July 9, 2020

Mufflers and Silencers
Advanced Topics

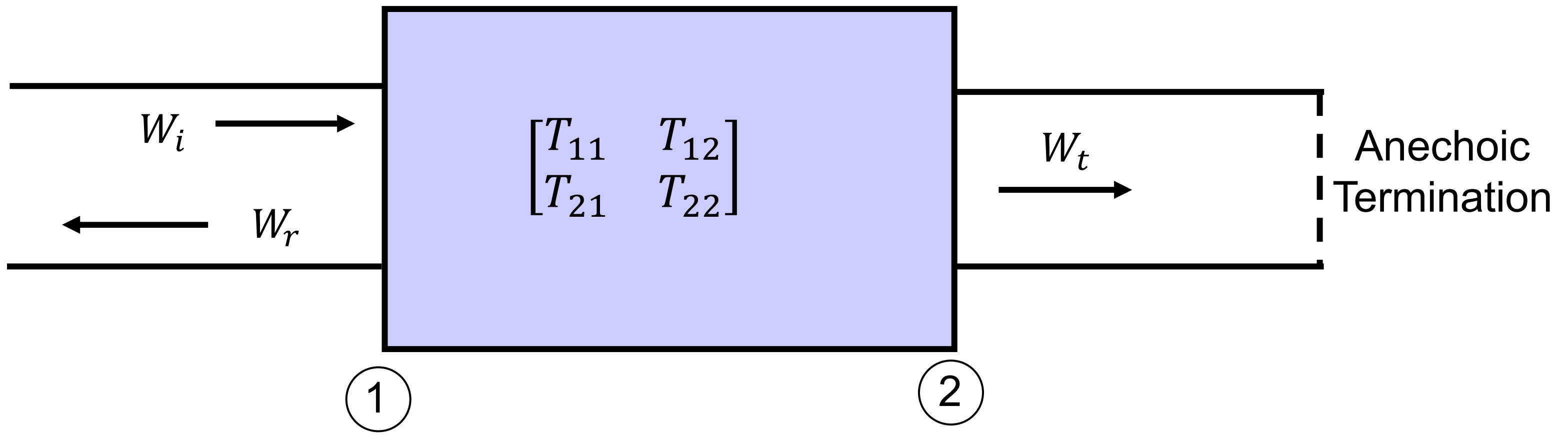
Vibro-Acoustics Consortium Web Meeting

University of Kentucky

Overview

- Muffler Metrics
- Transmission and Insertion Loss
- Source Impedance
- Simulation of Source Impedance
- Multi-Inlet Mufflers

Performance Measures Transmission Loss



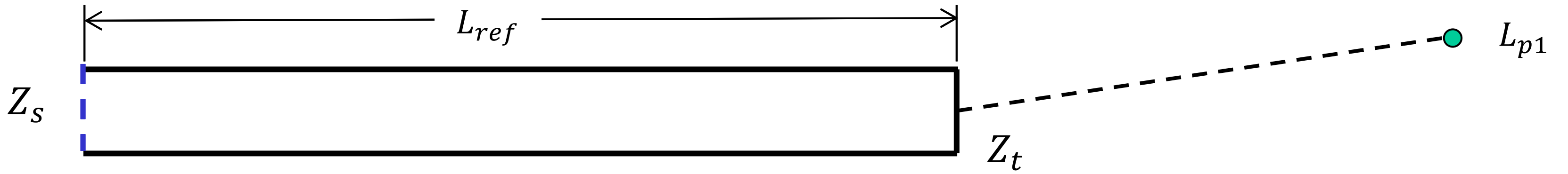
Transmission loss (TL) of the muffler:

$$TL = 10\log_{10}\frac{W_i}{W_t}$$

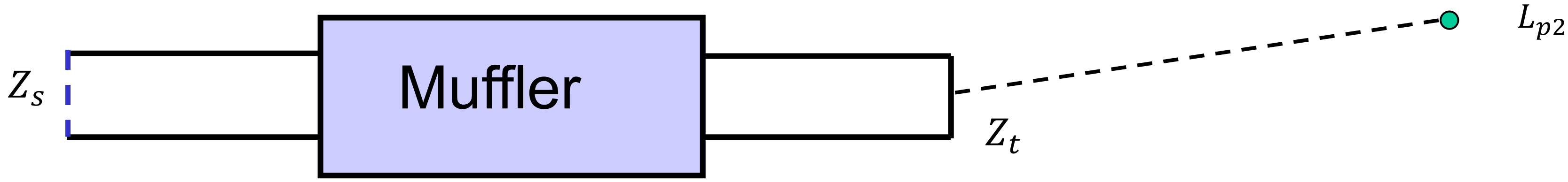
$$TL = 10\log_{10}\left\{\frac{S_{in}}{4S_{out}}\left|T_{11} + \frac{S_{out}T_{12}}{\rho c} + \frac{\rho c T_{21}}{S_{in}} + \frac{S_{out}}{S_{in}}T_{22}\right|^2\right\}$$

Performance Measures Insertion Loss

$$[T_0] = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

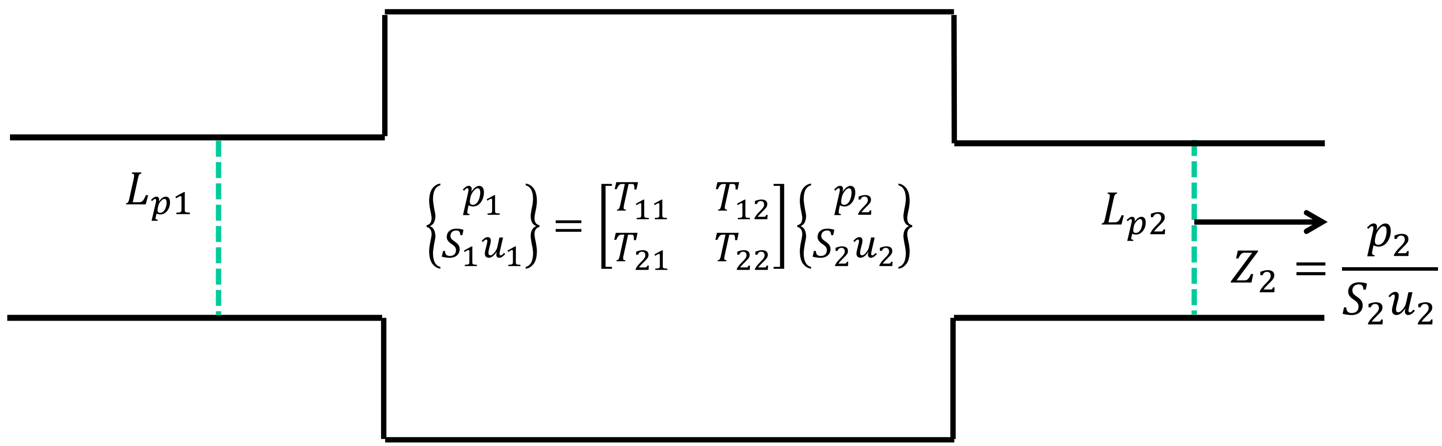


$$[T] = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}$$



$$IL = 20\log_{10}\left\{\left|\frac{\dfrac{T_{11}}{Z_s} + \dfrac{T_{12}}{Z_t Z_s} + T_{21} + \dfrac{T_{22}}{Z_t}}{\dfrac{A_{11}}{Z_s} + \dfrac{A_{12}}{Z_t Z_s} + A_{21} + \dfrac{A_{22}}{Z_t}}\right|\right\}$$

Performance Measures Noise Reduction



$$NR = L_{p1} - L_{p2} = 10 \log_{10} \left| T_{11} + \frac{T_{12}}{Z_2} \right|^2$$

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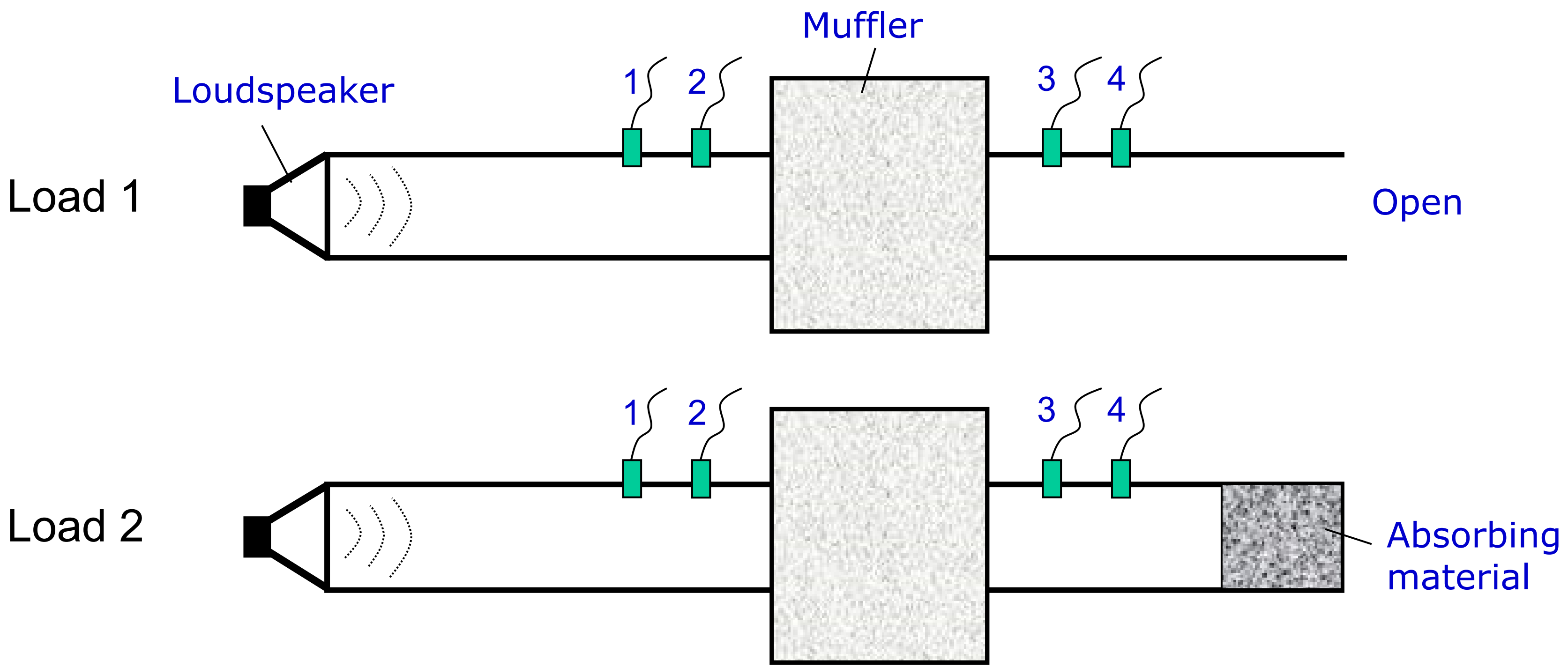
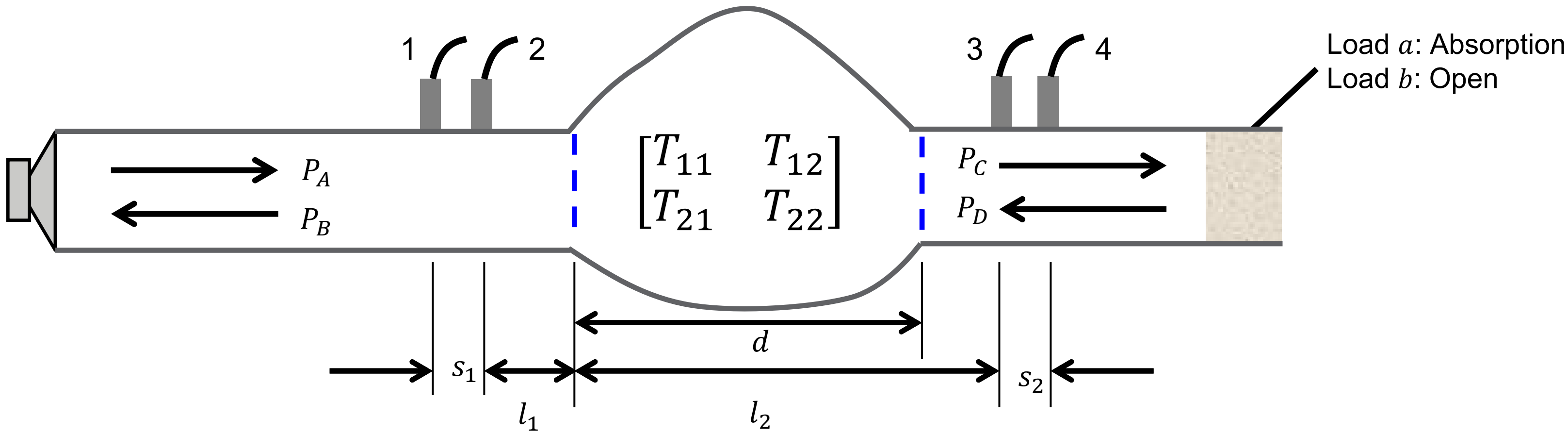
Transmission Loss Measurement

ASTM E2611



Transmission Loss Measurement



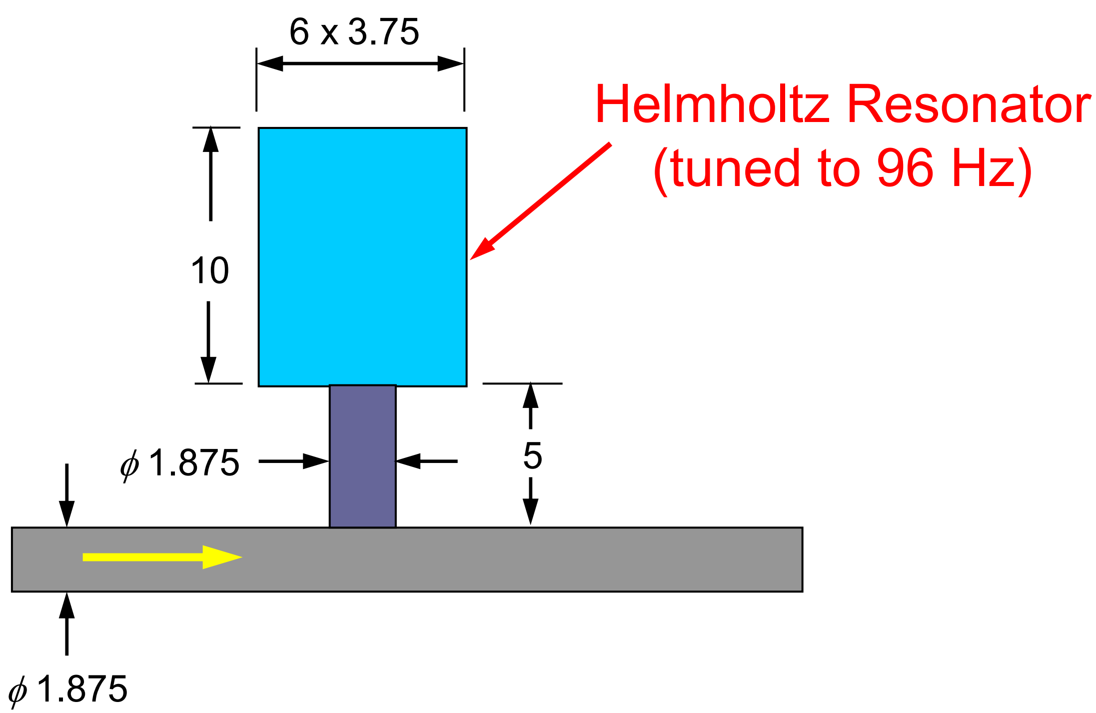
H: Transfer function

$$\begin{Bmatrix} p_1 \\ S_1 u_1 \end{Bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{Bmatrix} p_2 \\ S_2 u_2 \end{Bmatrix}$$

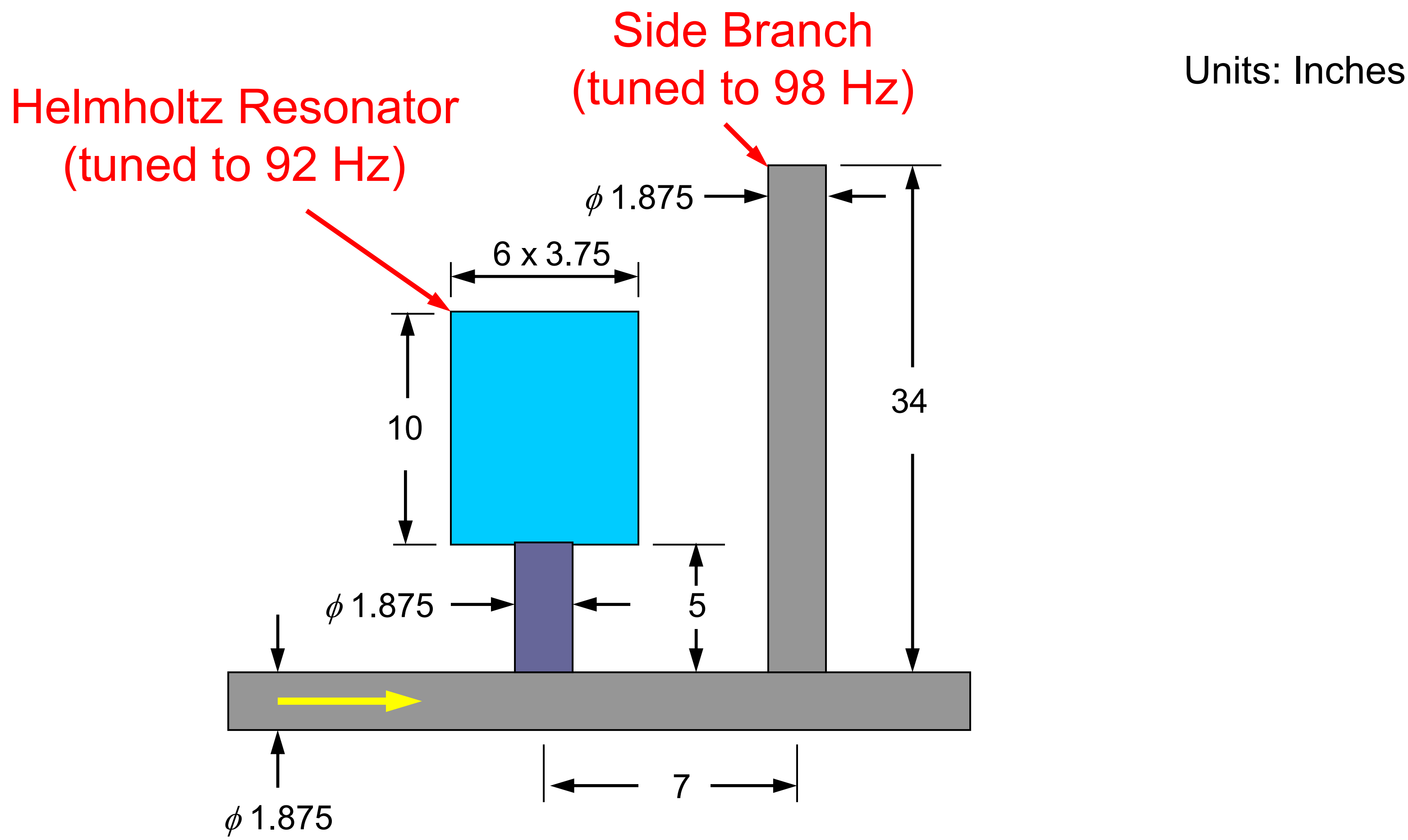
$$TL = 10\log_{10}\left\{\frac{S_{in}}{4S_{out}}\left|T_{11} + \frac{S_{out}T_{12}}{\rho c} + \frac{\rho c T_{21}}{S_{in}} + \frac{S_{out}}{S_{in}}T_{22}\right|^2\right\}$$

Design 1 Helmholtz Resonator

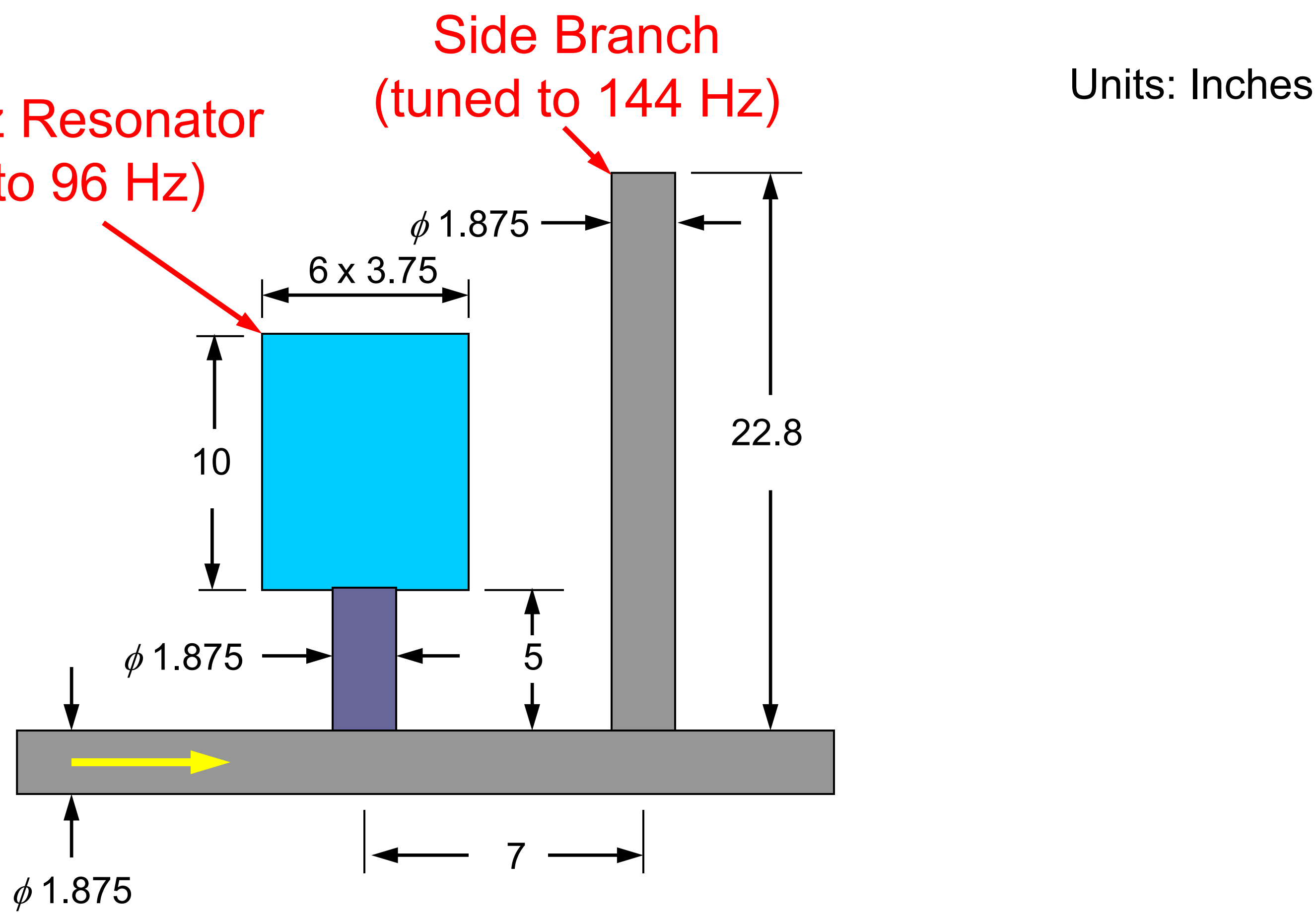
Units: Inches



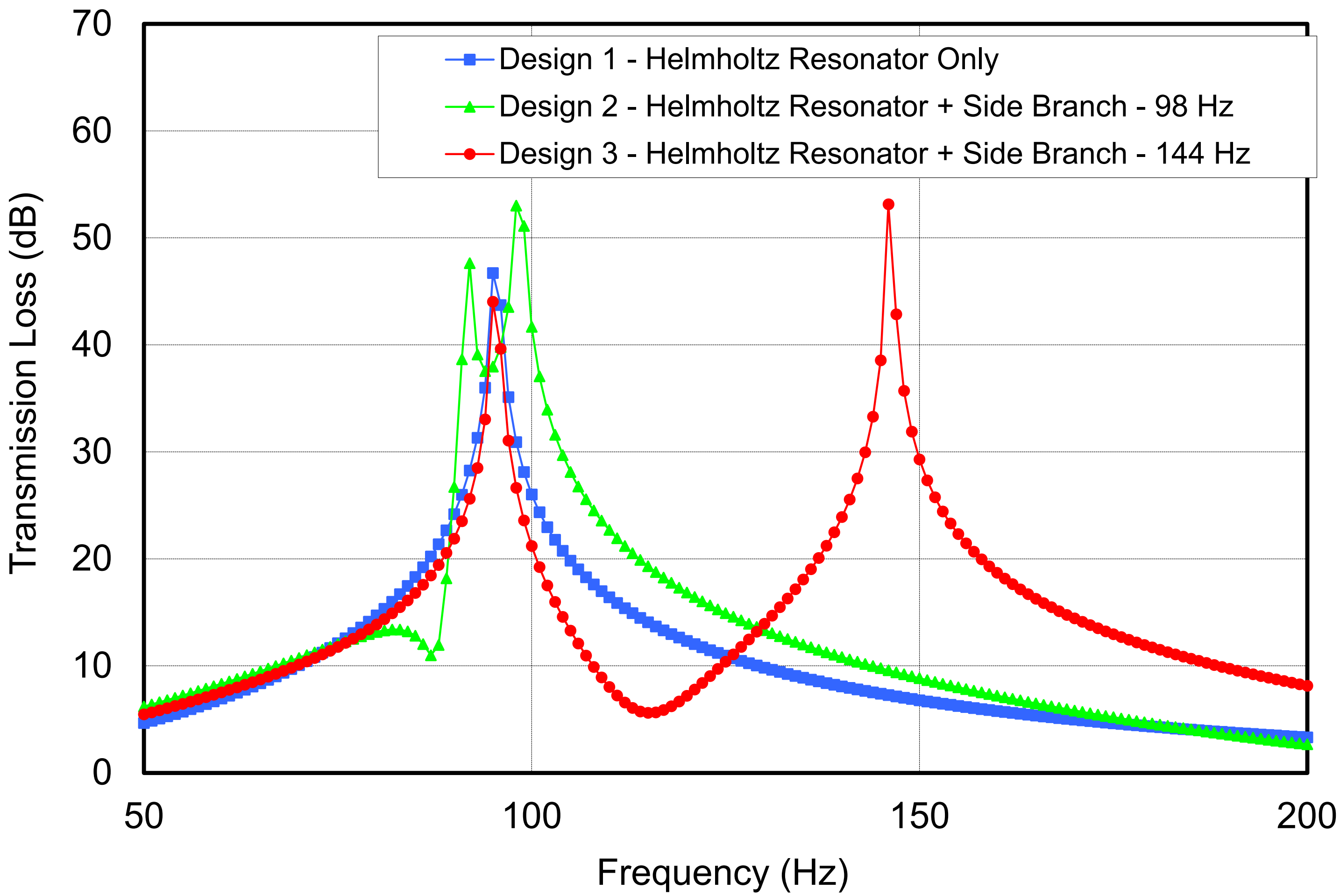
Design 2 Helmholtz Resonator + Side Branch



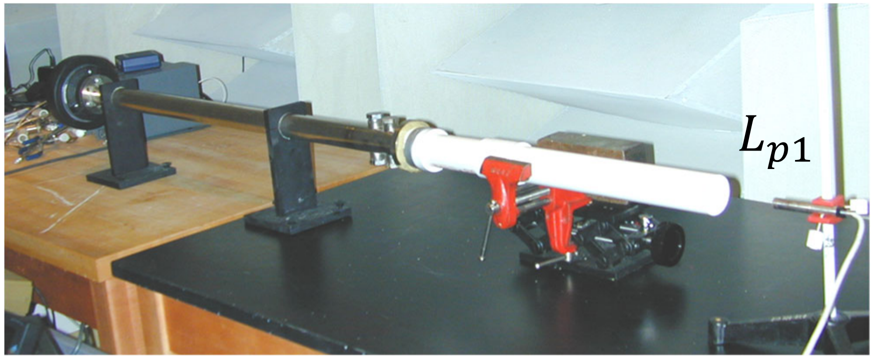
Design 3 Helmholtz Resonator + Side Branch

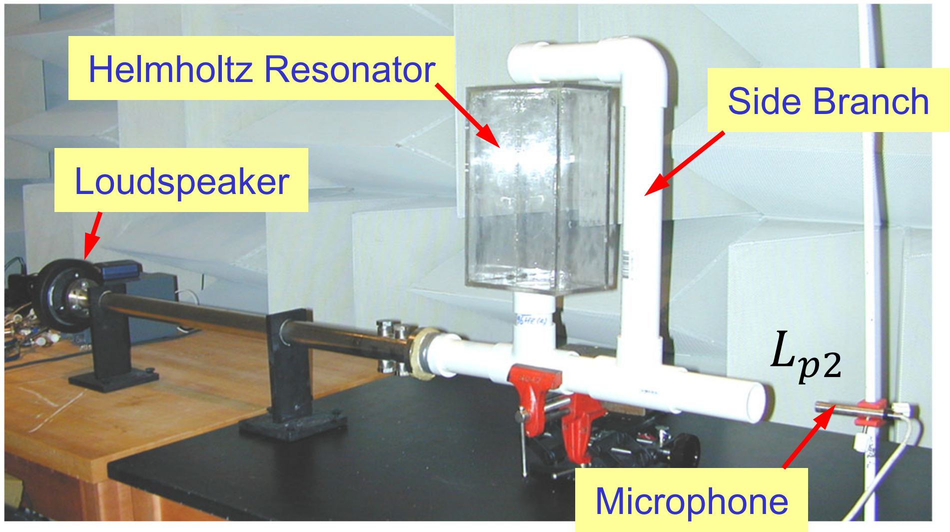


Transmission Loss Comparison



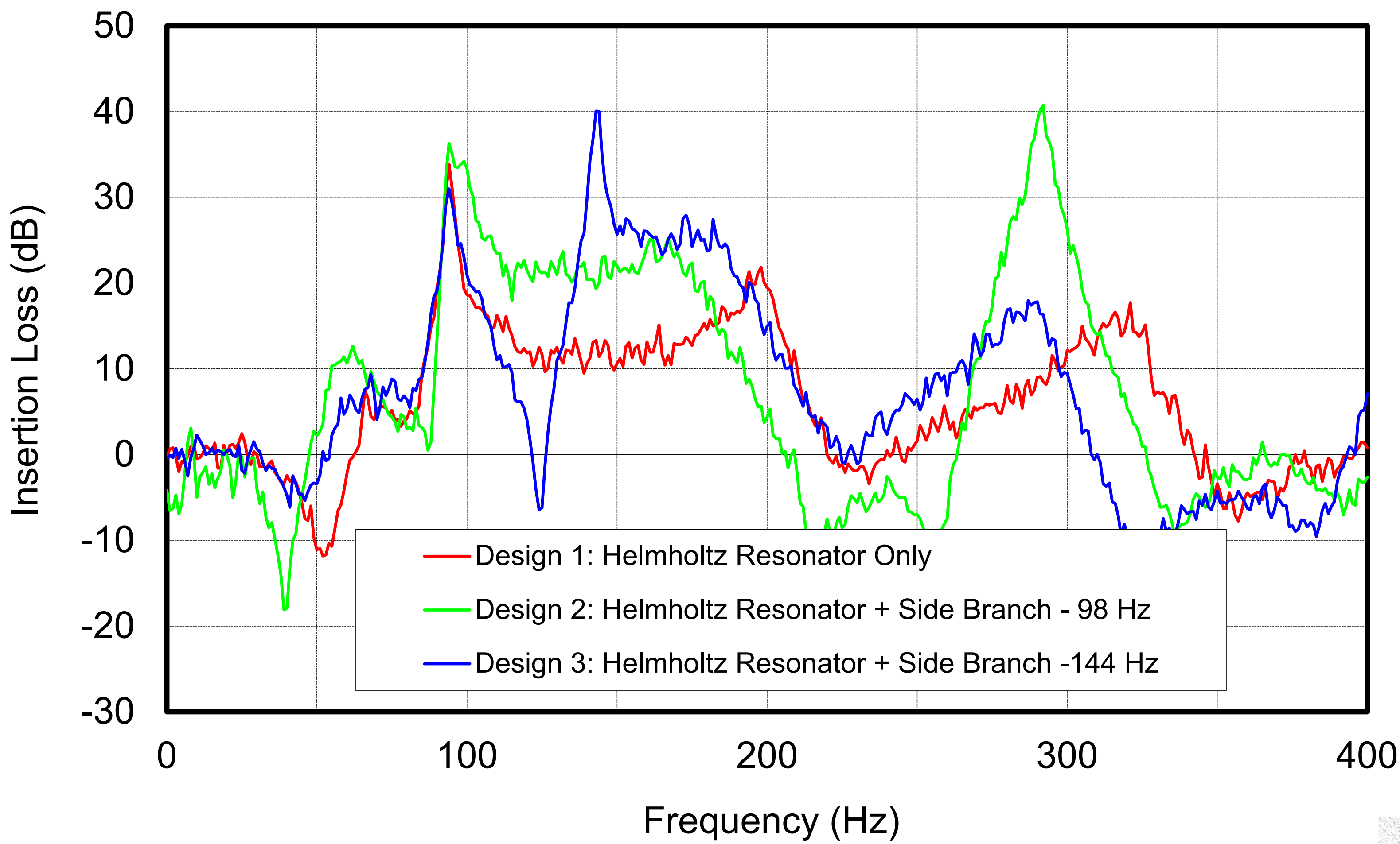
Insertion Loss Measurement



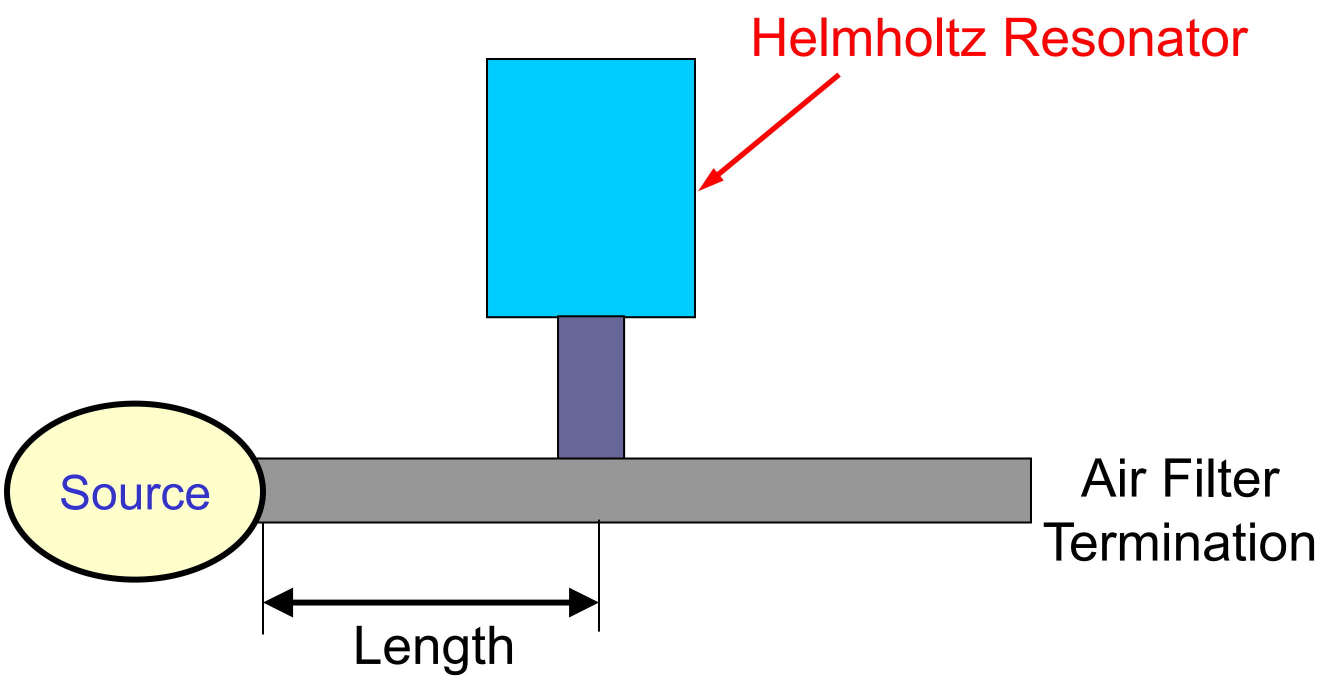


$$IL = L_{p1} - L_{p2} \quad \text{(dB)}$$

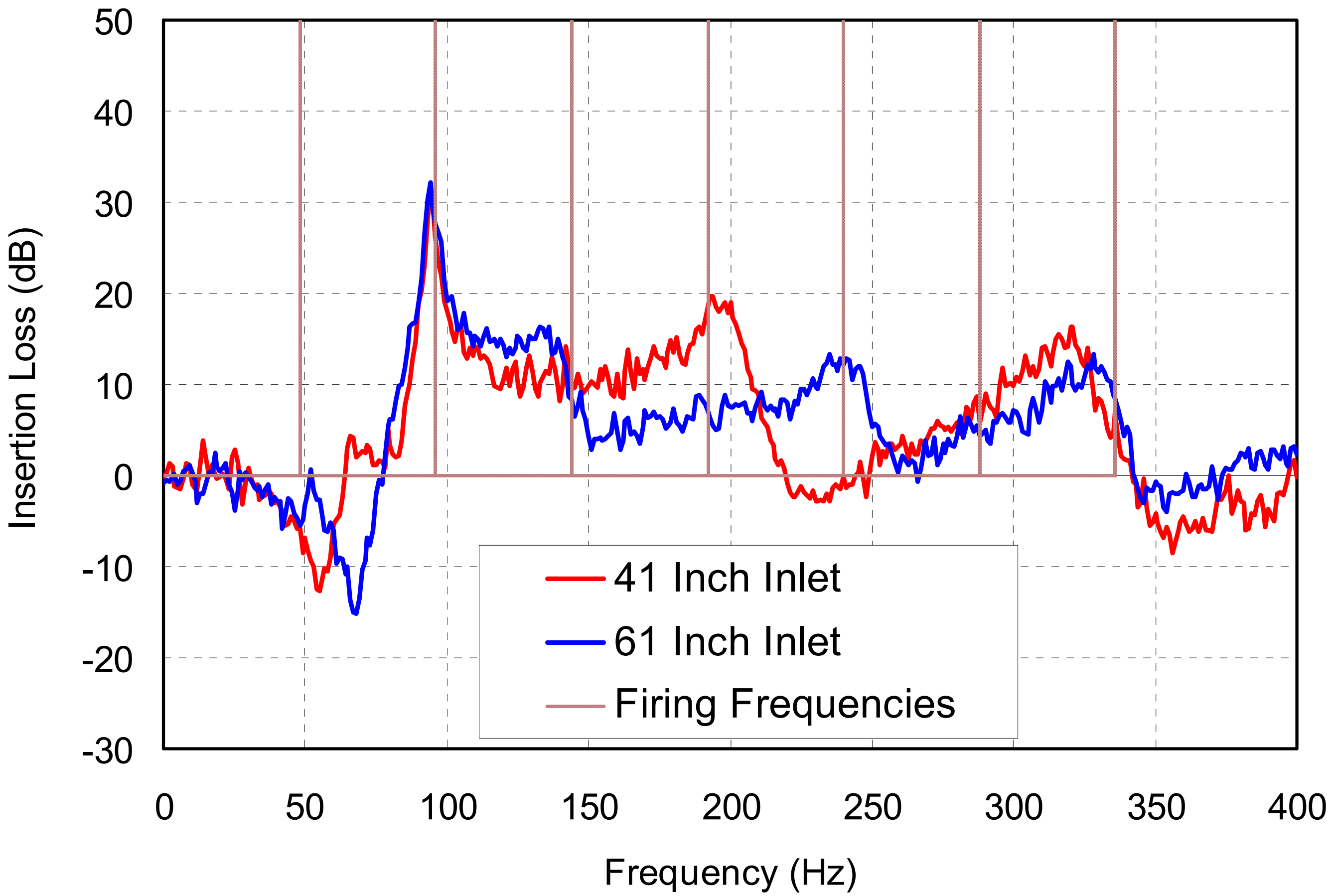
Insertion Loss Comparison



Design 1 Changing Inlet Length



Design 1 Changing Inlet Length



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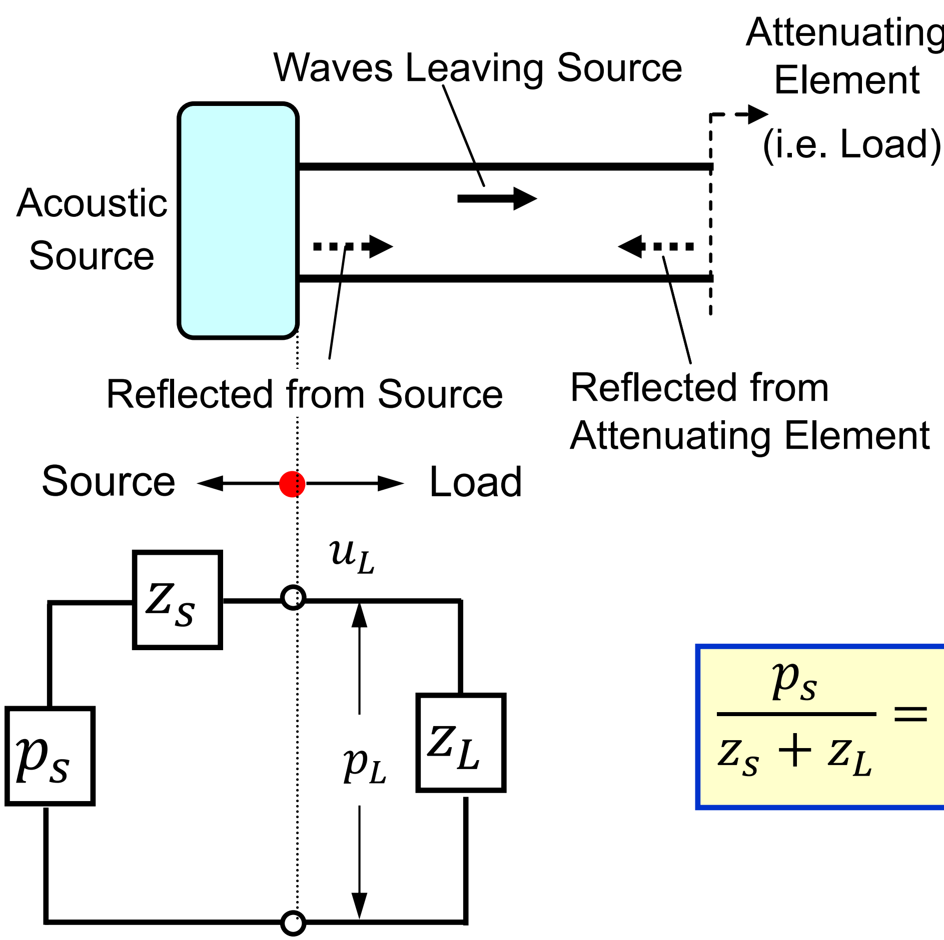
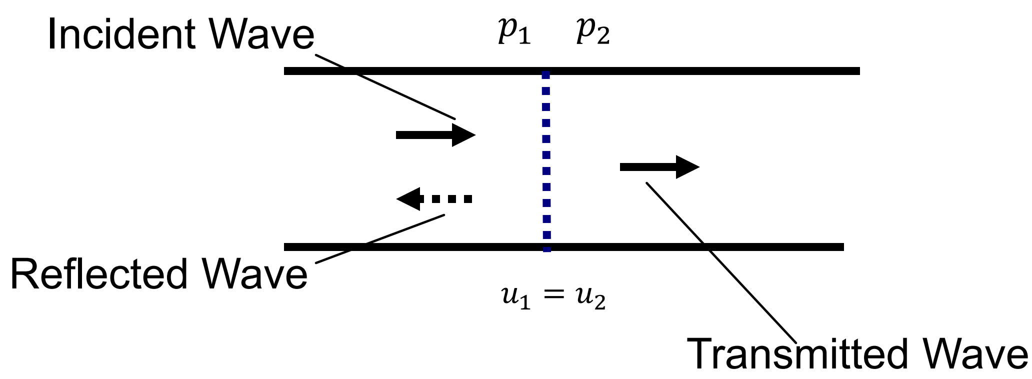
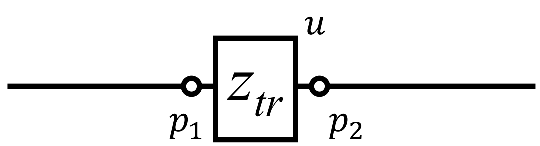
Source Impedance



$$\frac{p_S}{Z_S + Z_L} = \frac{p_L}{Z_L}$$

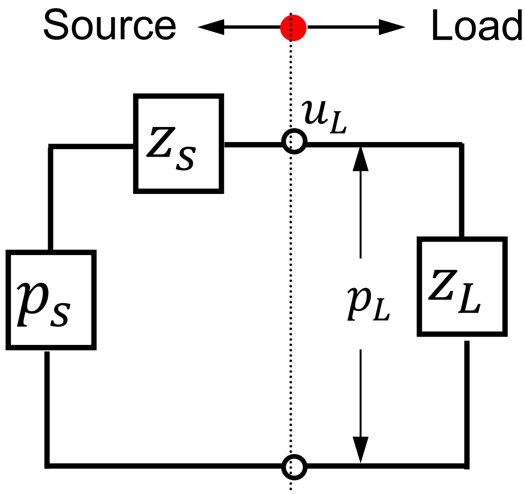
Series (Transfer) Impedance





$$z_{tr} = \frac{p_1 - p_2}{u}$$

Source Impedance



$$\frac{p_s}{z_s + z_L} = \frac{p_L}{z_L} = u_L$$

$$\Downarrow$$

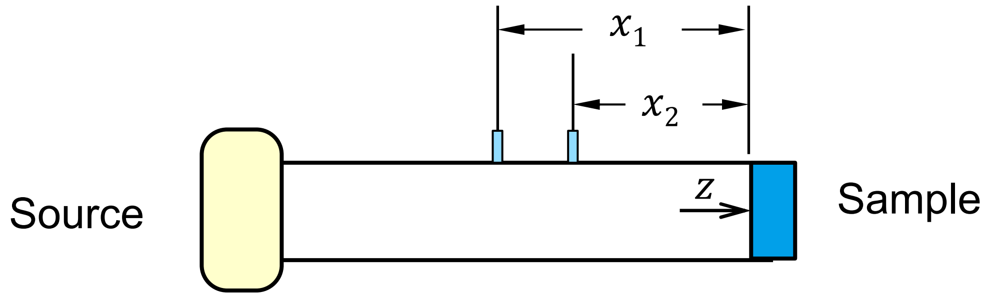
$$p_s = u_L z_s + p_L$$

$$\Downarrow$$

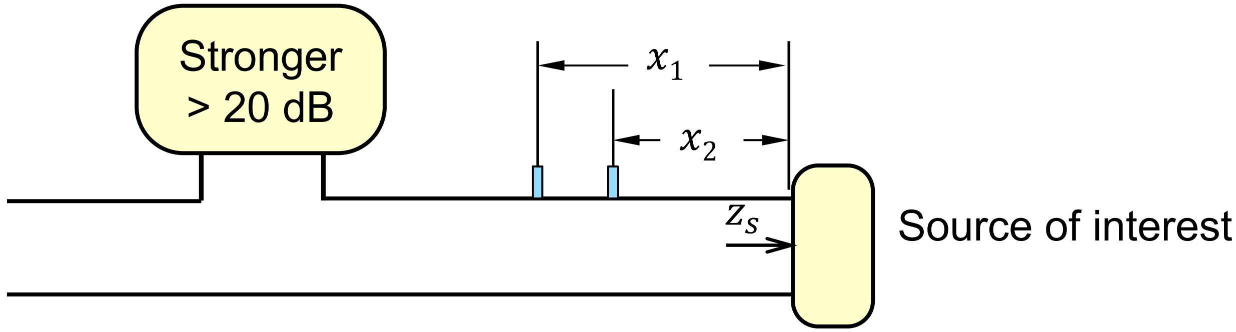
$$z_s = \frac{p_s - p_L}{u_L}$$

Source Impedance Direct Measurement

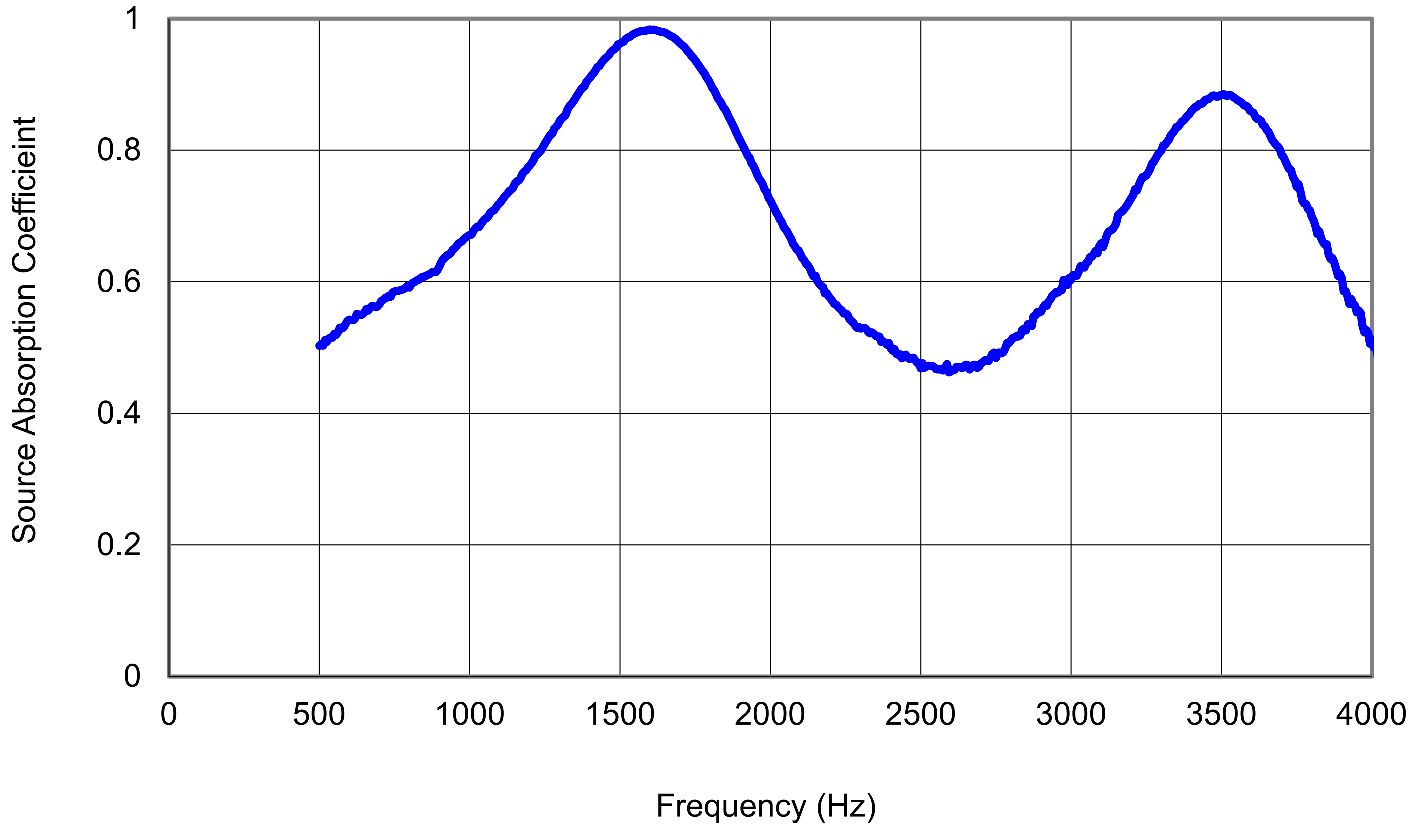
Measuring material sample :



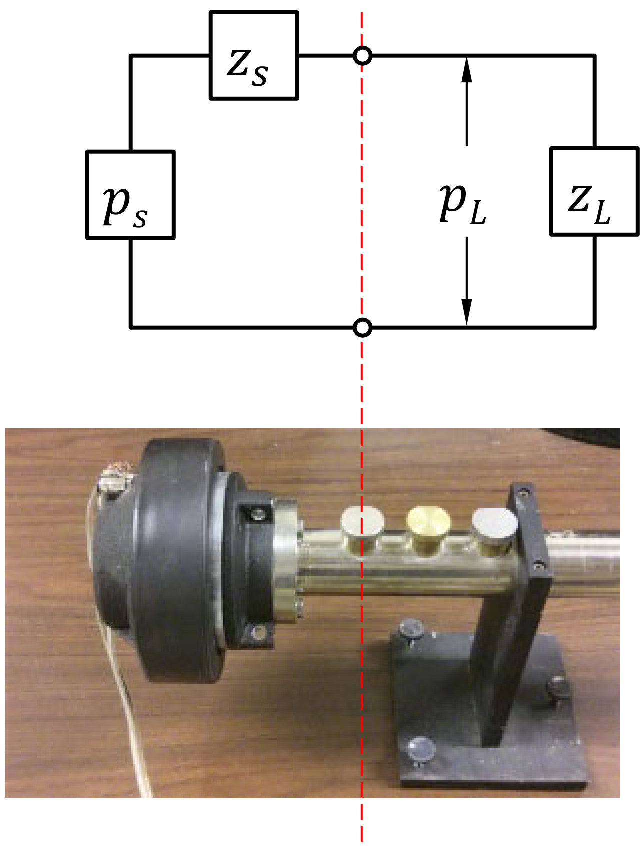
Measuring sound source:



Loudspeaker Source Absorption



Source Impedance Circuit Analogy



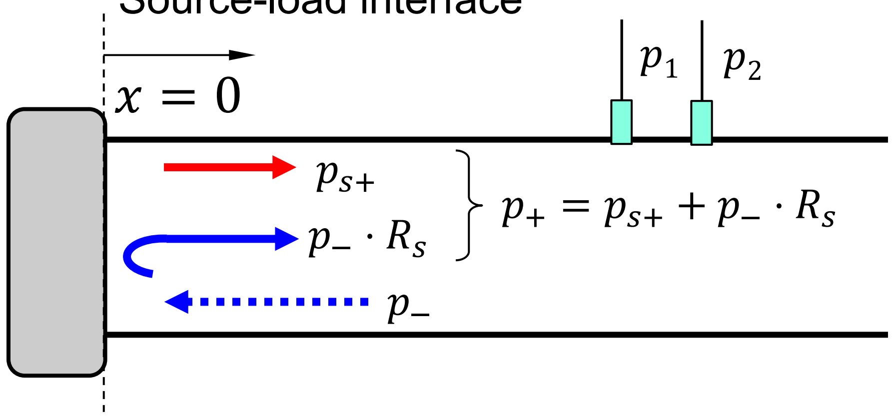
Source Strength ~ Voltage
Source Impedance ~ Internal resistor
Particle Velocity ~ Current

$$\frac{p_s}{Z_s + Z_L} = \frac{P_L}{Z_L}$$

Unknowns: p_s and z_s

Use 2 or more loads to solve source properties.

Source Impedance Wave Decomposition



Use 2 or more loads to solve source properties.

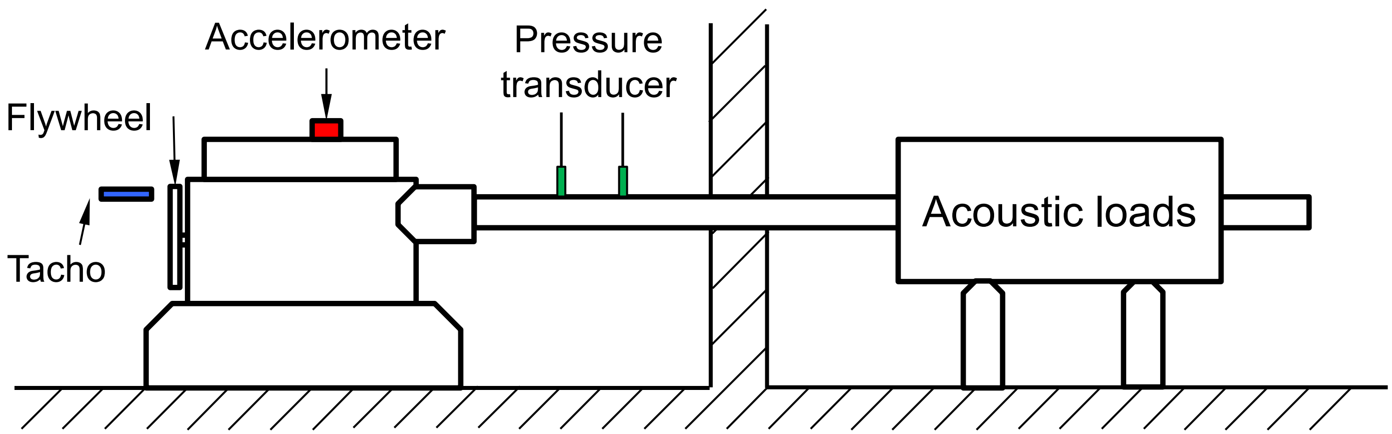
$$p_{1+} = p_{s+} + p_{1-} \cdot R_s$$
$$p_{2+} = p_{s+} + p_{2-} \cdot R_s$$

$\longrightarrow$ p_{s+} and R_s $\longrightarrow$

$$z_s = \frac{1 + R_s}{1 - R_s}$$

$$p_s = p_{s+}\left(\frac{1 - R_s}{2}\right)$$

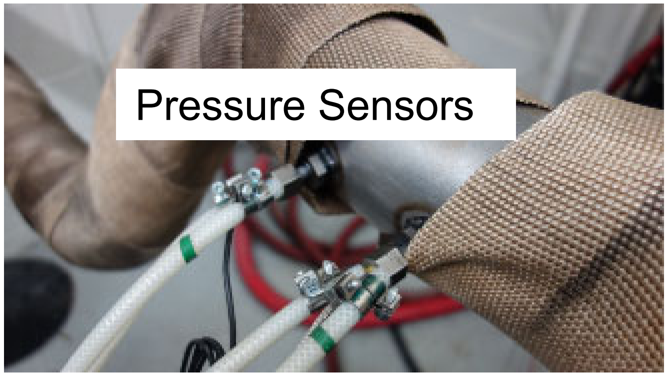
Measurement Setup



Use tachometer to detect the start of each rotation.

Example Diesel Engine

Layout	6 cylinder, 4 stroke
Fuel	Diesel
Exhaust Diameter	4 inches
Exhaust Flow Rate	0.12 Mach
Test RPM	2400
Test Output Torque	Around 500 N·m



Load number	Exhaust configuration
Load 1	ϕ0.10 m × 9.5 m
Load 2	ϕ0.10 m × 3.5 m ϕ0.25 m × 0.75 m SEC ϕ0.10 m × 5 m
Load 3	ϕ0.10 m × 7.2 m
Load 4	ϕ0.10 m × 8.1 m
Load 5	ϕ0.10 m × 6.2 m

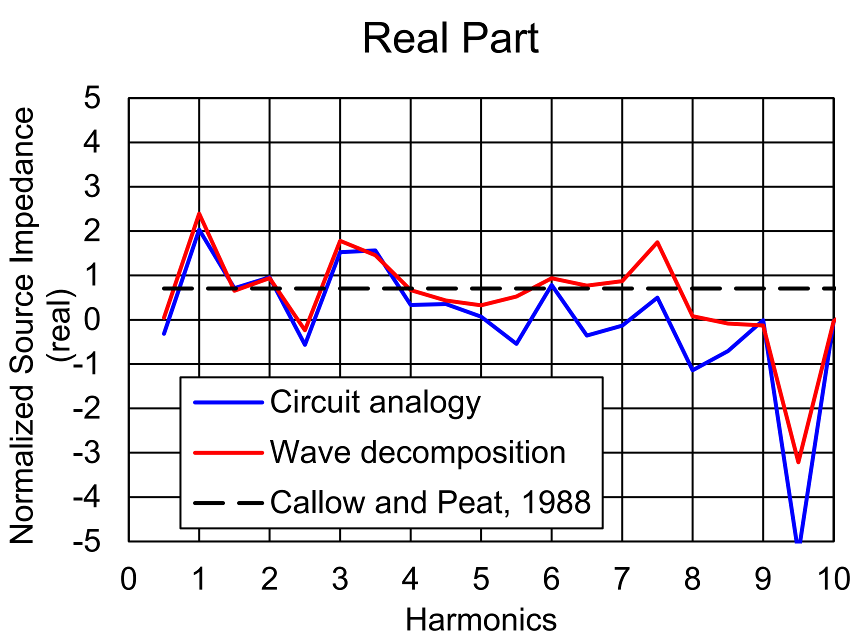
Load 1~4:
used to obtain source properties.

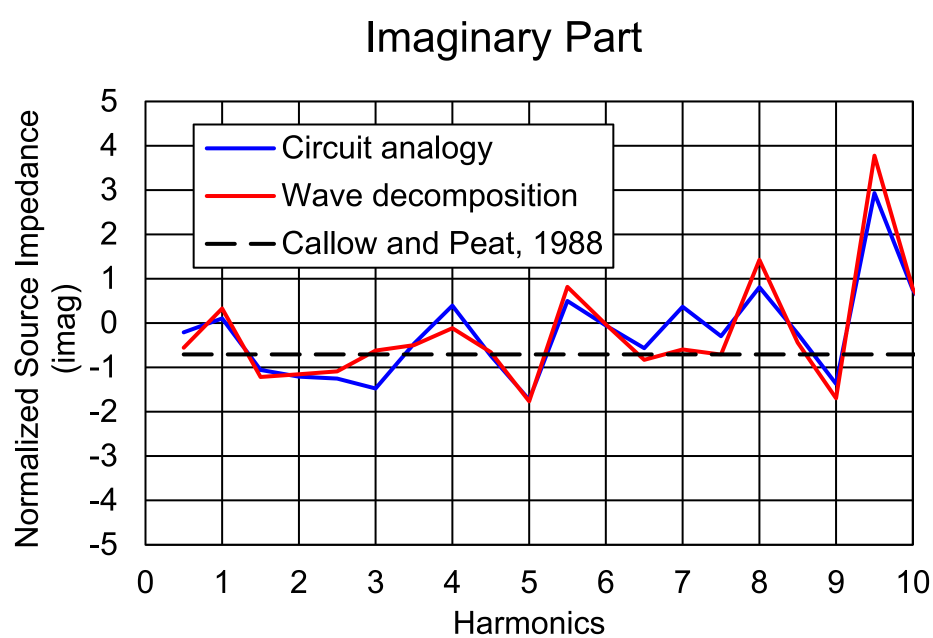
Load 5:
Used for validation.

Diesel Engine Source Impedance

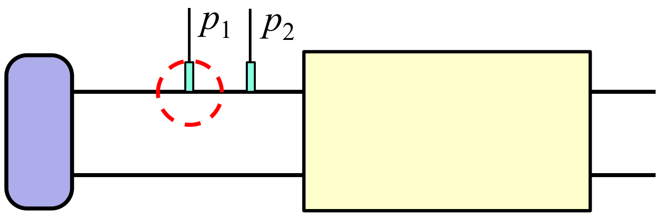
Mechanical working condition: 2400 RPM, throttle fully open

Constant source impedance: $z_s = \frac{\sqrt{2}}{2}(1 - 1i)$ → Callow and Peat, 1988

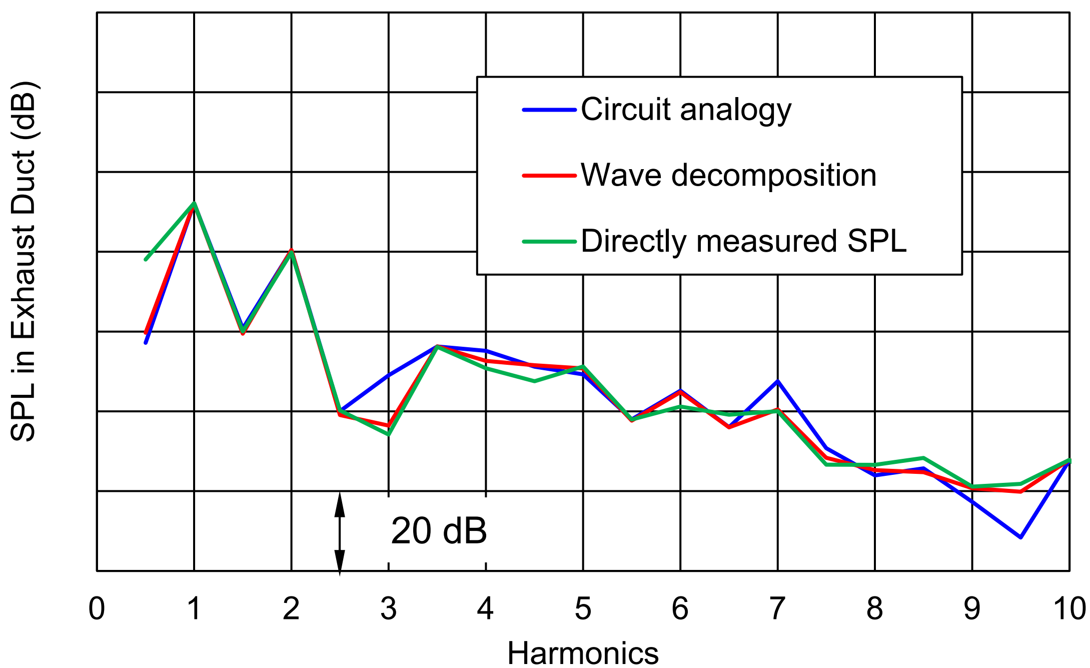




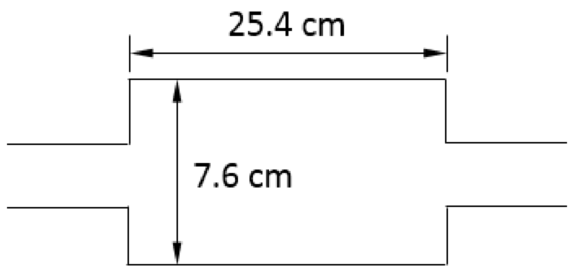
Diesel Engine SPL in Exhaust

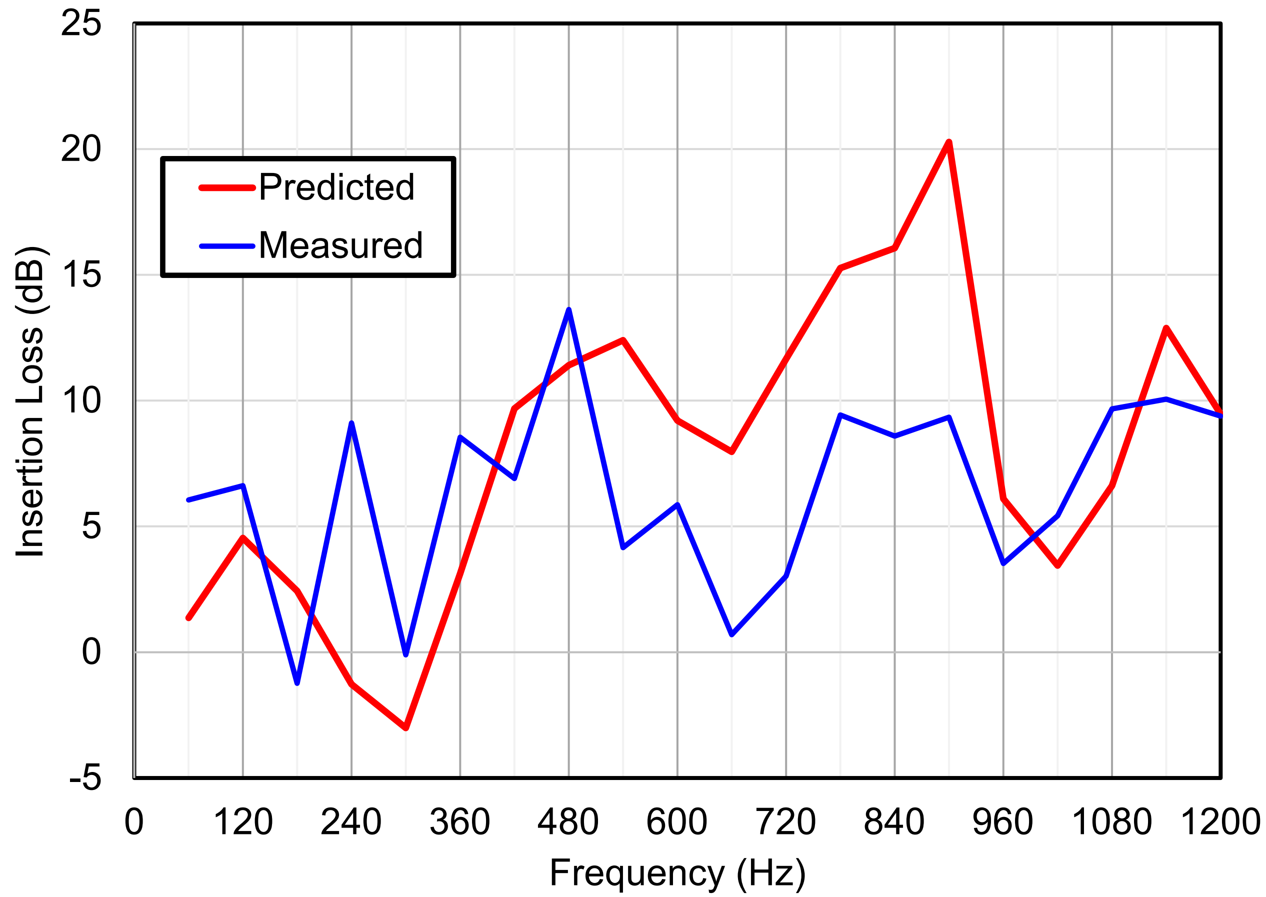


For Load 5, SPL at p_1 is predicted



Insertion Loss Muffler A





Source Impedance Measurement Difficulties

Difficulties of actual measurement:

- Extreme working conditions for pressure sensors (High temperature, turbulence…)
- Nonlinearity and time-variance of IC engines
- Choice of acoustic loads
- Cost

Sensitivity Study Muffler Insertion Loss

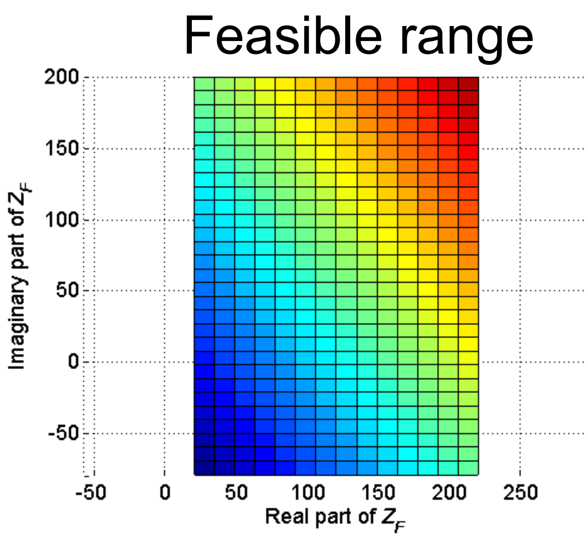
With the transfer matrix and source and transfer impedances known:

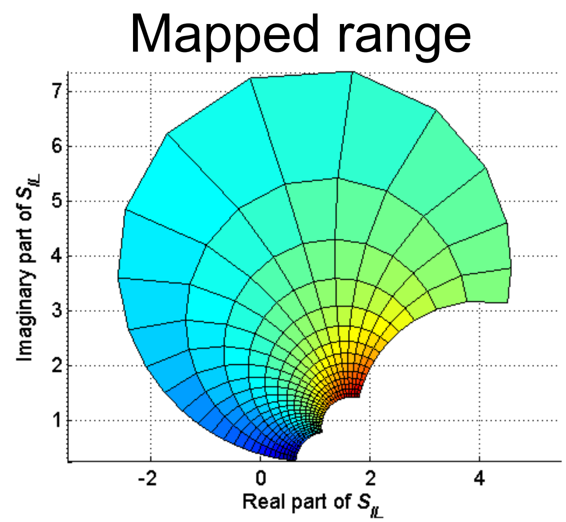
$$IL = 20\log_{10}\left\{\left|\frac{\frac{T_{11}}{Z_s}+\frac{T_{12}}{Z_t Z_s}+T_{21}+\frac{T_{22}}{Z_t}}{\frac{A_{11}}{Z_s}+\frac{A_{12}}{Z_t Z_s}+A_{21}+\frac{A_{22}}{Z_t}}\right|\right\}$$

A vector can be defined as

$$S_{IL} = \frac{\frac{T_{11}}{Z_s}+\frac{T_{12}}{Z_t Z_s}+T_{21}+\frac{T_{22}}{Z_t}}{\frac{A_{11}}{Z_s}+\frac{A_{12}}{Z_t Z_s}+A_{21}+\frac{A_{22}}{Z_t}}$$

Sensitivity Study Isolator Example





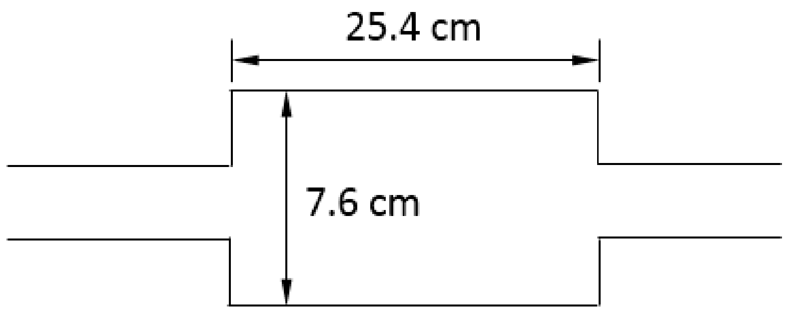
Zhang et al. (2018) showed that the boundary of the feasible range of z_S maps to the boundary of S_{IL} which can significantly speed up calculation of the range.

Process Effect of Source Impedance

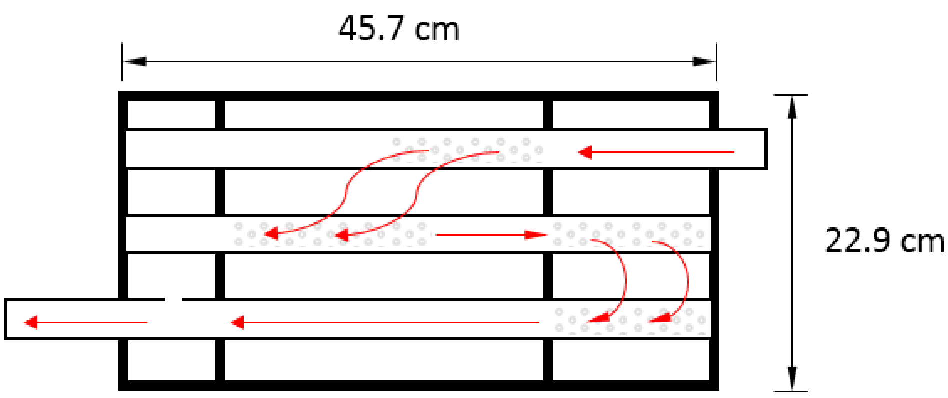
1. Measure source impedance for a typical case and condition.
2. Determine the μ and σ over all frequencies.
3. Suggest $\pm 2\sigma$ as a reasonable range.
4. Determine the range of insertion loss.

Example Muffler Designs

Muffler A

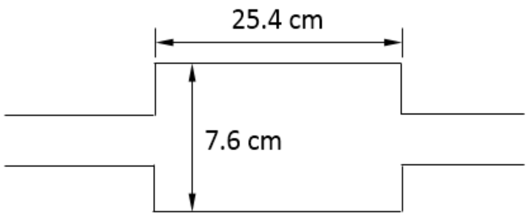


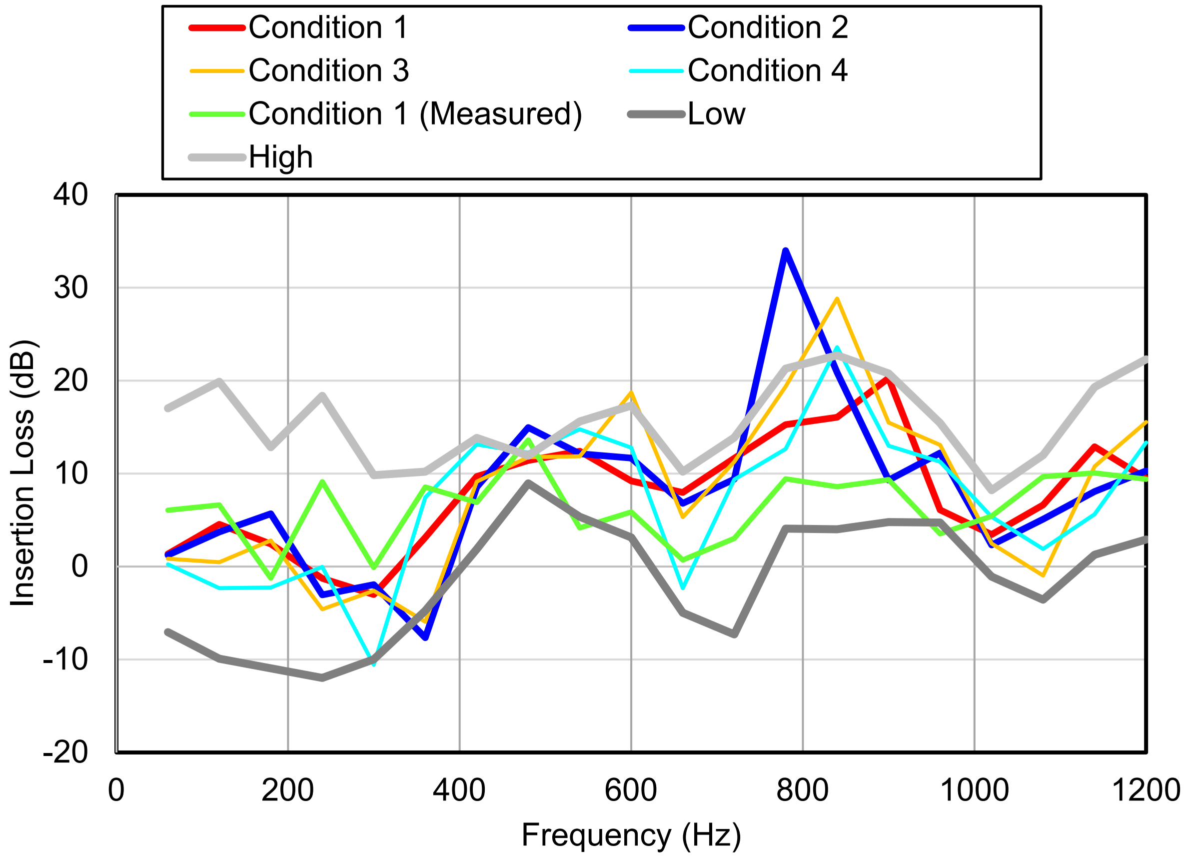
Muffler B



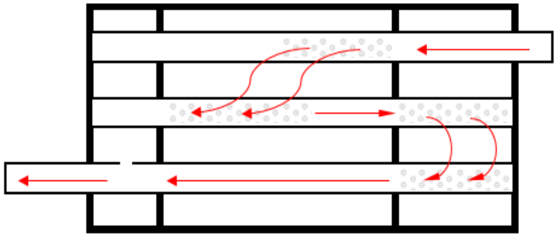
Condition	RPM	Load
Condition 1	2400	100% W.O.T.
Condition 2	2400	75% W.O.T.
Condition 3	2400	50% W.O.L.
Condition 4	2400	No Load

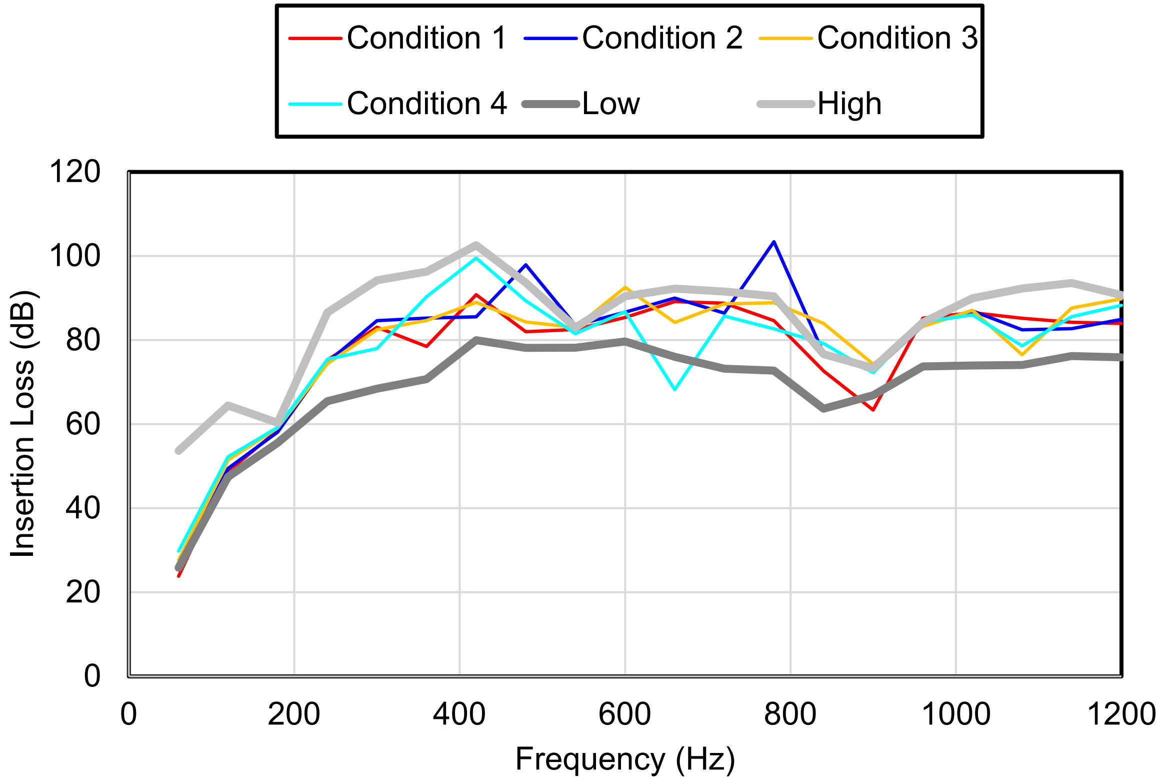
Insertion Loss Range Muffler A





Insertion Loss Range Muffler B





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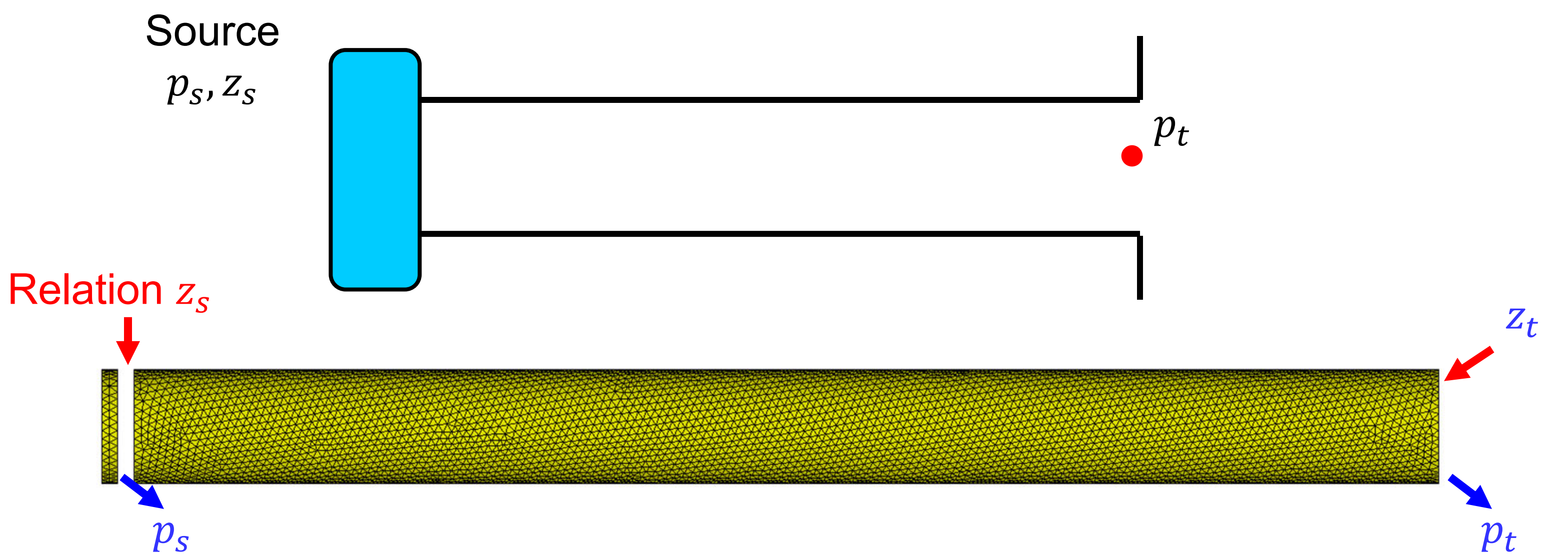
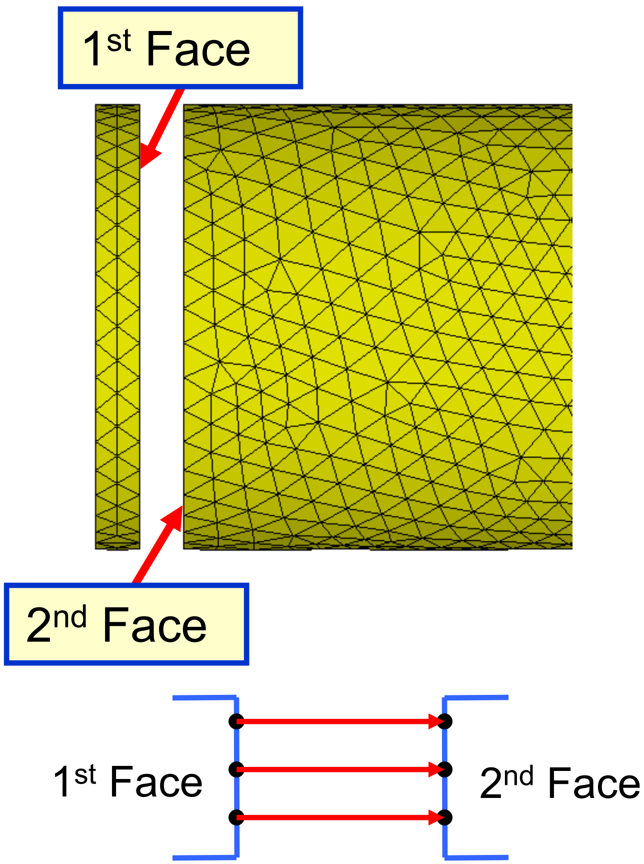
FEM Modeling Source Impedance?



- Source impedance is modeled as a transfer impedance
- The source pressure is input as a pressure BC with a short duct length

FEM Transfer Relation



$$\begin{bmatrix} v_{n1} \\ v_{n2} \end{bmatrix} = \begin{bmatrix} \alpha_1 & \alpha_2 \\ \alpha_4 & \alpha_5 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} + \begin{bmatrix} \alpha_3 \\ \alpha_6 \end{bmatrix}$$

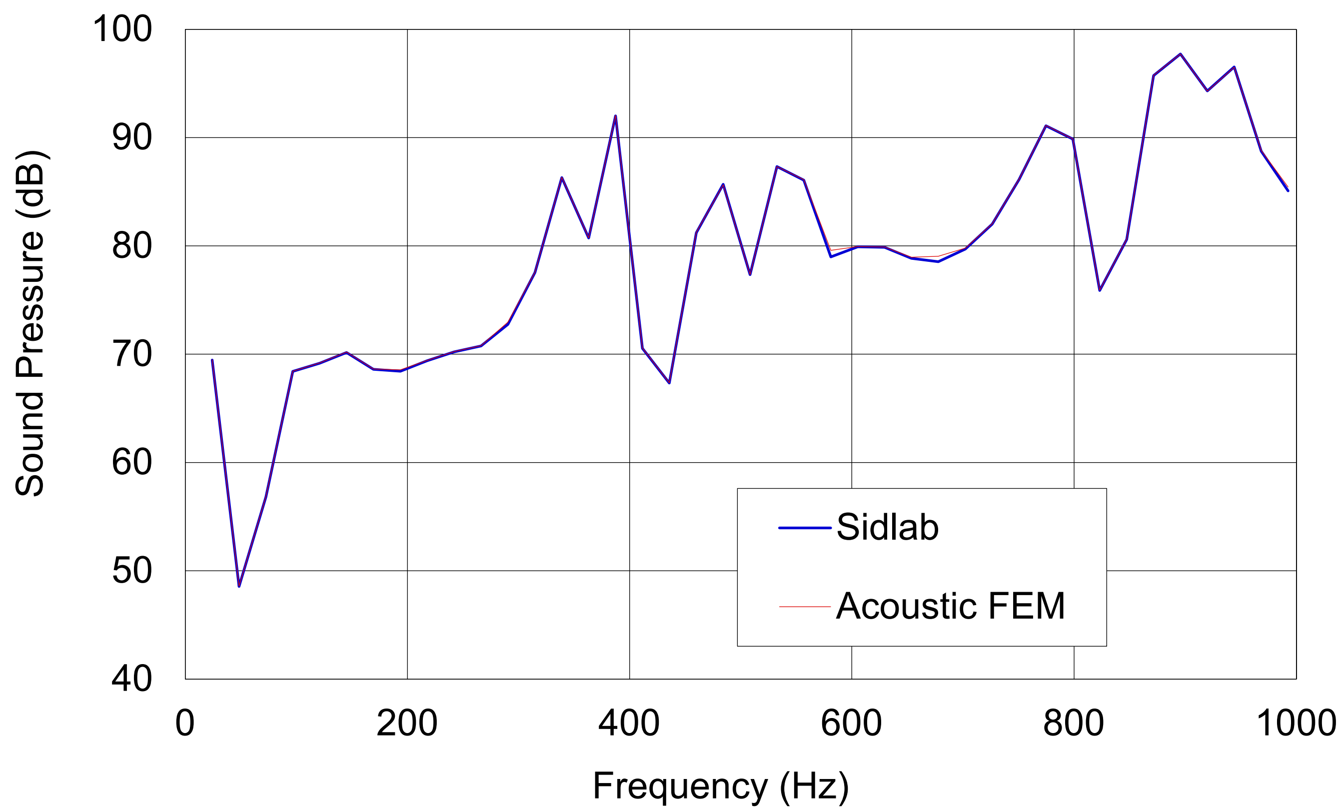
or,

$$\begin{bmatrix} v_{n1} \\ v_{n2} \end{bmatrix} = \begin{bmatrix} \beta & -\beta \\ -\beta & \beta \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix},$$

where β is the transfer admittance:

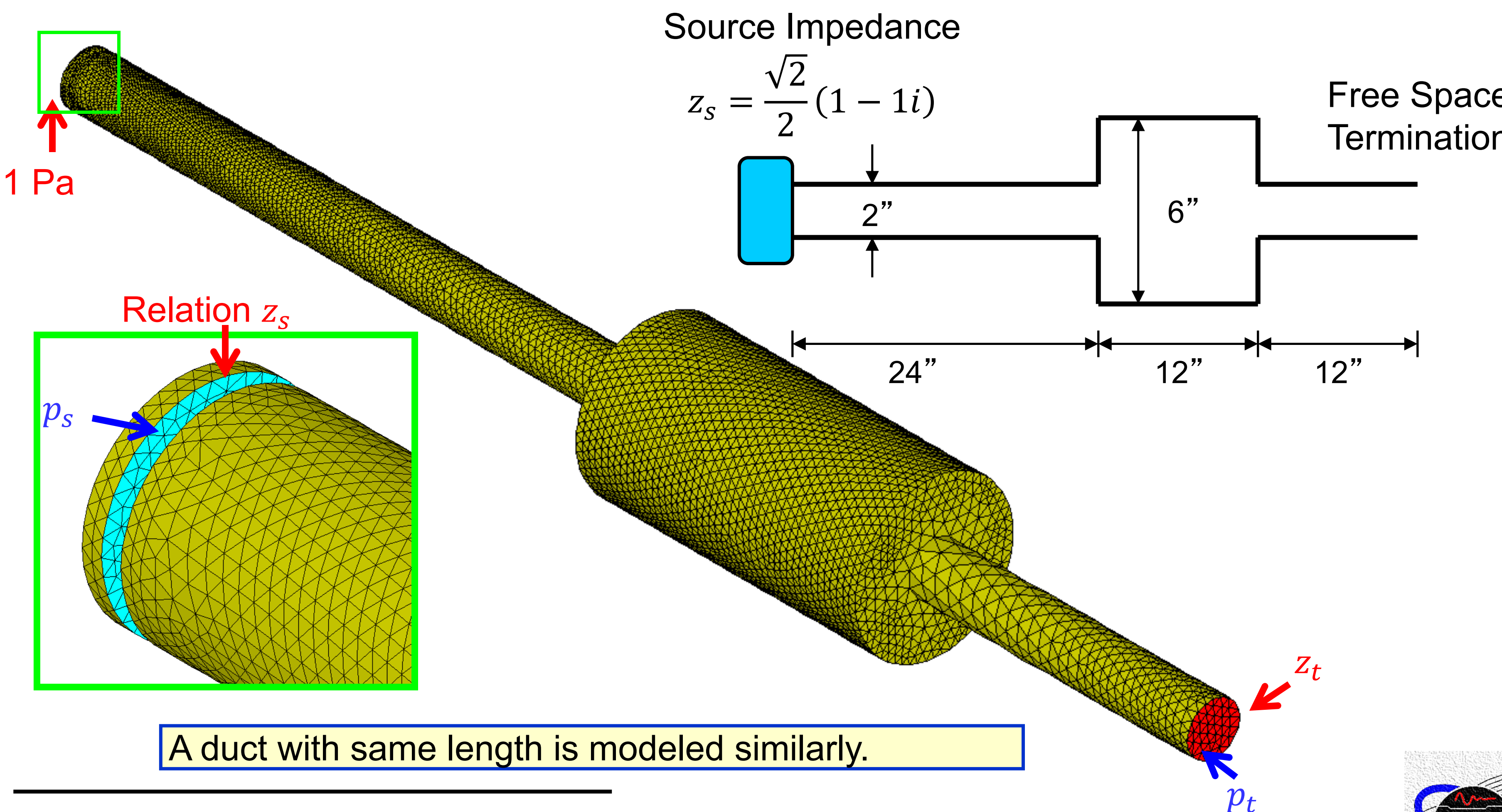
$$\beta = 1/z_s$$

FEM Sound Pressure at Termination



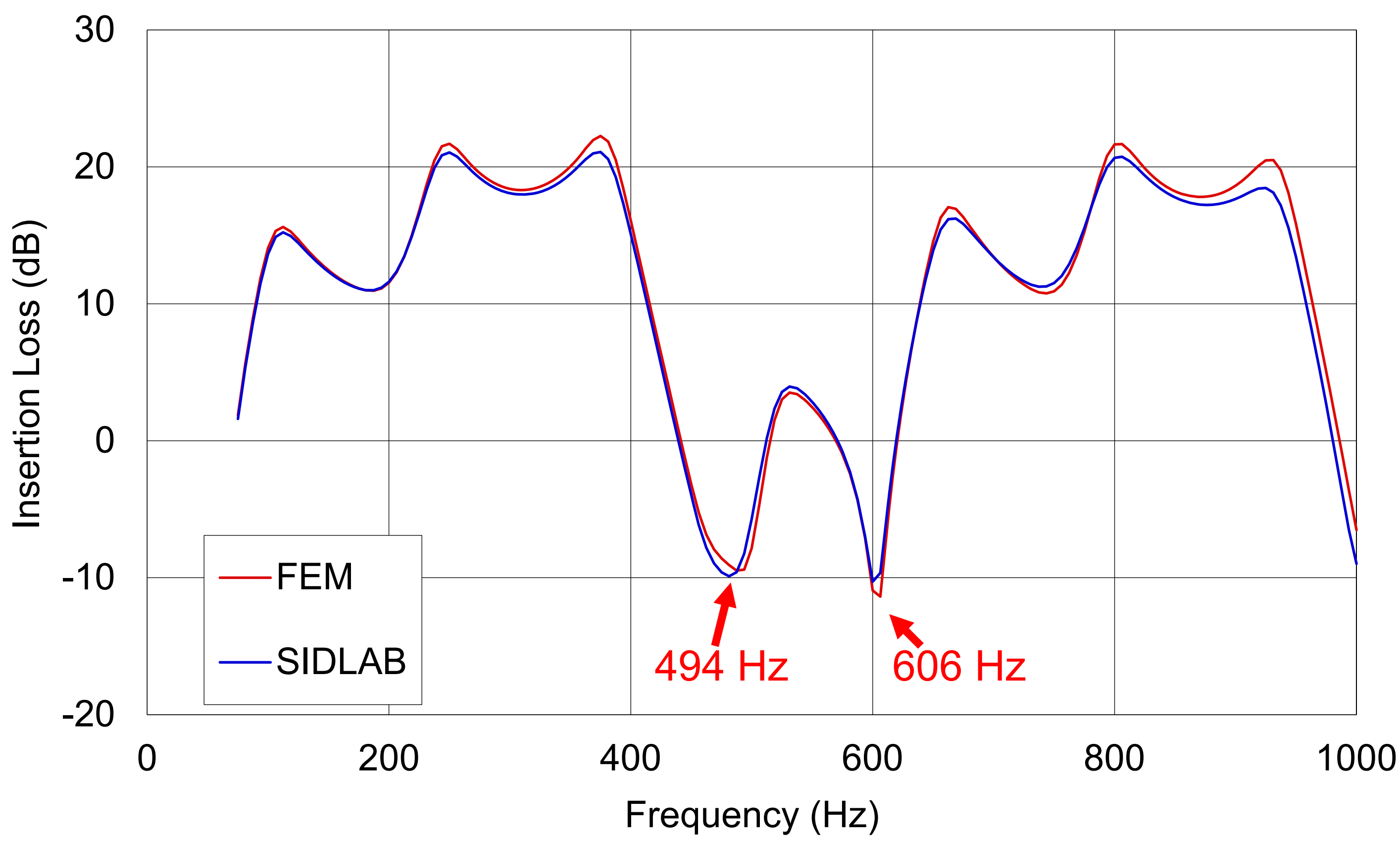
Source impedance for engine intake is specified and baffled termination is assumed.

Insertion Loss Modeling

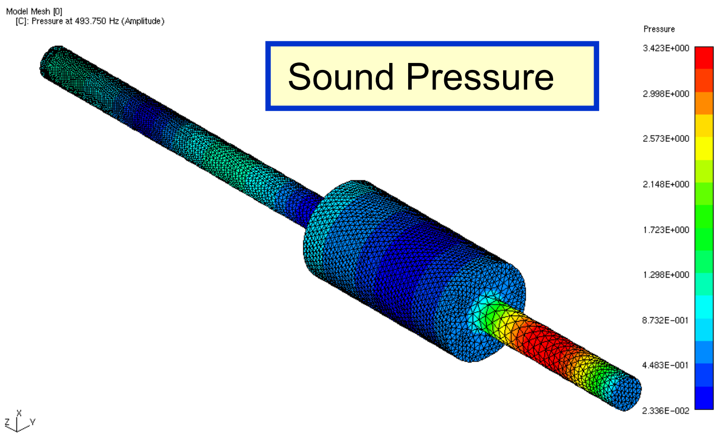


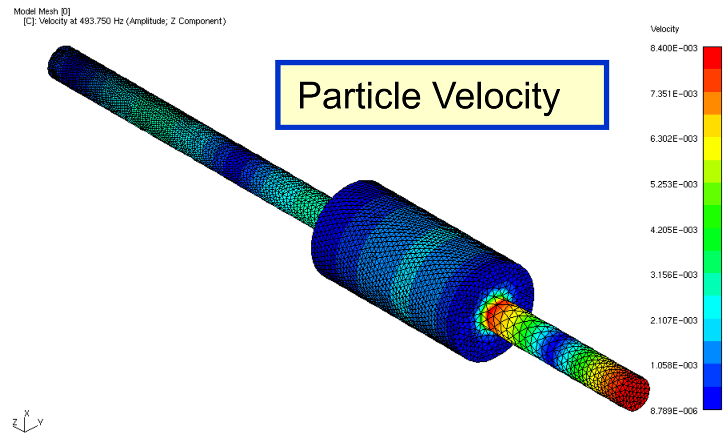
A duct with same length is modeled similarly.

Insertion Loss

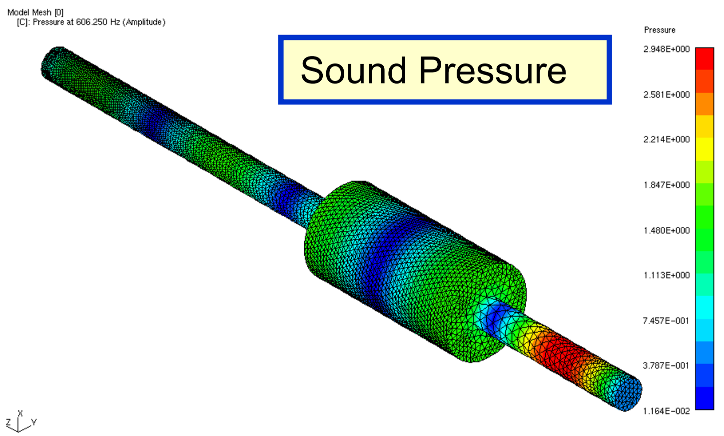


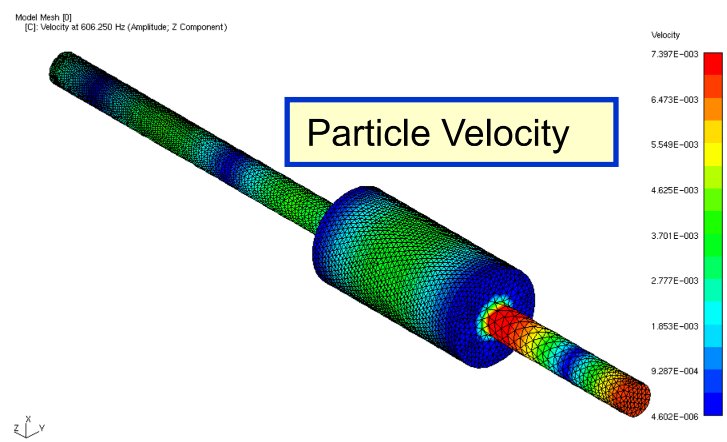
FEM Contour Plots 494 Hz





FEM Contour Plots 606 Hz





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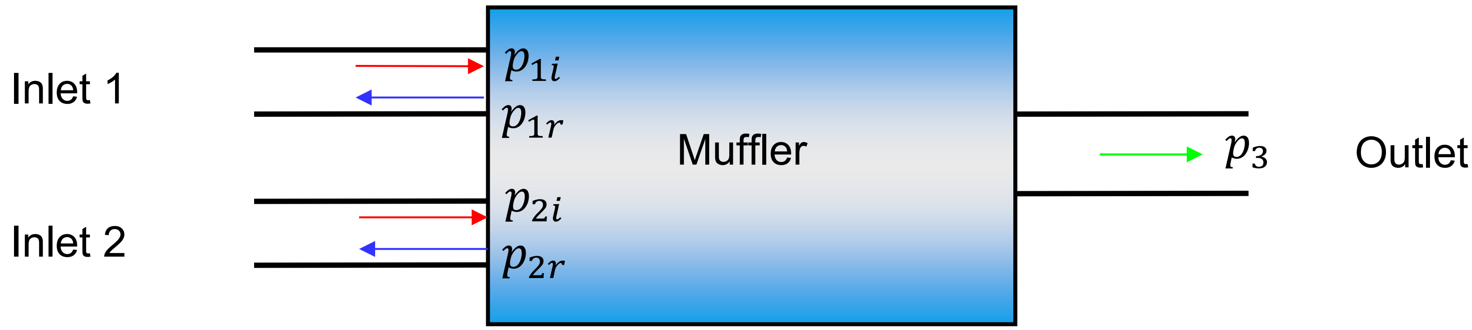
Transmission Loss Definition

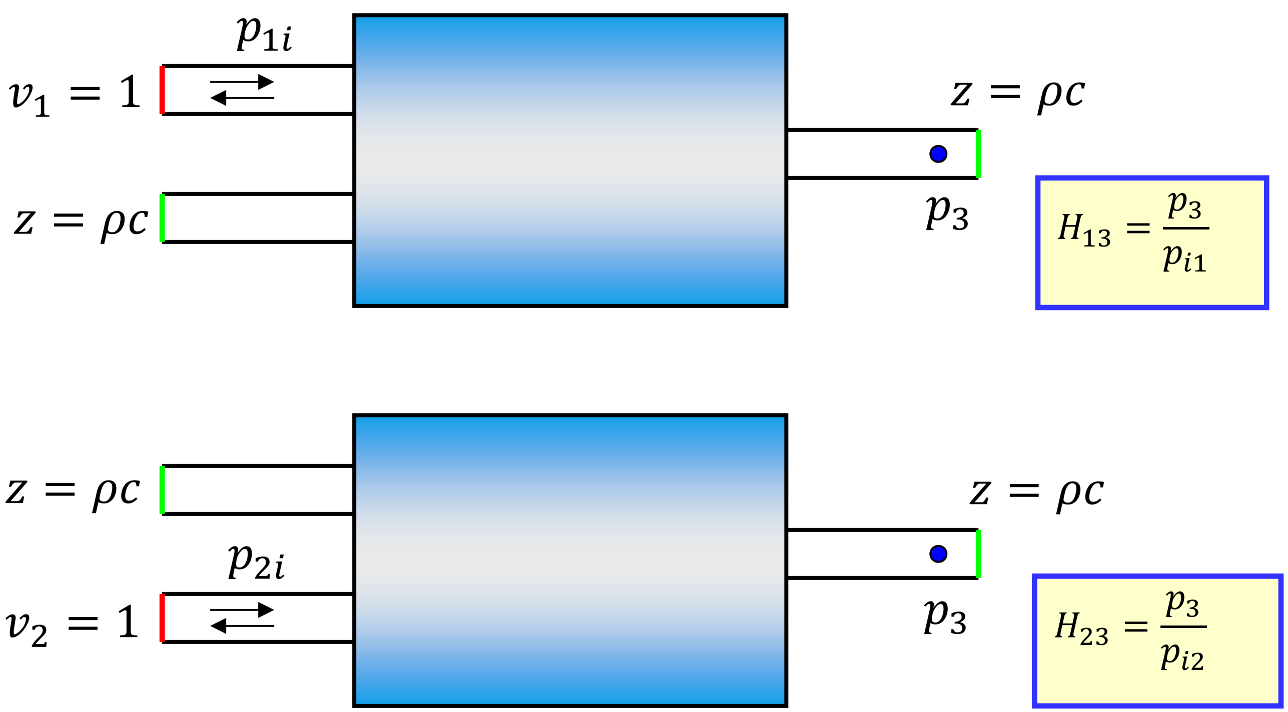


Define a complex ratio α between the two **incident pressures**:

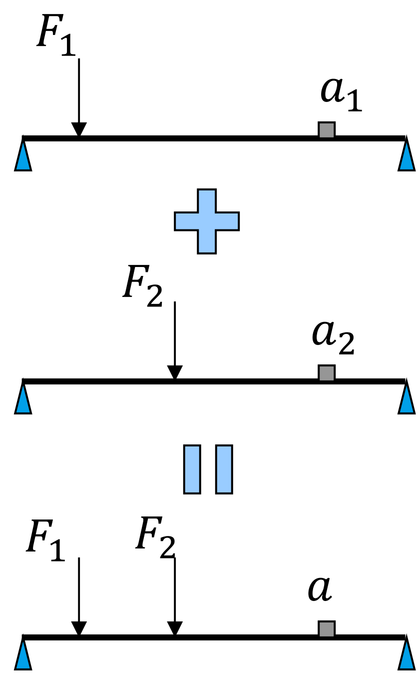
$$\beta = \frac{p_{2i}}{p_{1i}}$$

$$TL = 10\log_{10}\frac{W_{1i} + W_{2i}}{W_3} = 10\log_{10}\frac{|p_{1i}|^2 S_1 + |p_{2i}|^2 S_2}{|p_3|^2 S_3}$$

Transmission Loss Superposition Method



Vibration Analogy



Why anechoic? Avoid input coupling.

Transmission Loss Superposition Method

$$TL = 10\log_{10}\frac{W_{1i} + W_{2i}}{W_3} = 10\log_{10}\frac{|p_{1i}|^2 S_1 + |p_{2i}|^2 S_2}{|p_3|^2 S_3}$$

Transfer Functions (with other source and termination anechoic)

$$H_{13} = \frac{p_{3i}}{p_{1i}} \qquad H_{23} = \frac{p_{3i}}{p_{2i}}$$

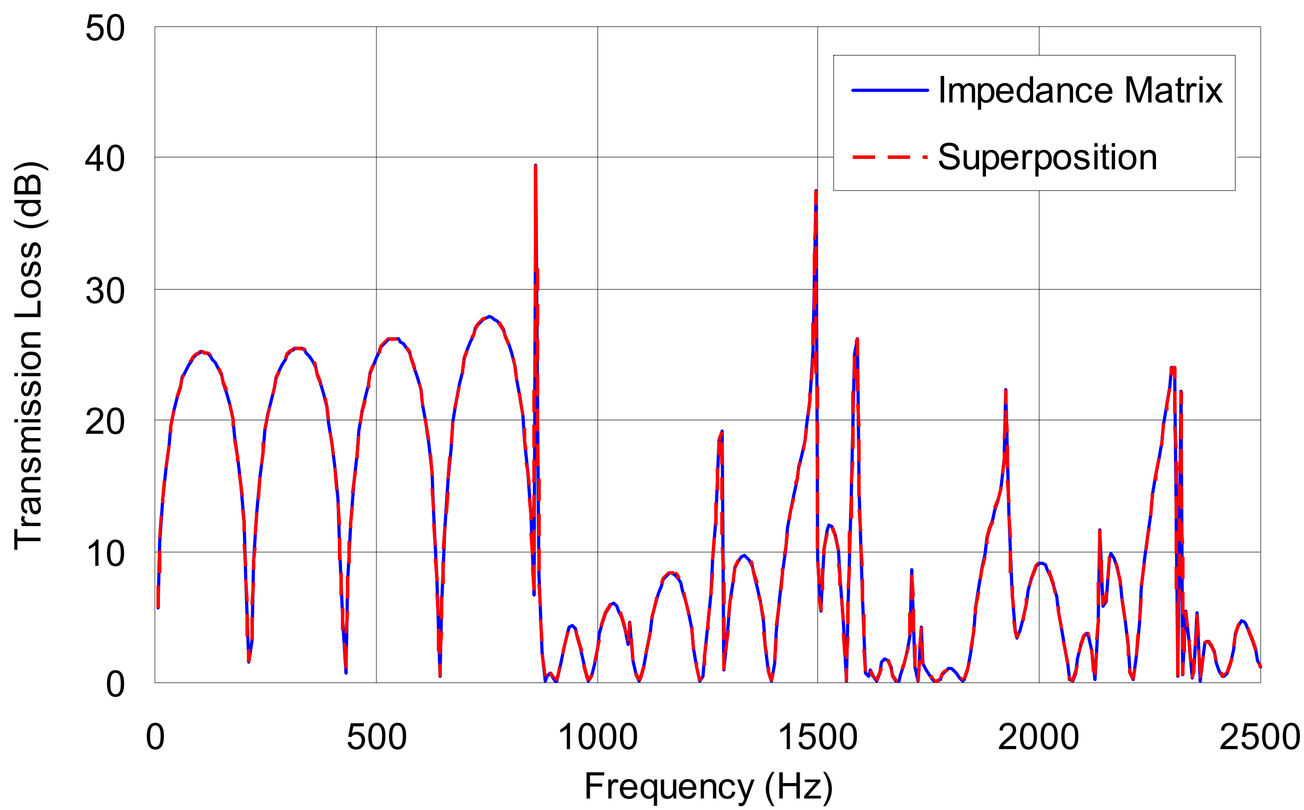
Transmission Loss

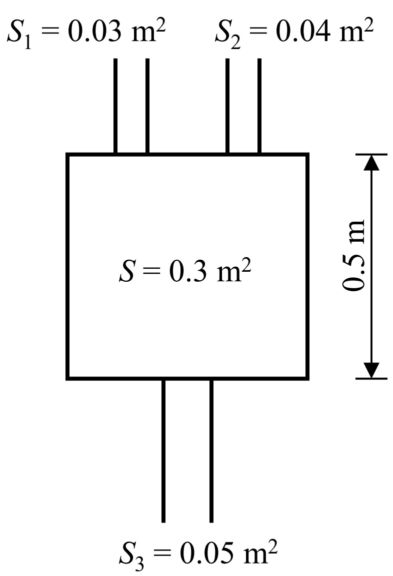
$$TL = 10\log_{10}\frac{S_1 + |\beta|^2 S_2}{|H_{13} + \beta H_{23}|^2 S_3}$$

$$\beta = \frac{p_{2i}}{p_{1i}}$$

Results 0° Phase Difference

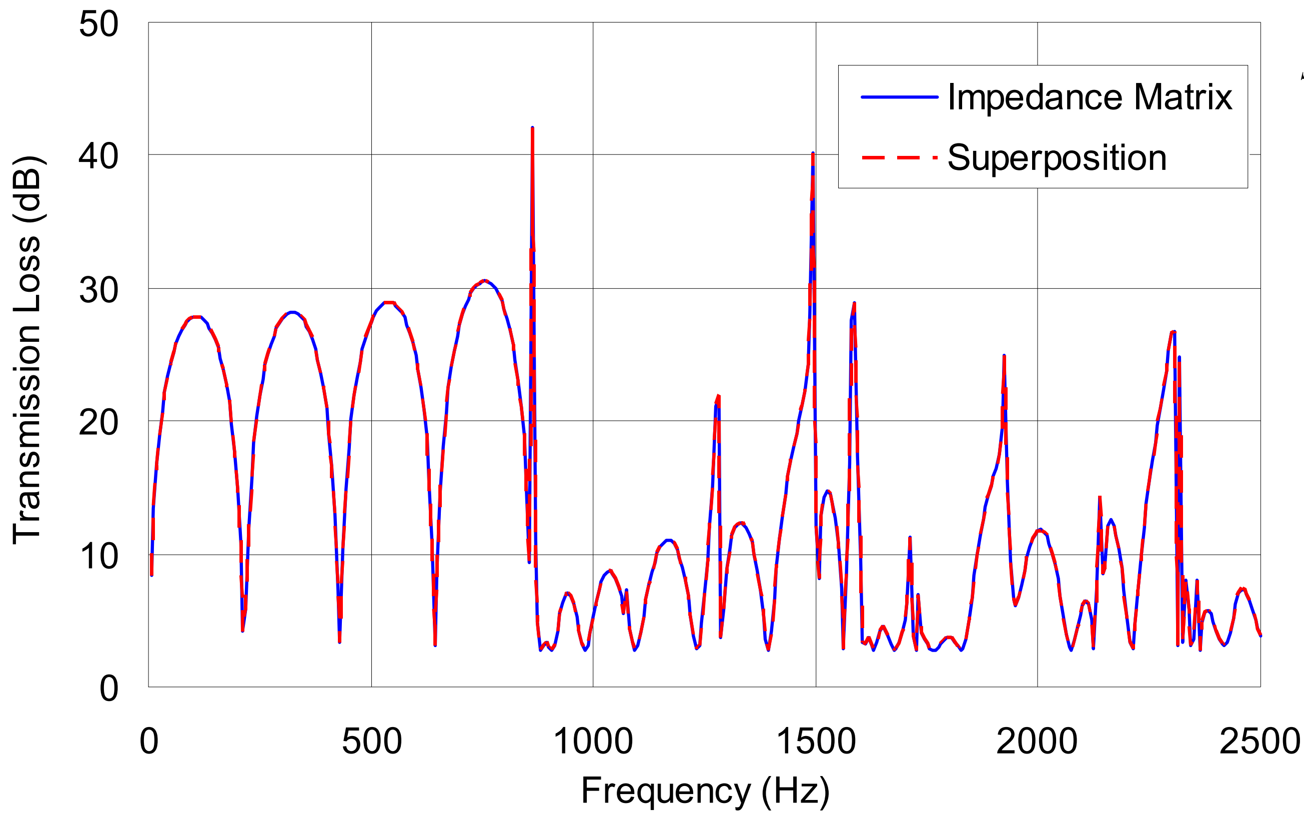
$\beta = 1$

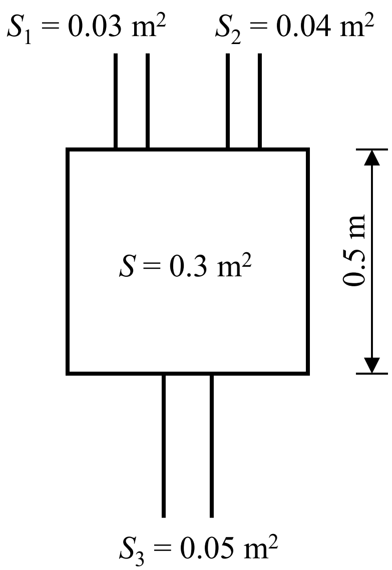




Results 90° Phase Difference

$\beta = j$





Multi-Inlet Muffler

$$TL = 10\log_{10}\frac{S_1 + |\beta|^2 S_2}{|H_{13} + \beta H_{23}|^2 S_3}$$

Two-Inlet Muffler

What about TL for three and more inlet muffler?

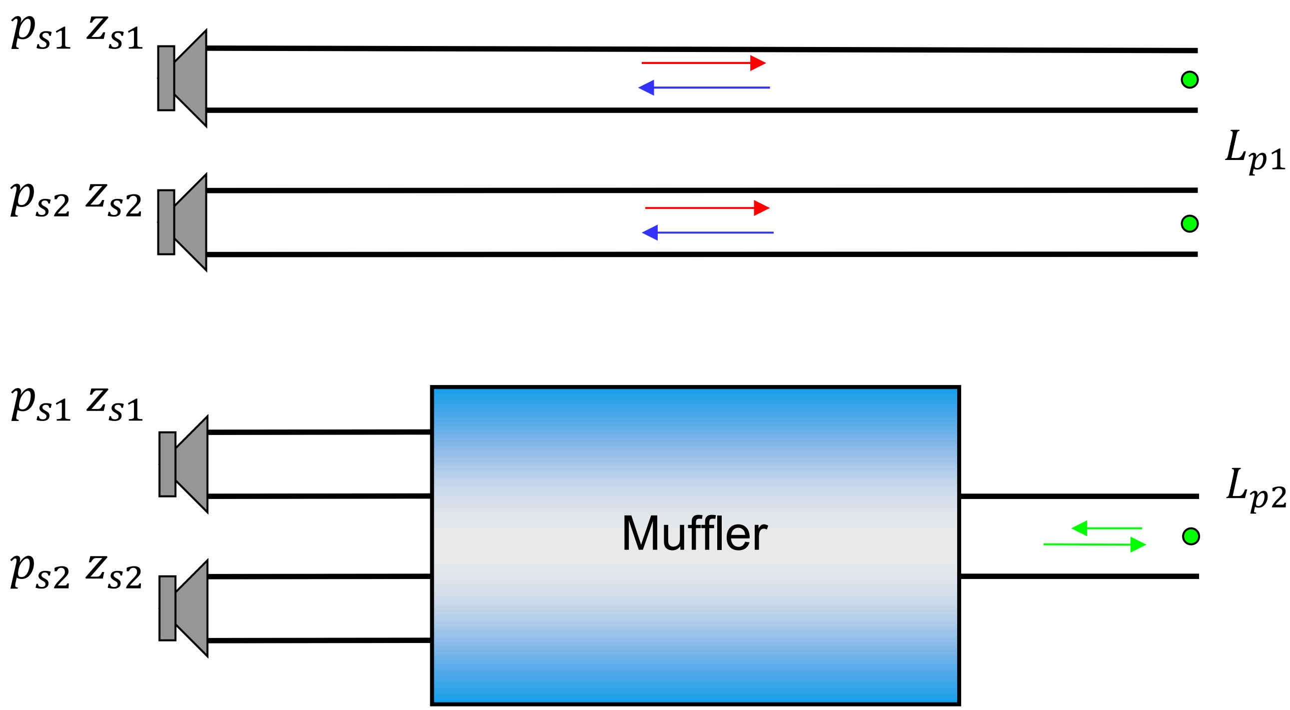
n-Inlet Muffler with Outlet o

$$TL = 10\log\frac{S_1 + |\beta_1|^2 S_2 + \ldots + |\beta_n|^2 S_n}{|H_{1o} + \beta_1 H_{2o} + \ldots + \beta_n H_{no}|^2 S_o}$$

$$\beta_m = \frac{p_{mi}}{p_{1i}}$$

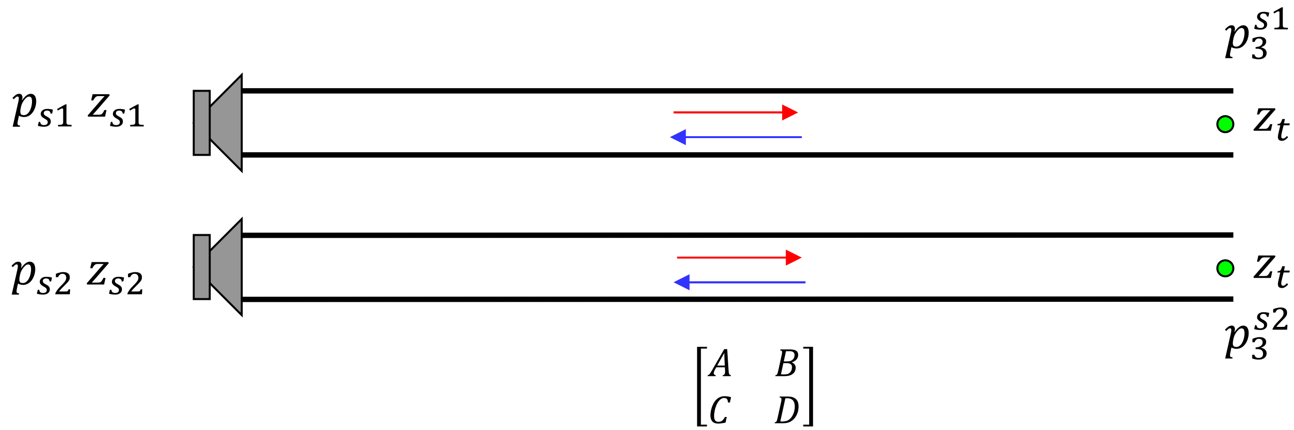
$\frac{\text{Incident wave strength of the } m\text{th inlet}}{\text{Incident wave strength of the 1st inlet}}$

Insertion Loss



$$IL = L_{p1} - L_{p2}$$

Insertion Loss

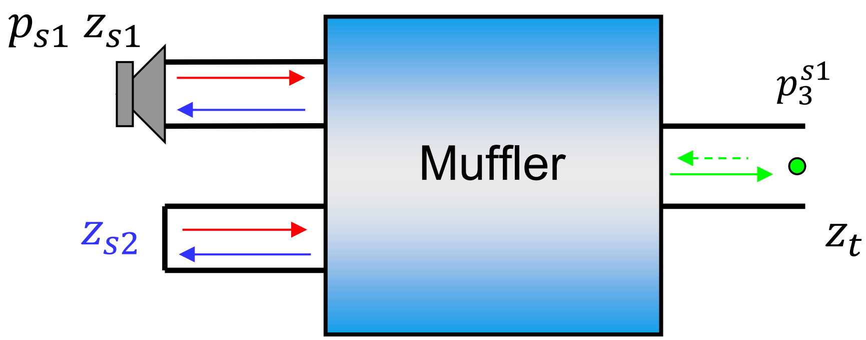


$$p_3^{s1} = \frac{p_{s1} z_t}{A_1 z_t + B_1 + C_1 z_{s1} z_t + D_1 z_{s1}} \qquad p_3^{s2} = \frac{p_{s2} z_t}{A_2 z_t + B_2 + C_2 z_{s2} z_t + D_2 z_{s2}}$$

$$L_{p_1} = 20 \log_{10}\left(\sqrt{(p_3^{s1})^2 + (p_3^{s2})^2 + 2 p_3^{s1} p_3^{s2} \cos\theta}\right)$$

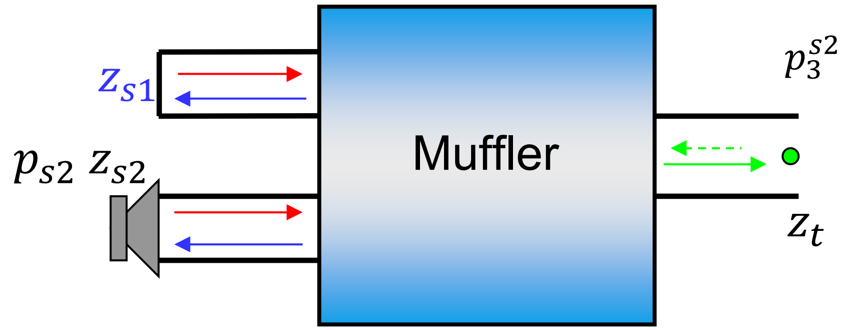
SPL with Muffler

Case 1



$$\begin{bmatrix} A_{13} & B_{13} \\ C_{13} & D_{13} \end{bmatrix}$$

Case 2



$$\begin{bmatrix} A_{23} & B_{23} \\ C_{23} & D_{23} \end{bmatrix}$$

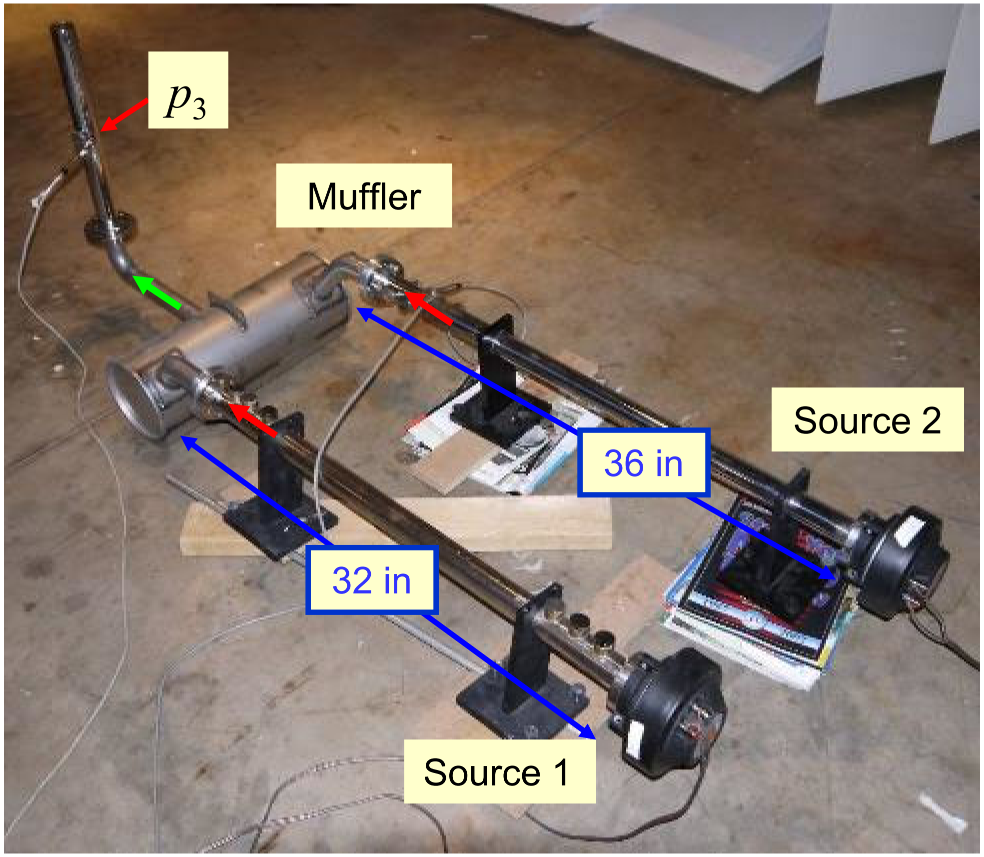
$$p_3^{s1} = \frac{p_{s1} z_t}{A_{13} z_t + B_{13} + C_{13} z_{s1} z_t + D_{13} z_{s1}}$$

$$p_3^{s2} = \frac{p_{s2} z_t}{A_{23} z_t + B_{23} + C_{23} z_{s2} z_t + D_{23} z_{s2}}$$

$$L_{p2} = 20 \log_{10}\left(\left| p_3^{s1} + p_3^{s2} \right| / p_{ref} \right)$$

* The similar technique can be applied to n-inlet muffler.

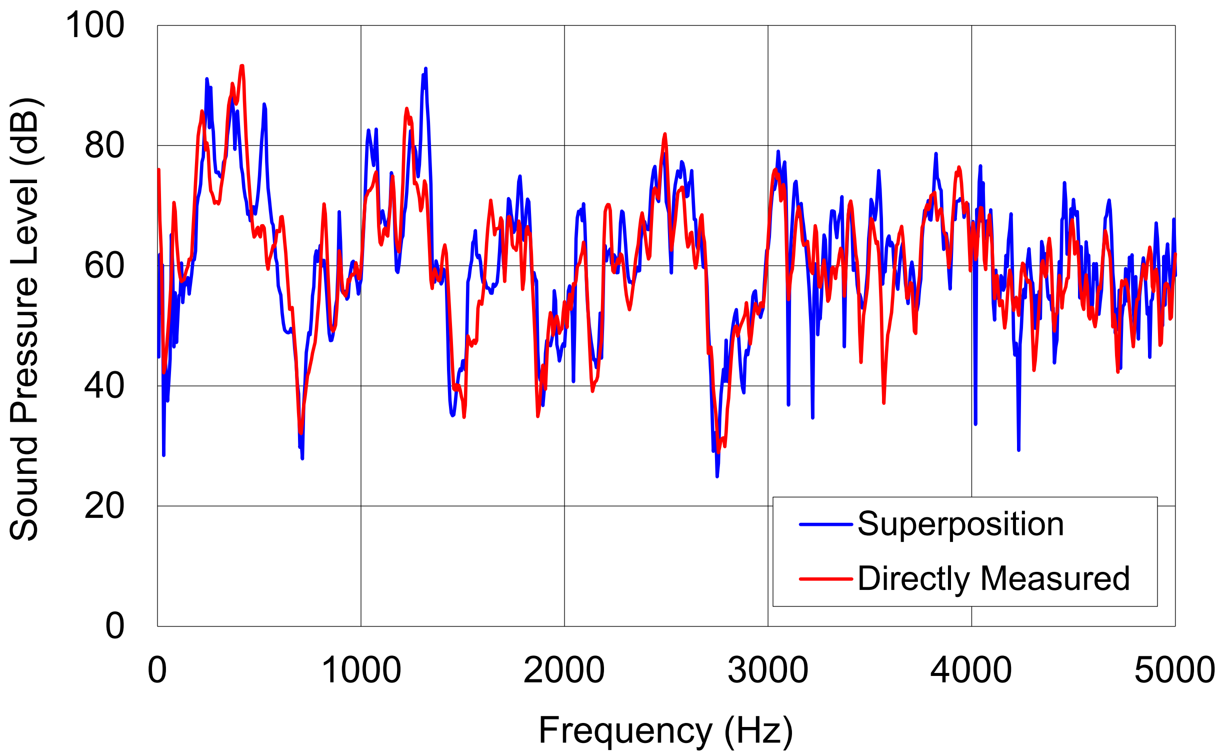
Experimental Validation



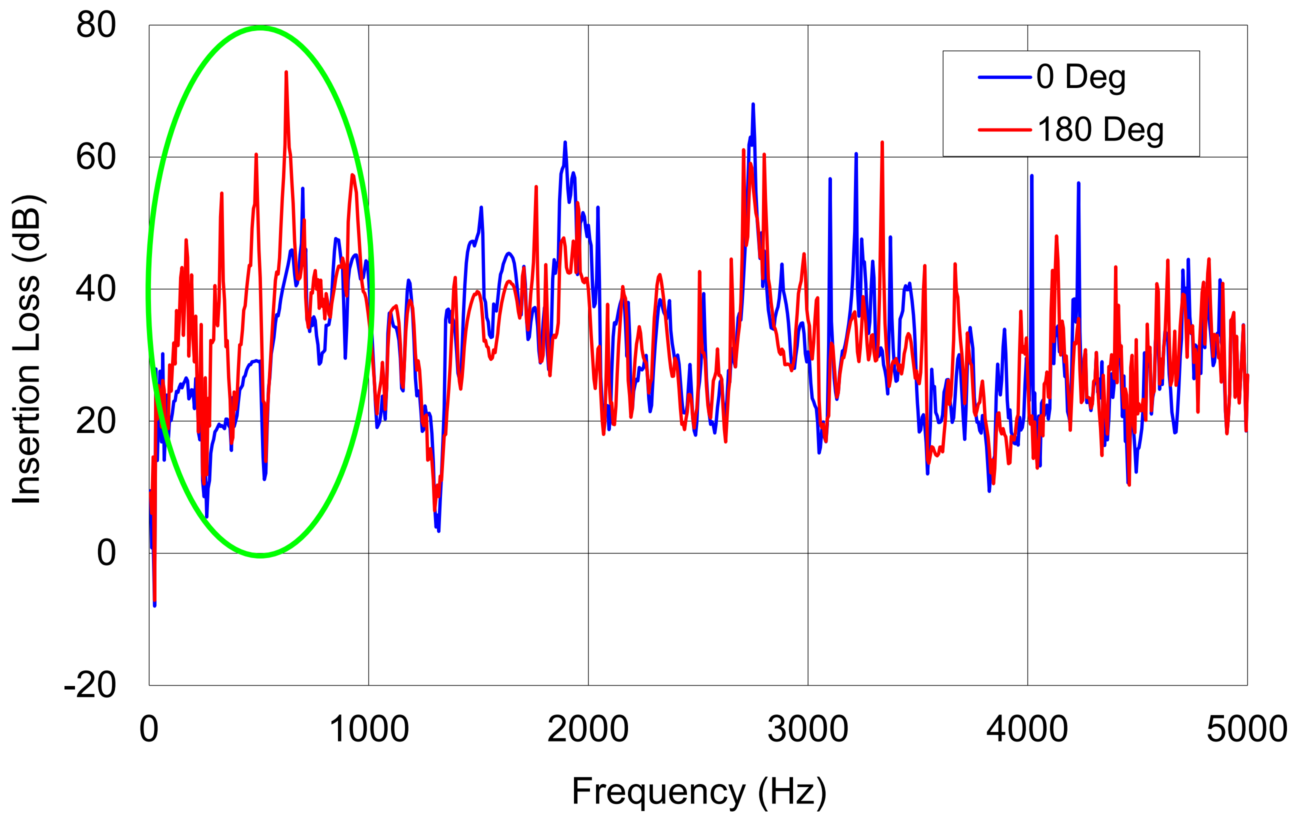
1. Measured source impedance and source strength for each loudspeaker.
2. Measured transfer matrix for muffler from each source to p_3 with other source “audible”.
3. Measure termination.
4. Use measured data from prior steps to predict the insertion loss.

Insertion Loss

Two sources in phase.



Effect of Source Phase



Overview

- Muffler Metrics
- Transmission and Insertion Loss
- Source Impedance
- Simulation of Source Impedance
- Multi-Inlet Mufflers

References

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