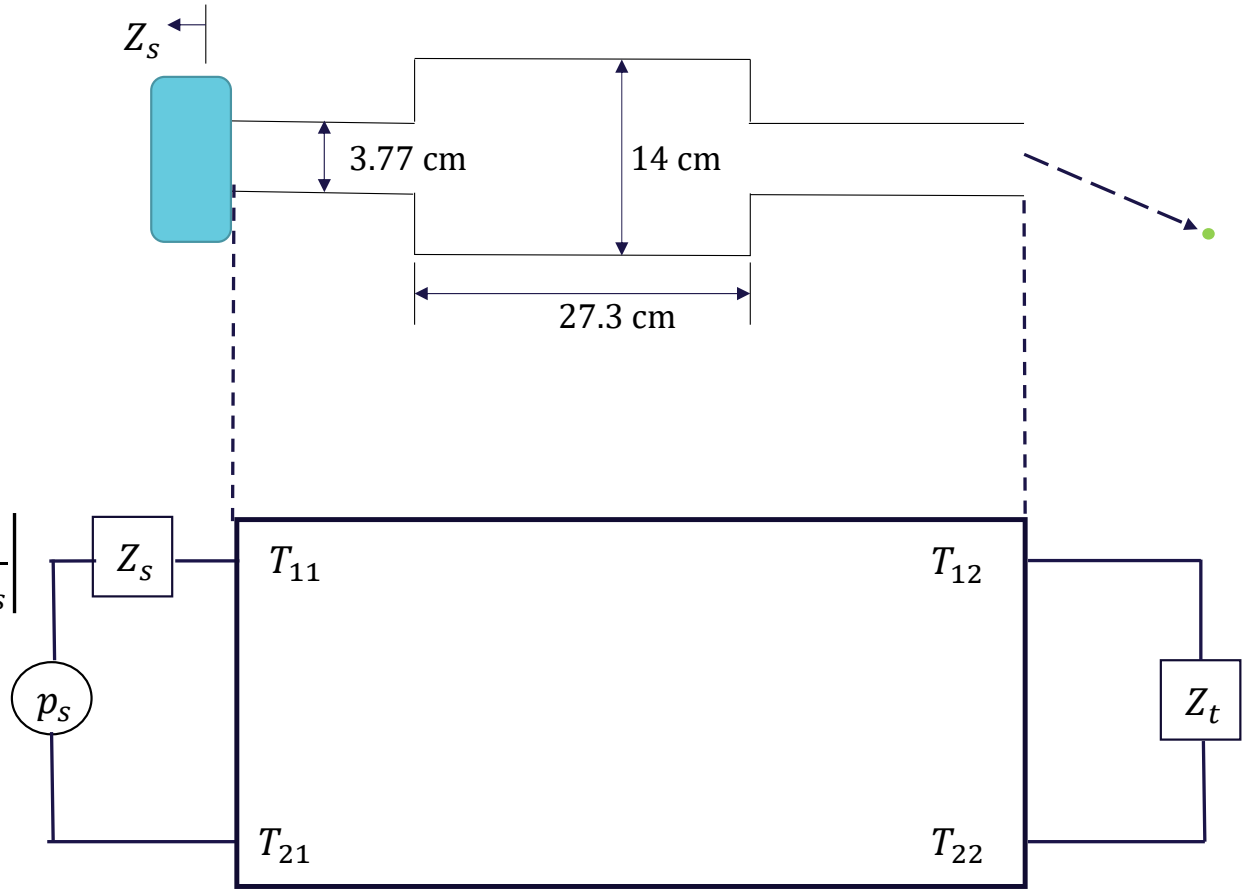
$$a_m = (1-\alpha_m^2) \qquad b_m = 2\left(R_{L_{m+1}} - \alpha_m^2 R_{L_m}\right)$$

$$d_m = 2\left(X_{L_{m+1}} - \alpha_m^2 X_{L_m}\right) \qquad c_m = 2\left(\alpha^2\left(R_{L_m}^2 + X_{L_m}^2\right) - \left(R_{L_{m+1}}^2 + X_{L_{m+1}}^2\right)\right)$$

Validation of Four Load method

- The measured and predicted insertion loss of a simple expansion chamber are compared
- Enclosed loudspeaker is used as the source
- Plane wave analysis is used to predict the IL

$$IL = 20\log_{10}\left|\frac{T_{11}Z_t + T_{12} + T_{21}Z_tZ_s + T_{22}Z_s}{A_{11}Z_t + A_{12} + A_{21}Z_tZ_s + A_{22}Z_s}\right|$$



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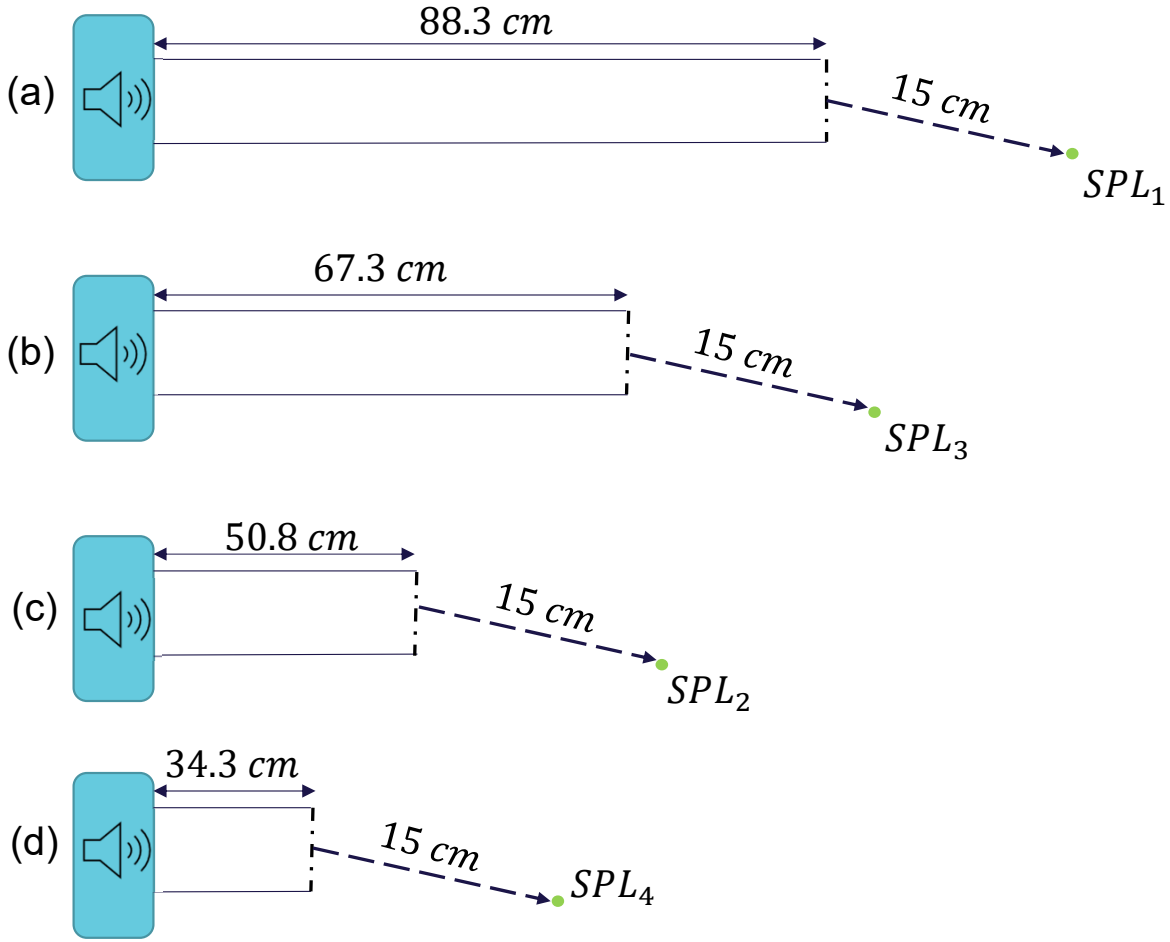

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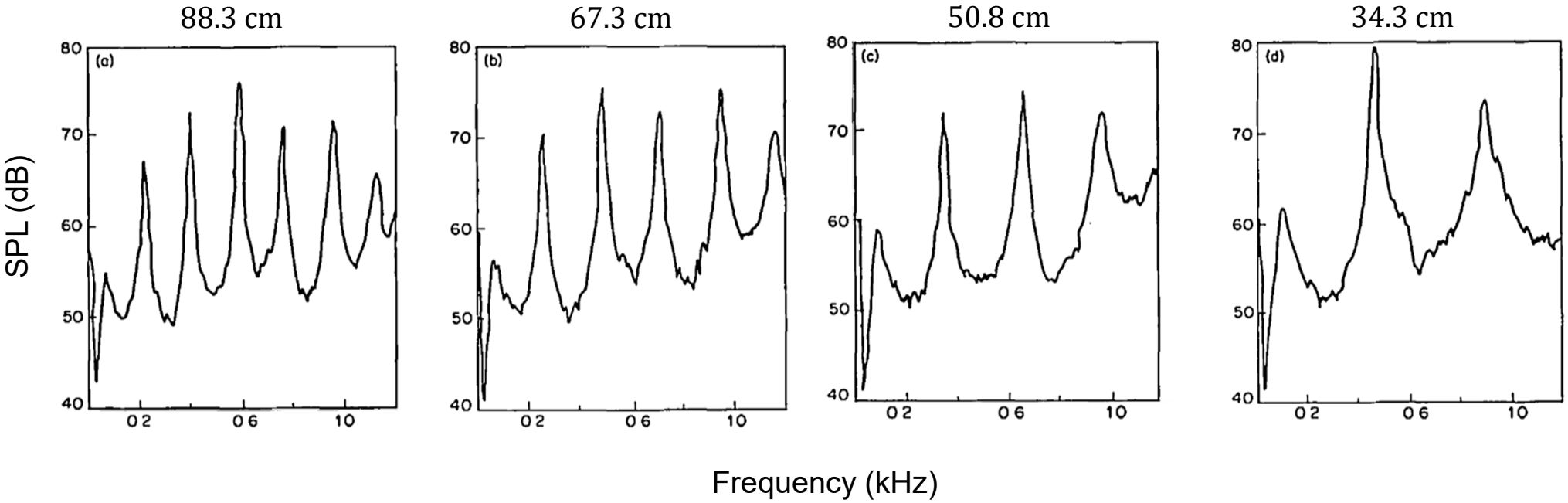

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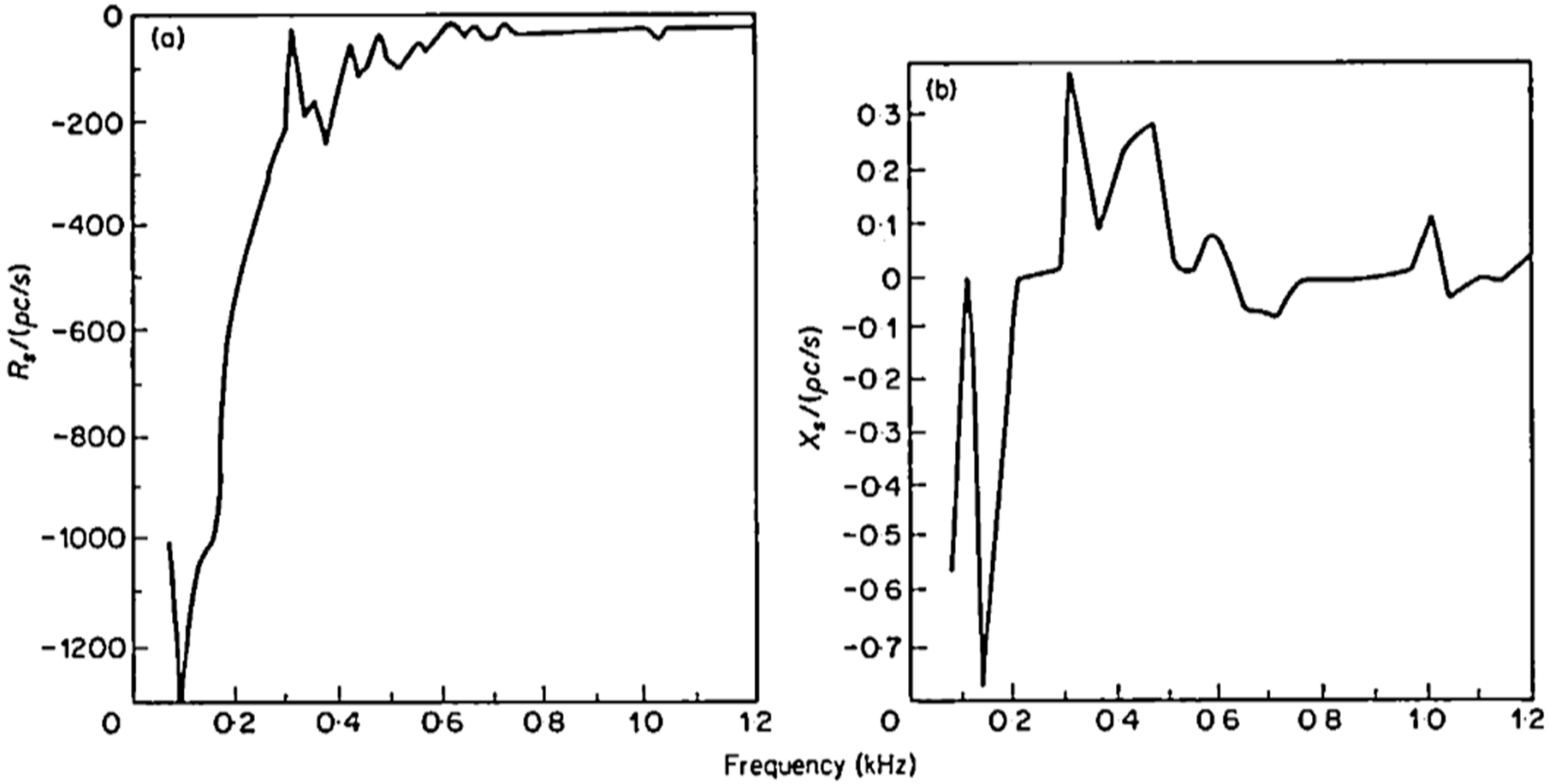

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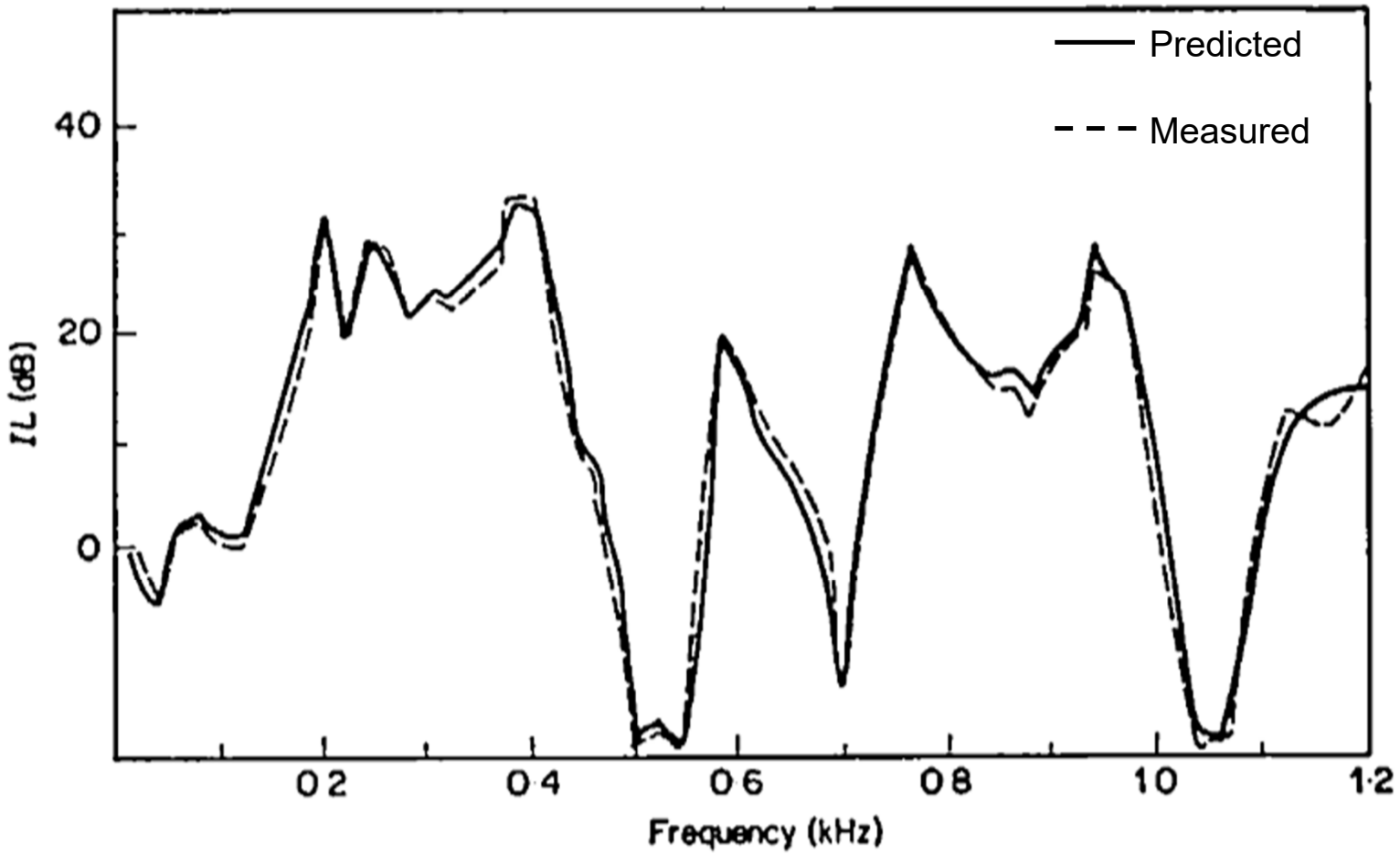

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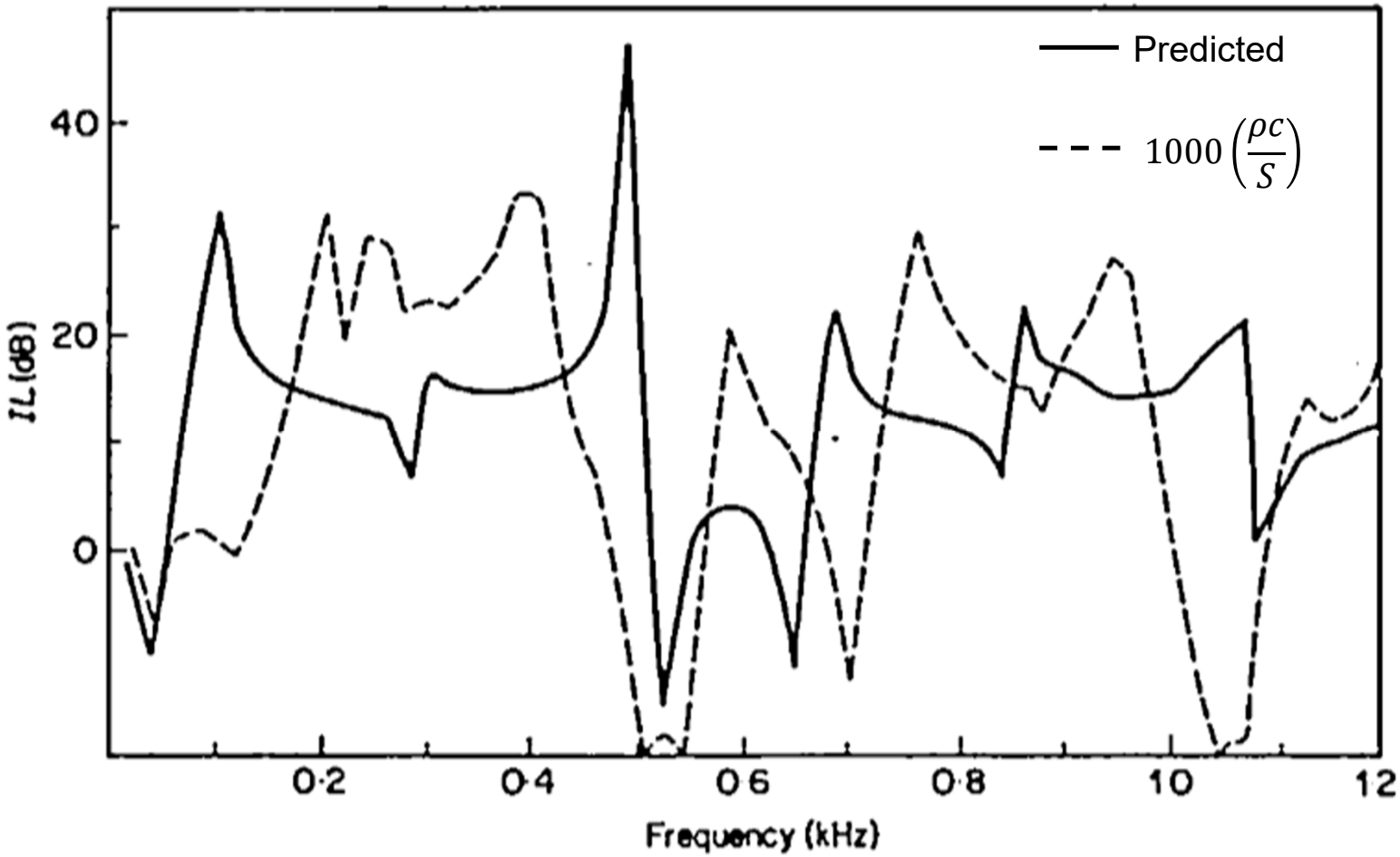

Conclusions and Results

- Developed first multi-load method to determine the source impedance.
- Four-load method is a much easier measurement than the wave decomposition approach.
- Bodén and Åbom built on the Prasad's four load method to develop multi-load methods for evaluating source characteristics.