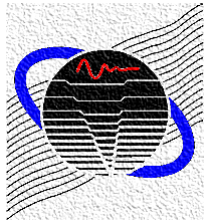
June 25, 2020

Muffler and Silencer Design

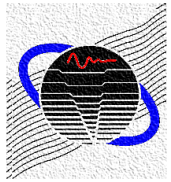
Vibro-Acoustics Consortium Web Meeting

University of Kentucky

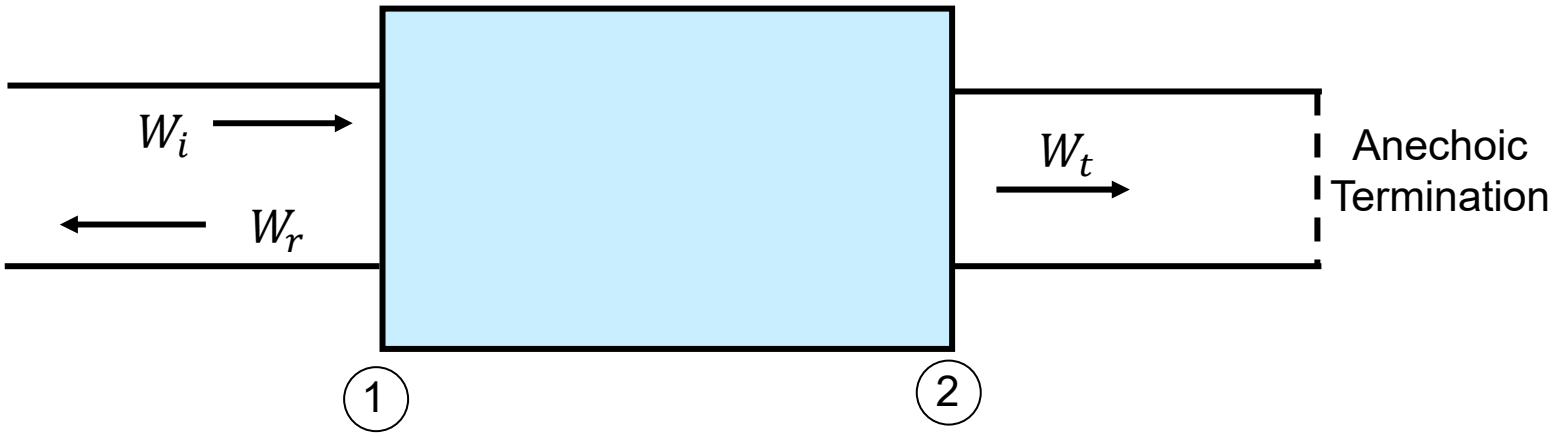


Overview

- Fundamental Concepts
- Illustrative Examples
- Conceptual Design
- Detailed Design
- Prototyping / Testing



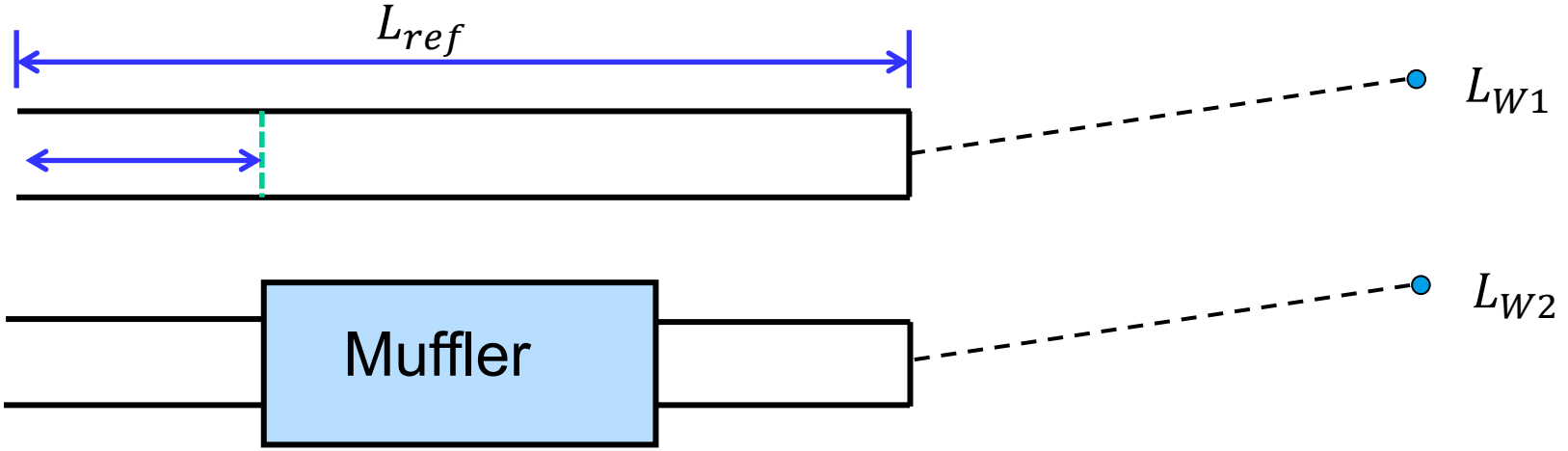
Performance Measures Transmission Loss



Transmission loss (TL) of the muffler:

$$TL = 10 \log_{10} \frac{W_i}{W_t}$$

Performance Measures Insertion Loss



$$IL(dB) = L_{W1} - L_{W2}$$

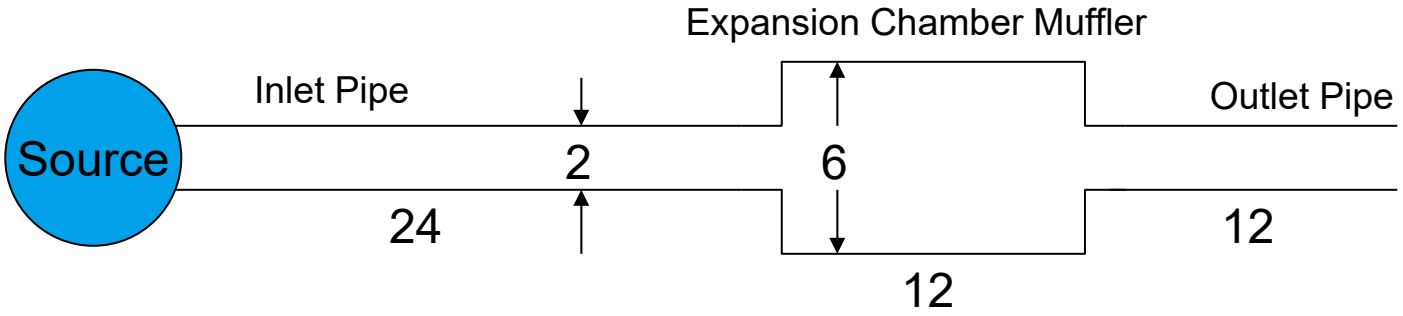
Insertion loss depends on :

- Muffler
- Lengths of pipes
- Termination (baffled vs. unbaffled)
- Source impedance

Note: TL is a property of the muffler; IL is a "system" performance measure.

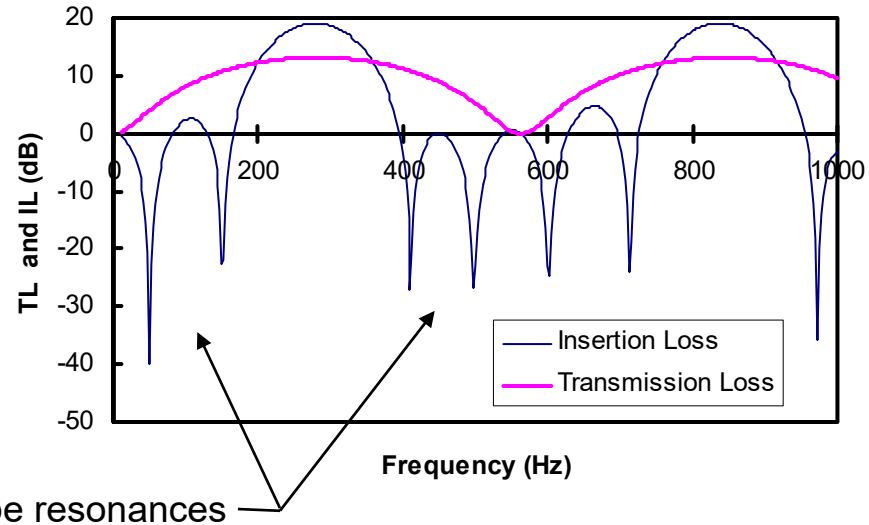
Example Transmission and Insertion Loss

Dimensions in Inches

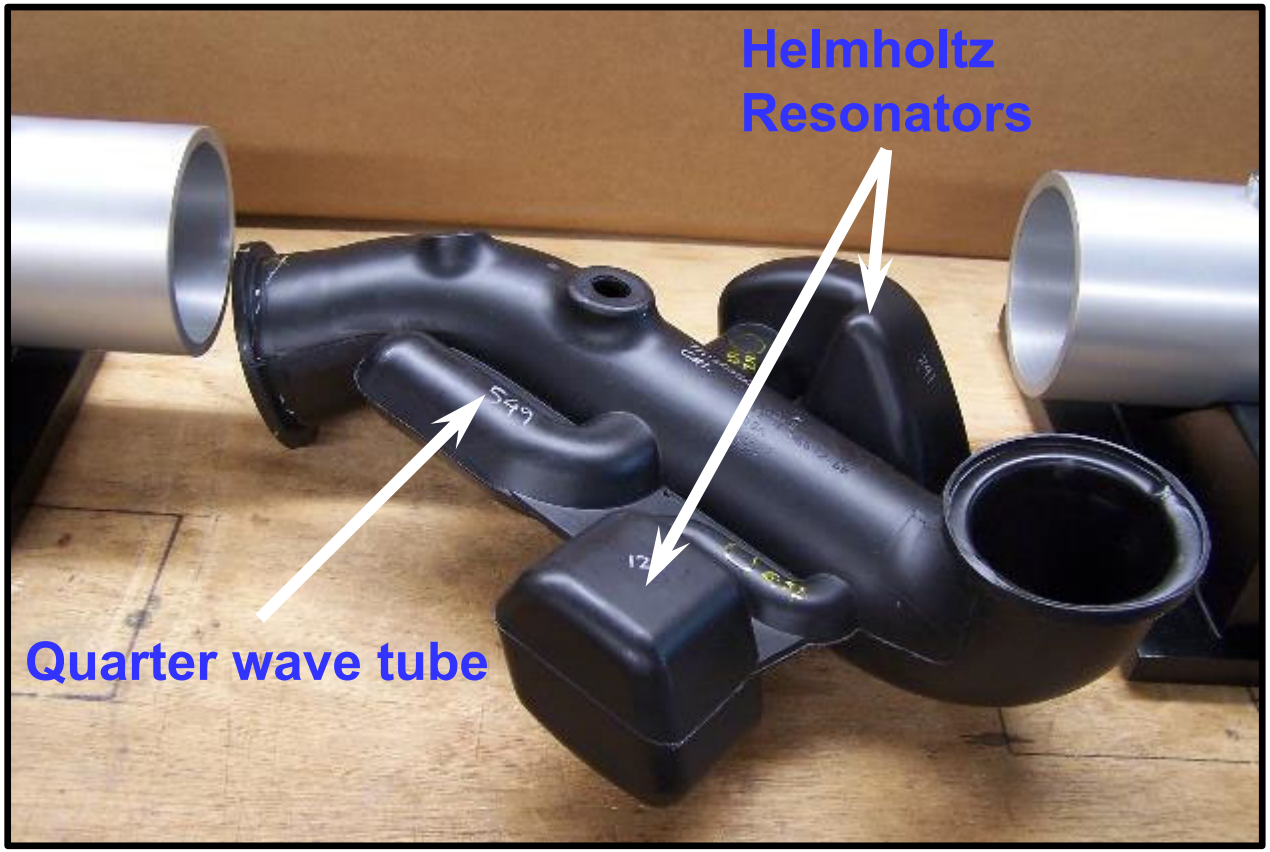


Acoustic Wavelength

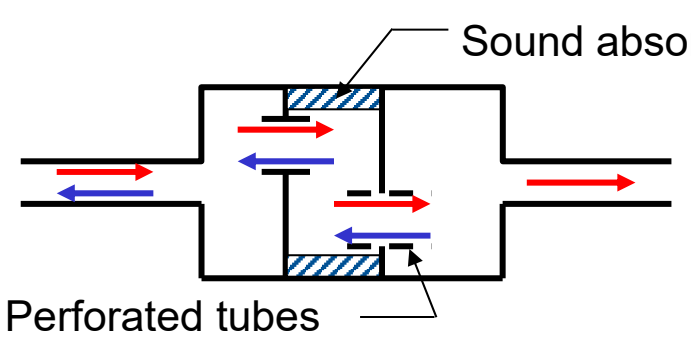
$$\lambda = \frac{c}{f} = \frac{343\ \mathrm{m/s}}{f}$$

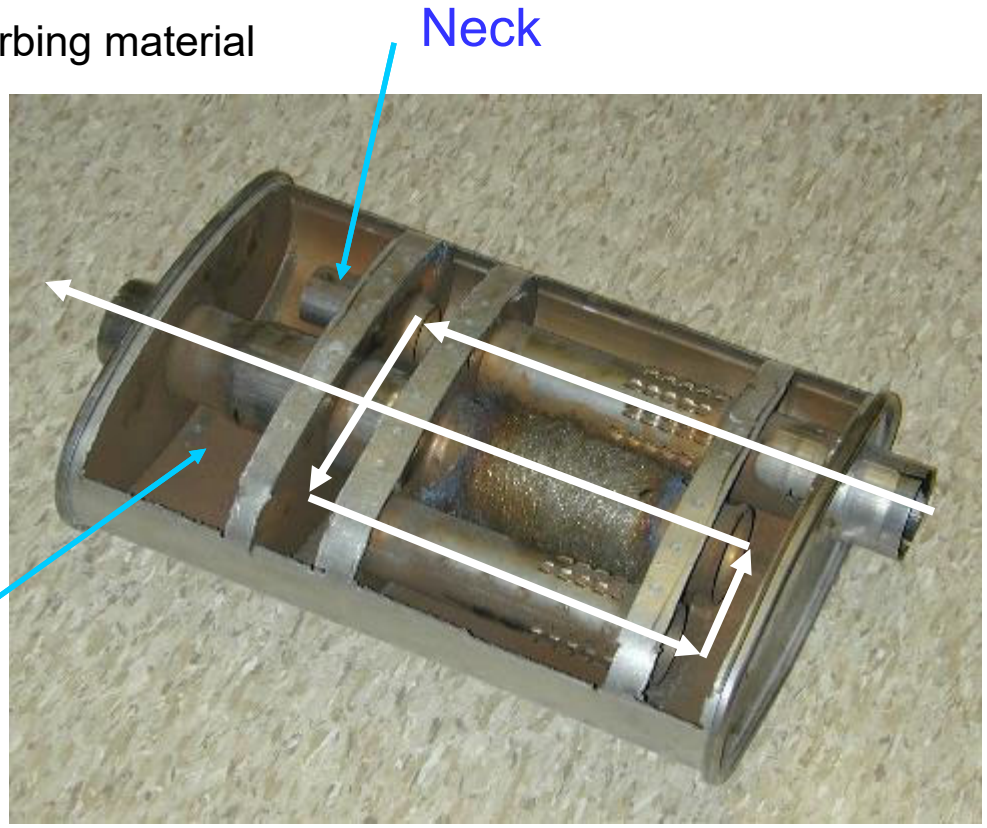


Reactive Muffler

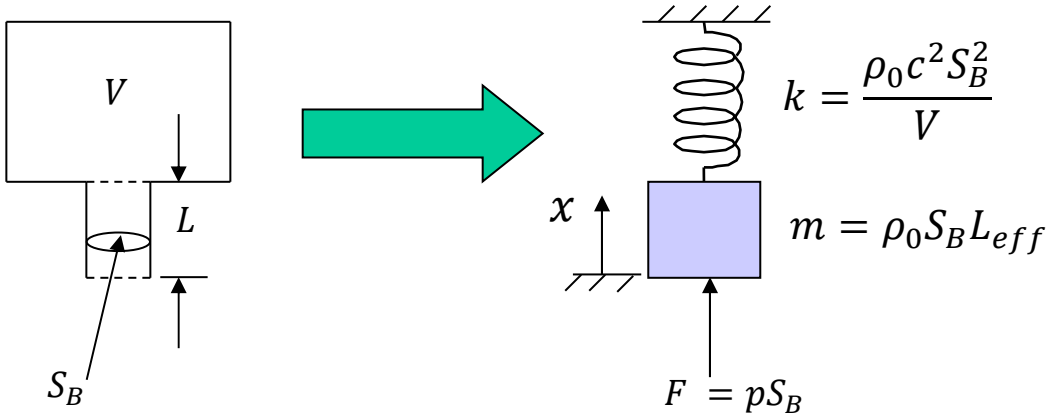


Reactive and Dissipative Muffler





The Helmholtz Resonator



L_{eff} is the equivalent length of the neck (some air on either end also moves).

$$m\ddot{x} + kx = pS_B \qquad \ddot{x} = j\omega u_B \qquad x = \frac{u_B}{j\omega}$$

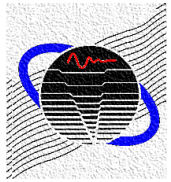
Damping due to viscosity in the neck are neglected

$$j\left(\omega m - \frac{k}{\omega}\right) u_B = pS_B$$

$$z_B = \frac{p}{S_B u_B} = j\left(\frac{1}{S_B^2}\right)\left(\omega m - \frac{k}{\omega}\right)$$

$$z_B \to \omega = \sqrt{\frac{k}{m}} = c\sqrt{\frac{S_B}{L_{eff} V}}$$

(resonance frequency of the Helmholtz resonator)



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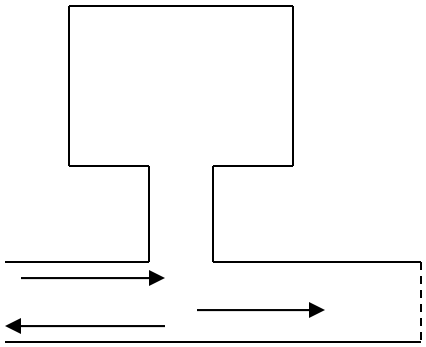
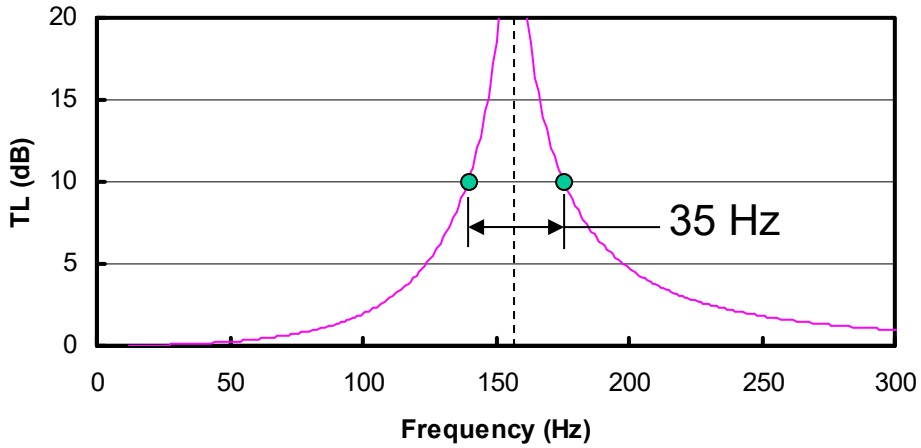


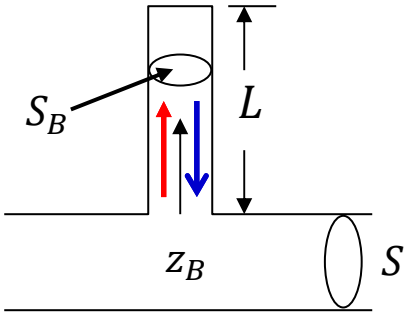
$$TL = 10\log_{10}\left(1+\left(\frac{c/2S}{\omega L_{eff}/S_B - c^2/\omega V}\right)^2\right)$$

Anechoic termination

$V = 0.001\ \text{m}^3$
$L = 25\ \text{mm}$
$S_B = 2 \times 10^{-4}\ \text{m}^2$
$S = 8 \times 10^{-4}\ \text{m}^2$



Quarter Wave Resonator



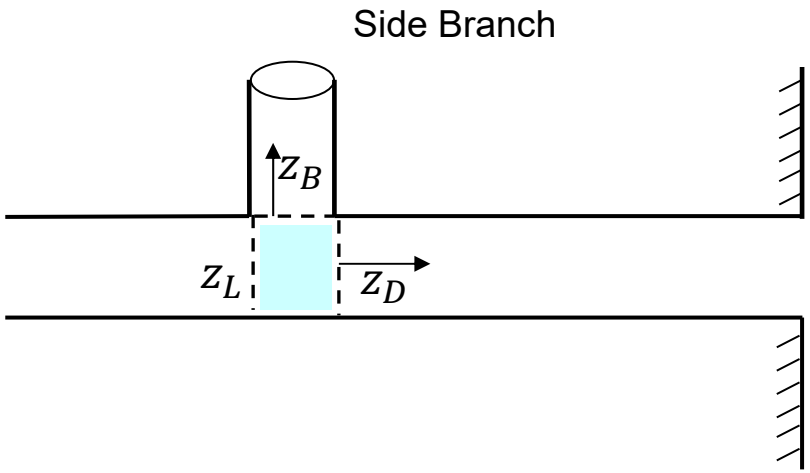
$$TL = 10\log_{10}\left(\frac{\tan^2(kl) + 4(S/S_B)^2}{4(S/S_B)^2}\right)$$

$$z_B = -\frac{j\rho_0 c}{S_B}\cot\left(\frac{\omega L}{c}\right) = 0 \quad \text{and} \quad \frac{\omega L}{c} = \frac{n\pi}{2} \quad n = 1,3,5,\ldots$$

$$\omega_n = \frac{n\pi c}{2L} \quad n = 1,3,5,\ldots$$

$$f_n = \frac{nc}{4L} \quad \text{or} \quad L = \frac{nc}{4f} = n\left(\frac{\lambda}{4}\right) \quad n = 1,3,5,\ldots$$

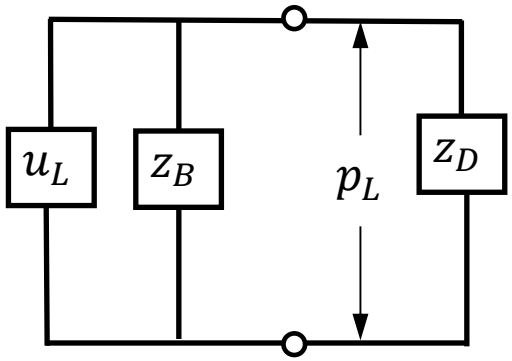
Parallel Impedance



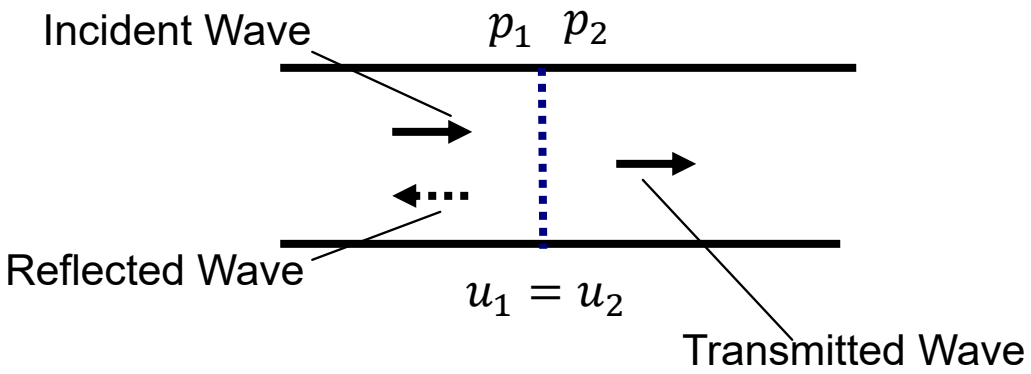
$$z_L = \frac{p_L}{Su_L} = \left(\frac{z_B z_D}{z_B + z_D}\right)$$

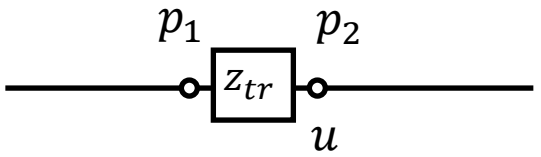
$$z_B = 0$$

$$z_L = 0$$



Series (Transfer) Impedance

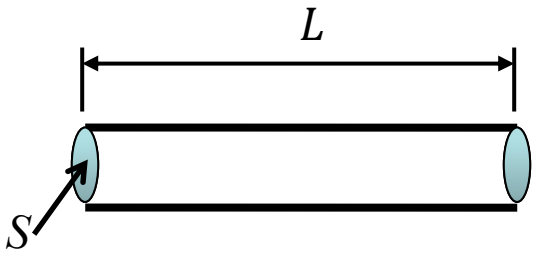




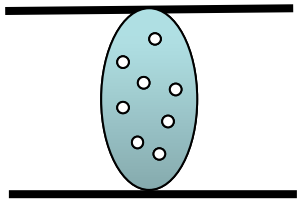
$$z_{tr} = \frac{p_1 - p_2}{Su}$$

Muffler Elements

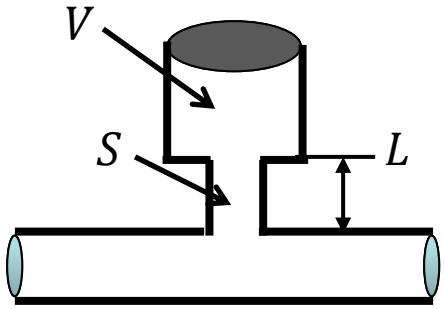
Straight Pipe



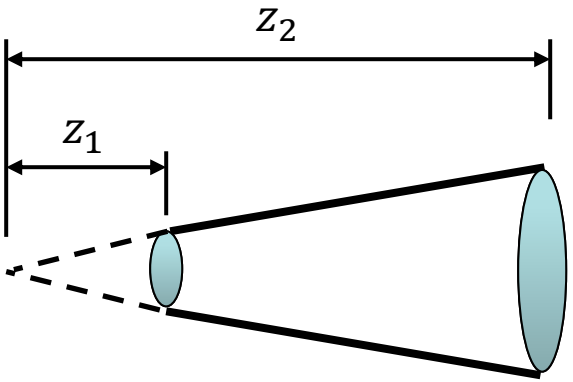
Perforate



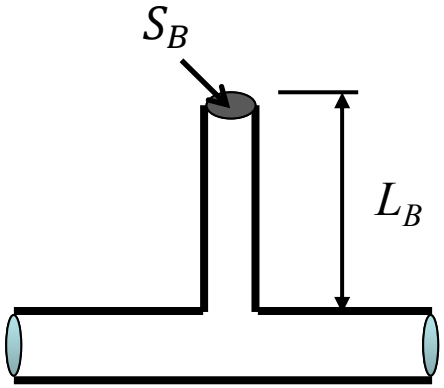
Helmholtz Resonator



Cone



Quarter Wave Tube



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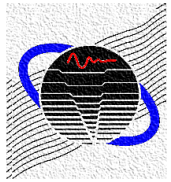
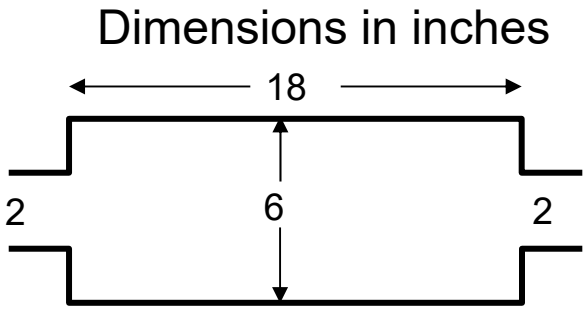
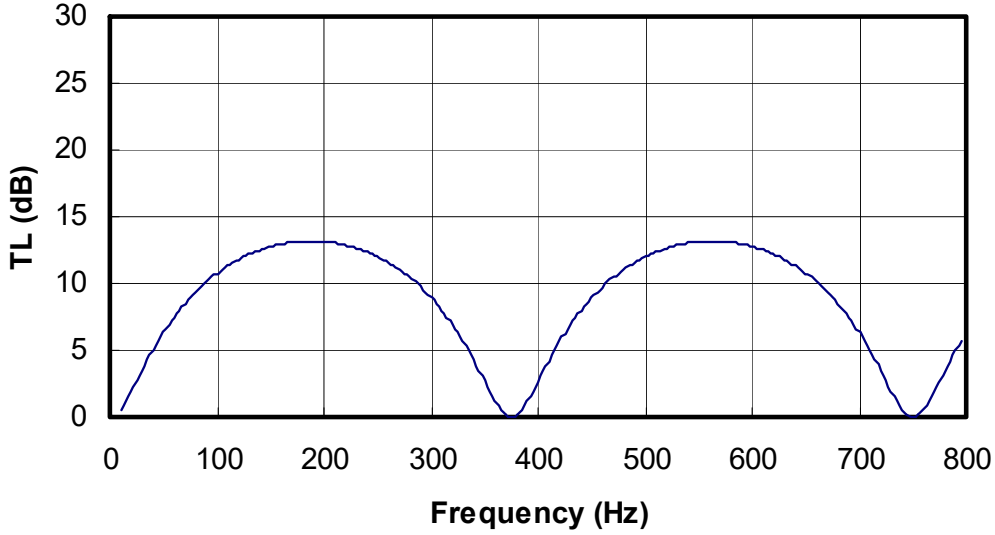


Muffler Elements Simple Expansion Chamber



$$TL = 10\log_{10}\left(\frac{1}{4}\left(4\cos^2(kL) + \left(m + \frac{1}{m}\right)^2 \sin^2(kL)\right)\right)$$

where m is the expansion ratio (chamber area/pipe area) = 9 in this example and *L* is the length of the chamber.



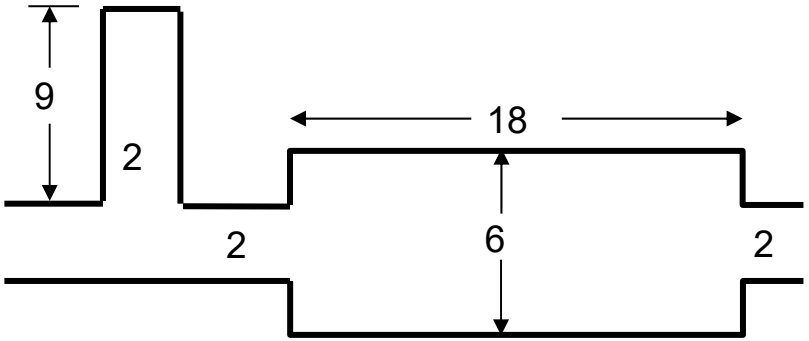
$$m = d_o^2/d_i^2$$

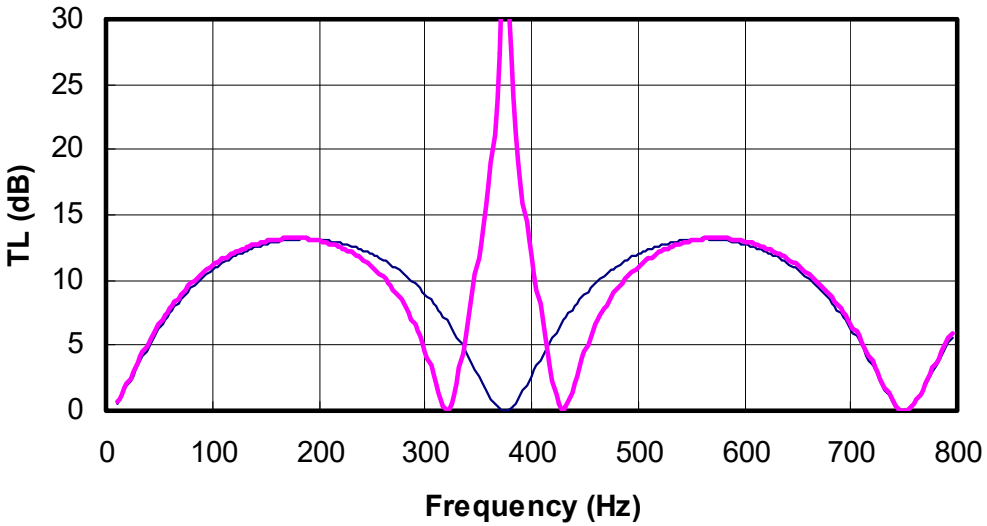
If $m > 10$

$$TL \approx 10\log_{10}\left(\frac{1}{4}(m^2 \sin^2(kL))\right)$$

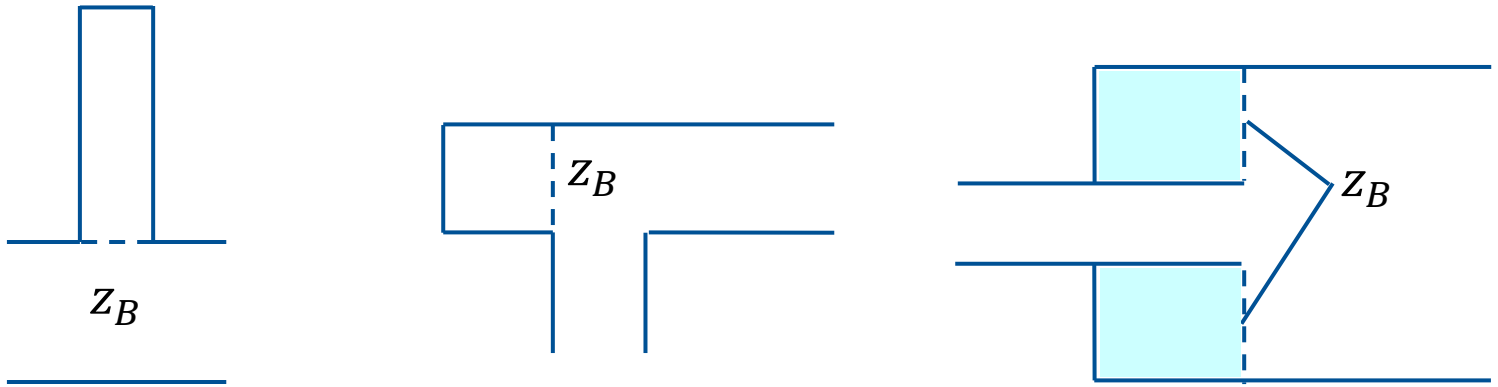
Muffler Elements Quarter Wave Tube

Dimensions in inches



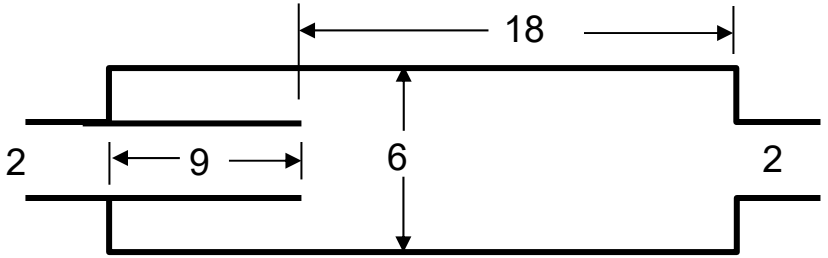


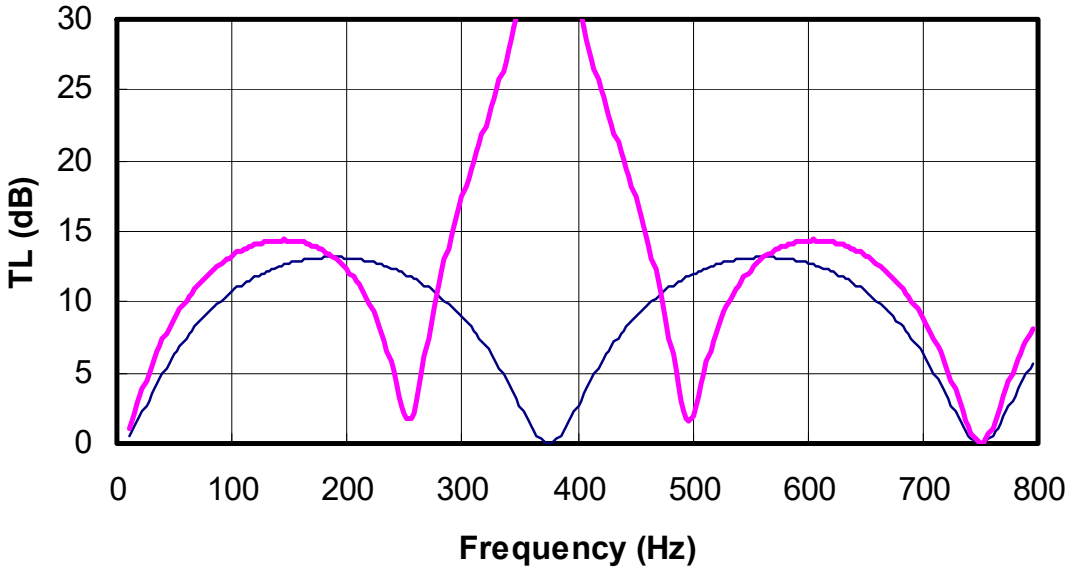
A Note on Quarter Wave Tubes



Muffler Elements Extended Inlet

Dimensions in inches

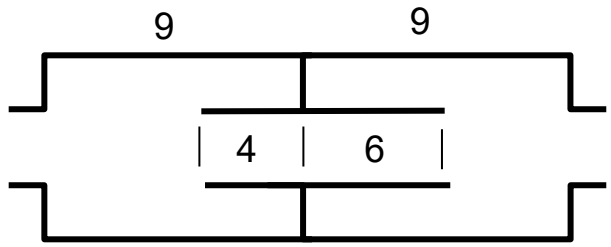


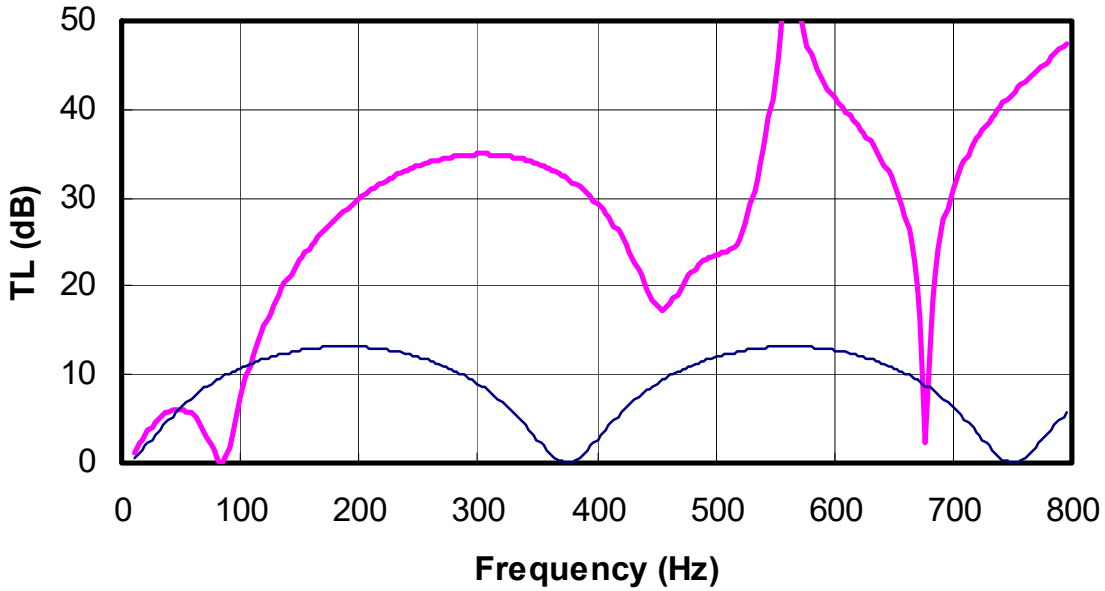


(similar for extended outlet)

Muffler Elements Extended Inlet and Outlet

Dimensions in inches

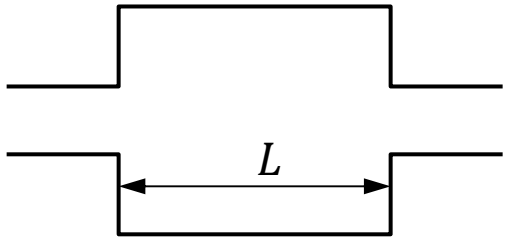




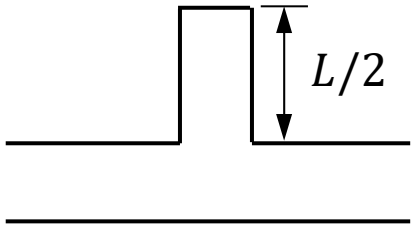
Muffler Elements Extended Inlet and Outlet

Dimensions in inches

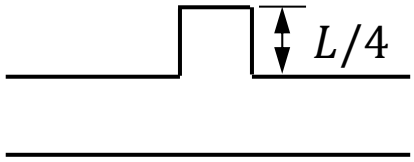
Expansion Chamber

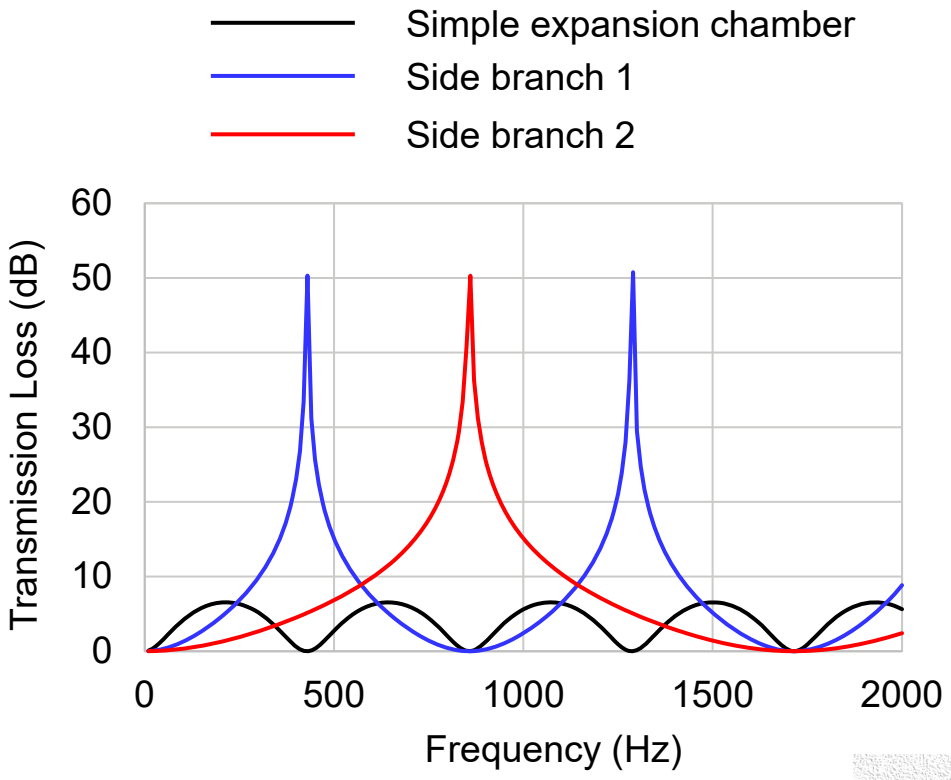


Side Branch 1



Side Branch 2

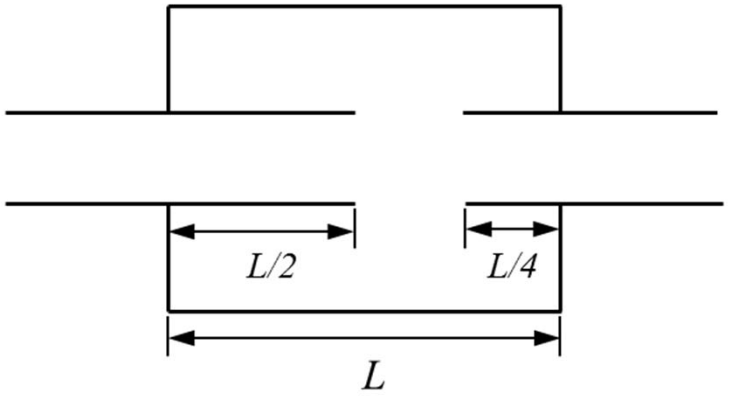


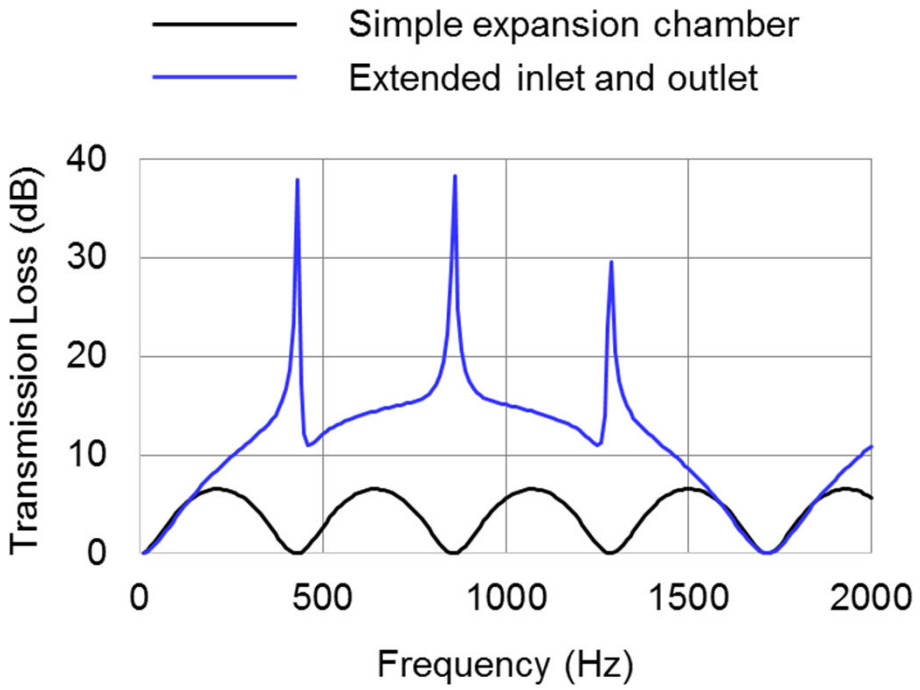


Double Tuned Expansion Chamber

Munjal, 2011

Extend inlet and outlet into the expansion chamber:





Three-fourths of the troughs are nullified.

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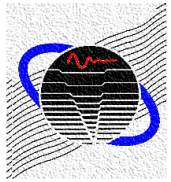
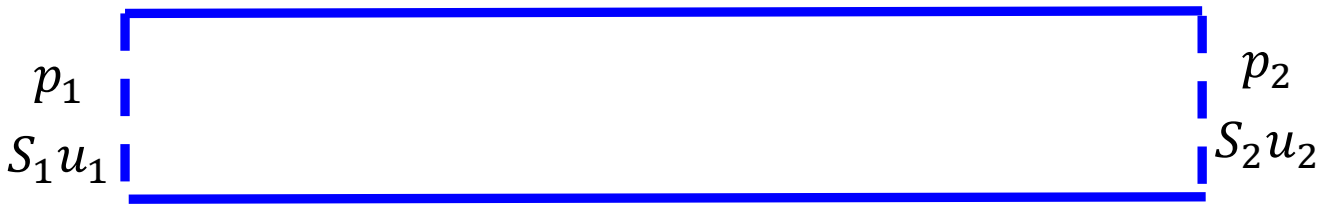
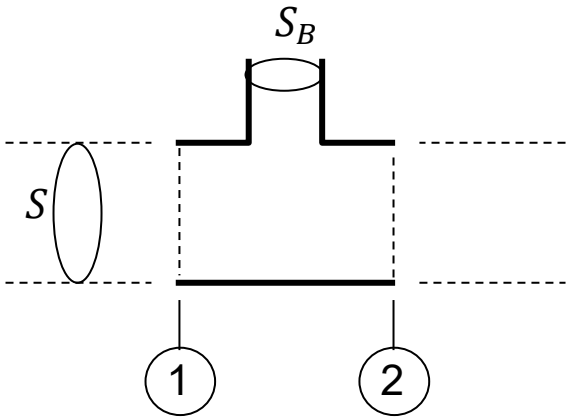


Plane Wave Modeling

$$\begin{Bmatrix} p_1 \\ S_1 u_1 \end{Bmatrix} = \begin{bmatrix} \cos(kL) & \frac{j\rho_0 c}{S_2}\sin(kL) \\ \frac{jS_1}{\rho_0 c}\sin(kL) & \frac{S_1}{S_2}\cos(kL) \end{bmatrix} \begin{Bmatrix} p_2 \\ S_2 u_2 \end{Bmatrix}$$



Parallel Impedance



$$p_1 = p_2 = p_B$$

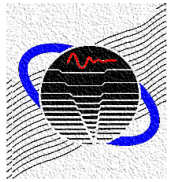
$$Su_1 = S_B u_B + Su_2$$

$$z_B = \frac{p_B}{S_B u_B} = \frac{p_2}{S_B u_B}$$

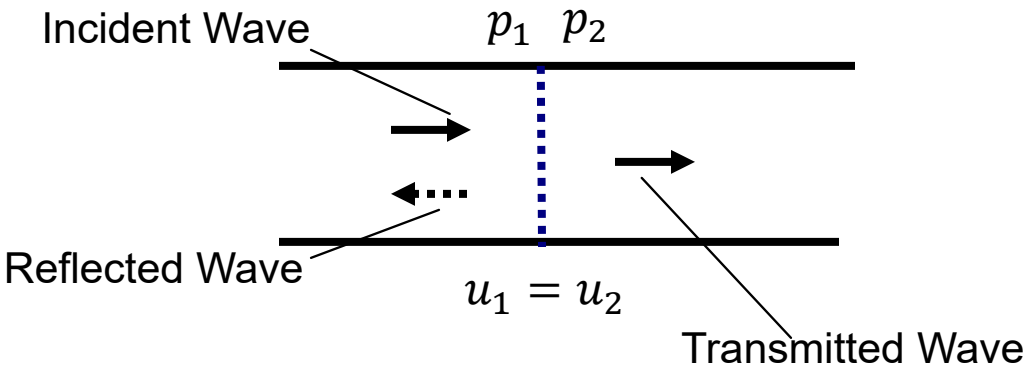
$$Su_1 = \frac{p_2}{z_B} + Su_2$$

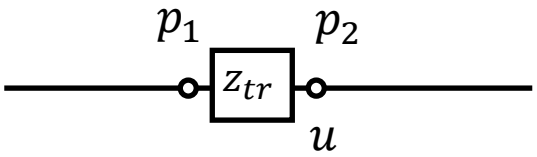
Transfer Matrix

$$\begin{Bmatrix} p_1 \\ S_1 u_1 \end{Bmatrix} = \begin{bmatrix} 1 & 0 \\ 1/z_B & 1 \end{bmatrix} \begin{Bmatrix} p_2 \\ S_2 u_2 \end{Bmatrix}$$



Series (Transfer) Impedance

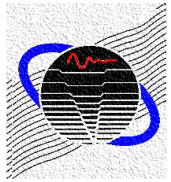




$$z_{tr} = \frac{p_1 - p_2}{Su}$$

Transfer Matrix

$$\begin{Bmatrix} p_1 \\ S_1 u_1 \end{Bmatrix} = \begin{bmatrix} 1 & z_{tr} \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} p_2 \\ S_2 u_2 \end{Bmatrix}$$



Empirical Equations Perforated Panel

Sullivan and Crocker (1978)

$$z_{tr} = (1/\rho c\sigma)(2.4 + j0.02f)$$

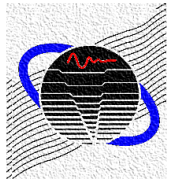
Sullivan and Crocker (1978))

$$z_{tr} = (1/\sigma)\left(0.006 + jk(t + 0.75d_h)\right)$$

ρ	density of air
c	speed of sound in air
f	frequency
k	wave number ($^{2\pi f}/_{c}$)
σ	porosity
t	panel thickness
d_h	hole diameter
M	flow Mach number

Rao and Munjal (1986)

$$z_{tr} = (1/\sigma)\left(7.337 \times 10^{-3}(1 + 72.23M) + j2.2245 \times 10^{-5} f(1 + 51t)(1 + 204d_h)\right)$$



Combining Elements

For each element:

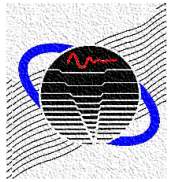
$$\begin{Bmatrix} p_i \\ S_i u_i \end{Bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}_i \begin{Bmatrix} p_{i+1} \\ S_{i+1} u_{i+1} \end{Bmatrix}$$

Overall:

$$[T_{total}] = [T_1][T_2][T_3] \cdots [T_n] = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}_{total}$$

Transmission Loss

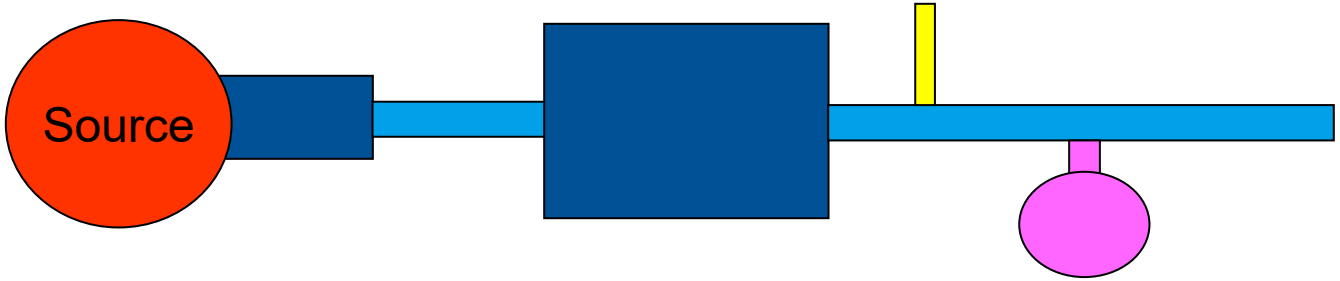
$$TL = 10 \log_{10} \left\{ \frac{S_{in}}{4S_{out}} \left| T_{11} + \frac{S_{out} T_{12}}{\rho c} + \frac{\rho c T_{21}}{S_{in}} + \frac{S_{out}}{S_{in}} T_{22} \right|^2 \right\}$$



Plane Wave Modeling Virtual Design

Plane Wave Modeling (Virtual Design)

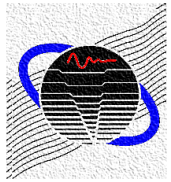
In course work



A real muffler

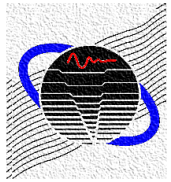
Plane Wave Modeling Tips

1. Examine the muffler to be modeled and determine a direction of sound propagation. This will normally be the direction having the longest cross-dimension and is also usually the same as the flow path.
2. Details like welds, seams, and fillets that are small compared to an acoustic wavelength can be ignored. However, leaks should be included.
3. Identify and include quarter wave tubes. Often in the case of right-angle turns, quarter wave tubes are unintentionally included. Additionally, be sure to include mass effects in the case of extended inlets or outlets.

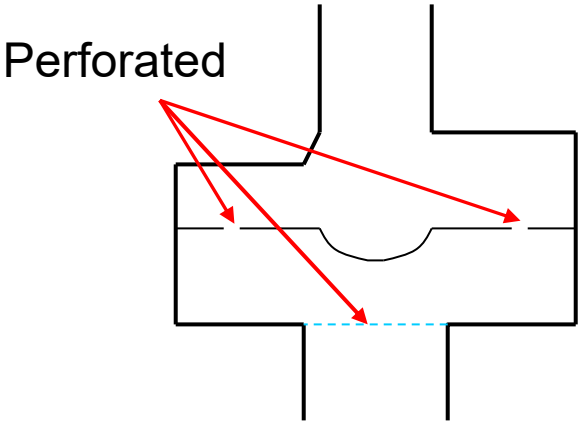


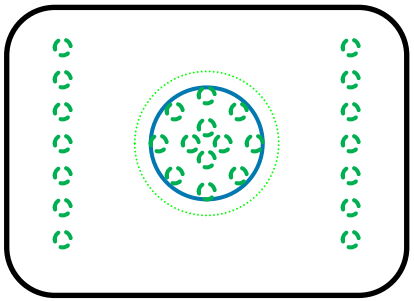
Plane Wave Modeling Tips

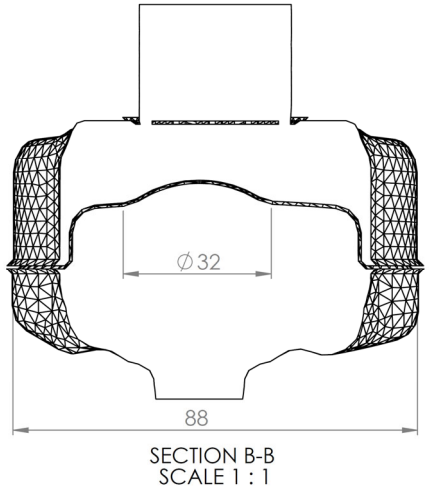
4. The shape of the cross-section, whether circular, elliptical, or rectangular will have no effect.
5. Sound will be reflected due to changes in the cross-sectional area. Significant area changes should be accounted for. Gradual changes in cross-sectional area can be modeled as cones.
6. If the porosity (perforation rate) exceeds 30%, a perforate can generally be ignored and treated as open.



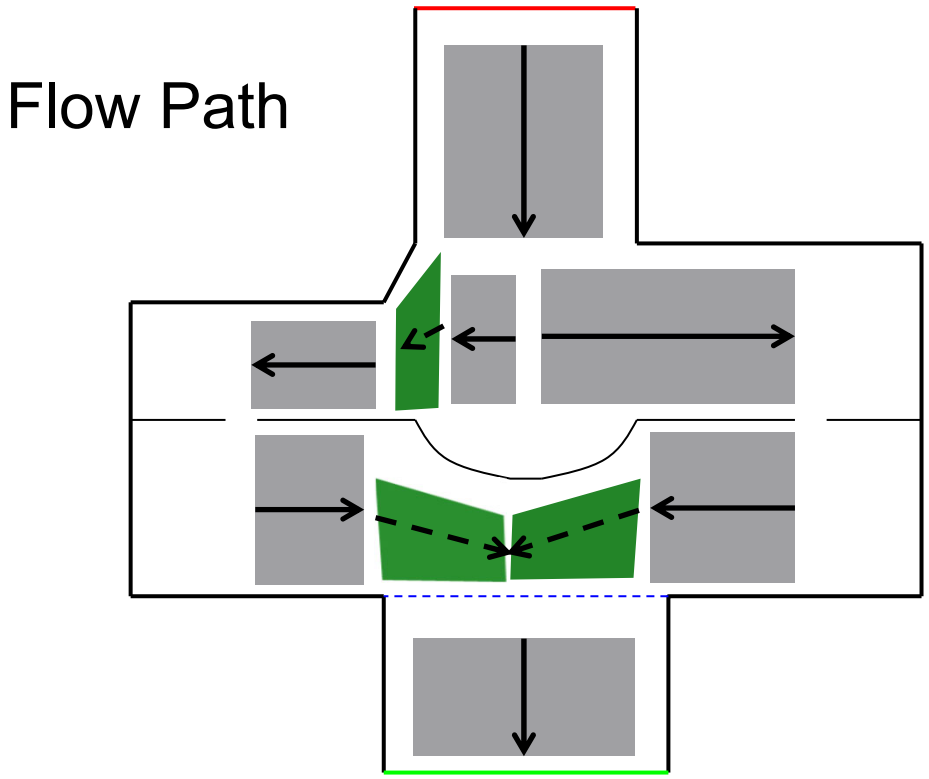
Example Small Engine Muffler



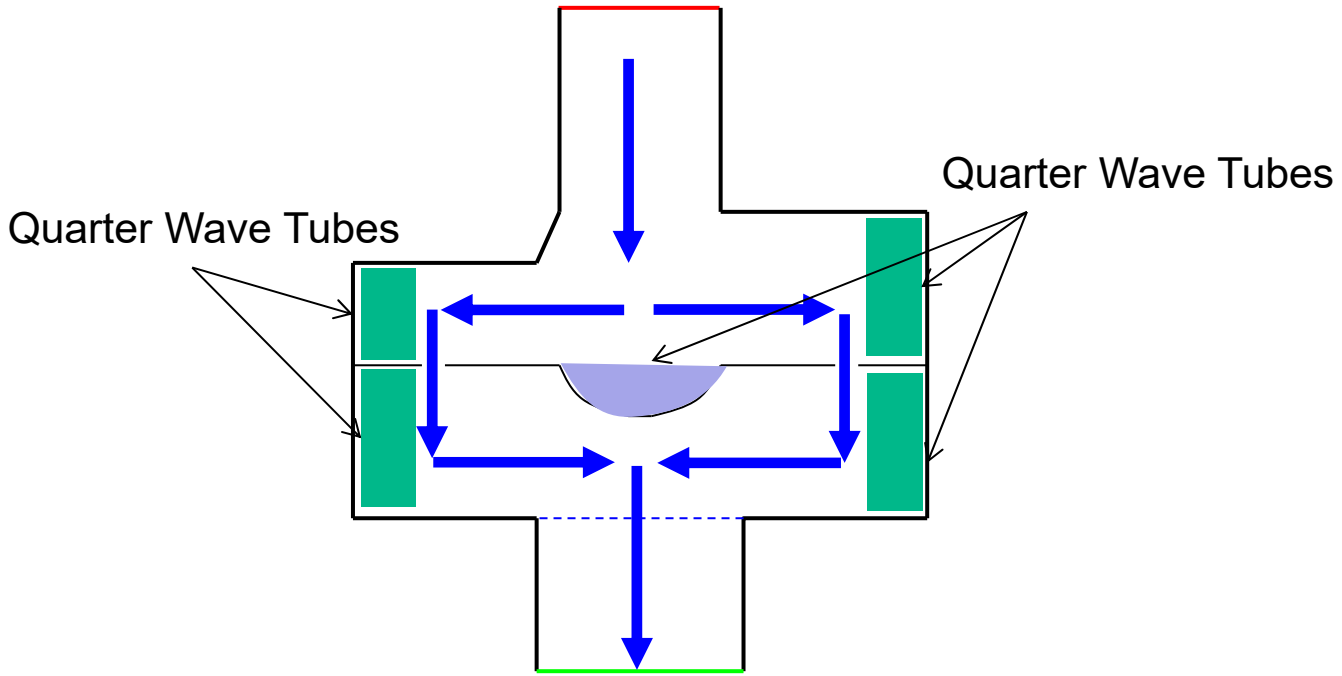




Example Small Engine Muffler

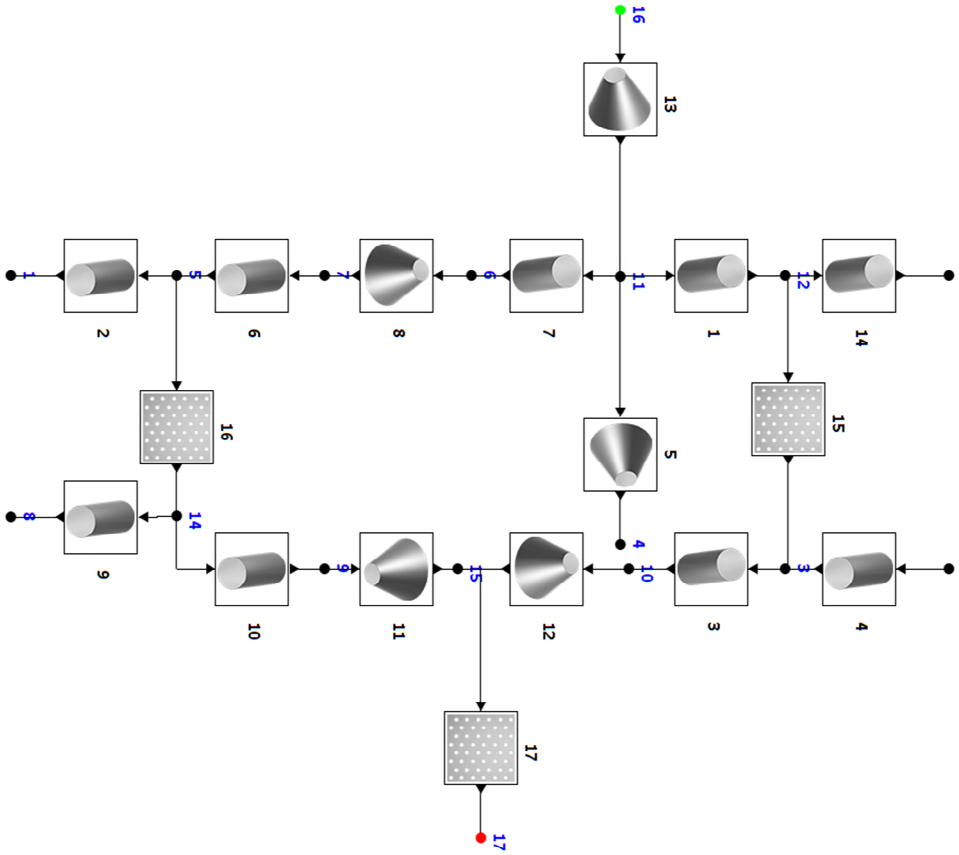


Example Small Engine Muffler

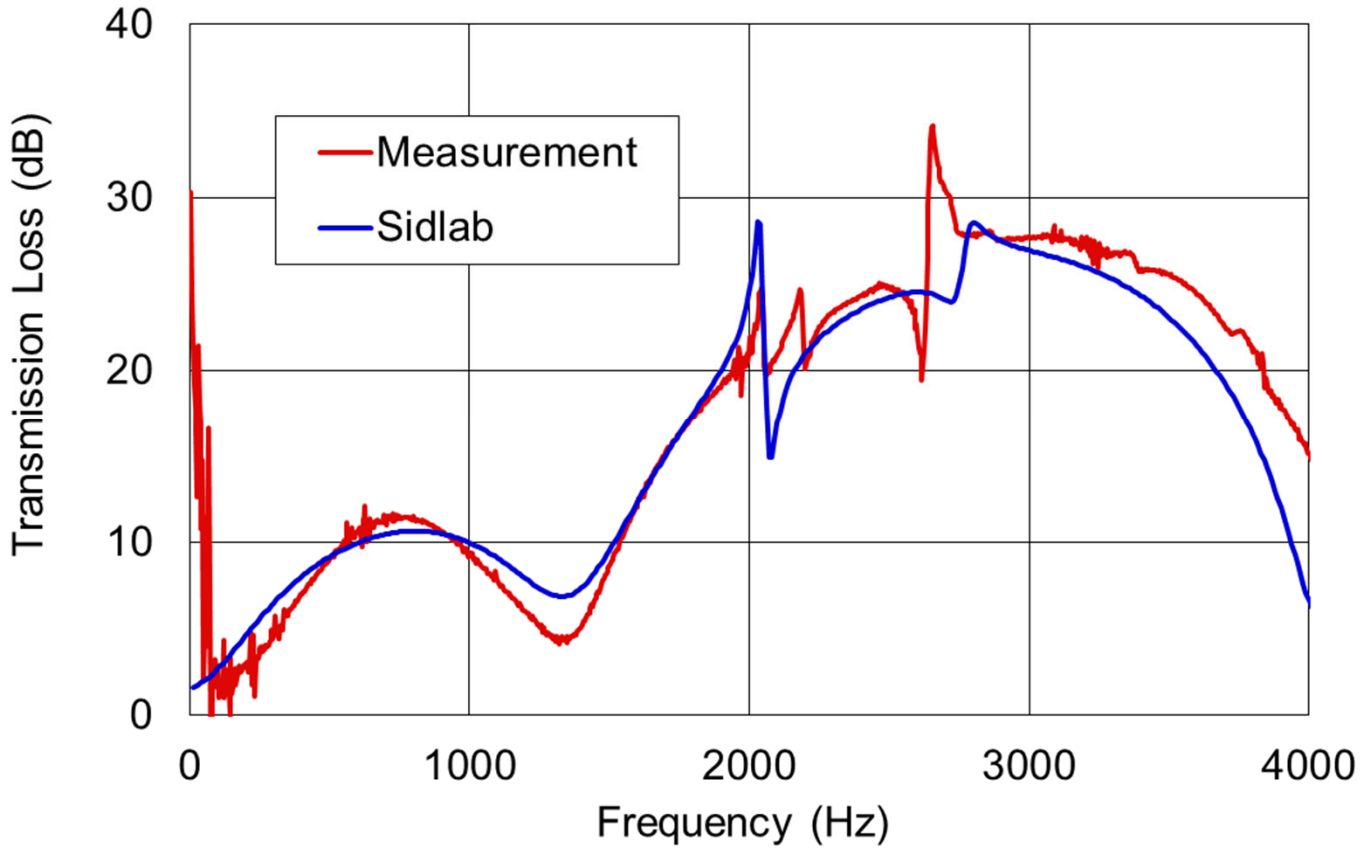


Example Small Engine Muffler

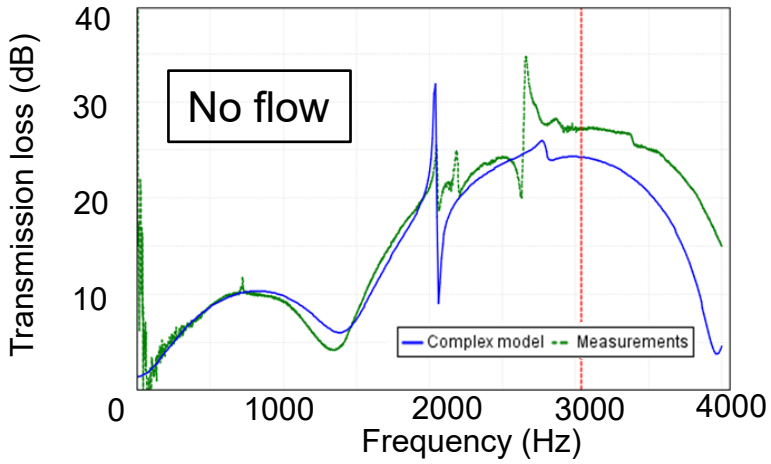
SIDLAB Model

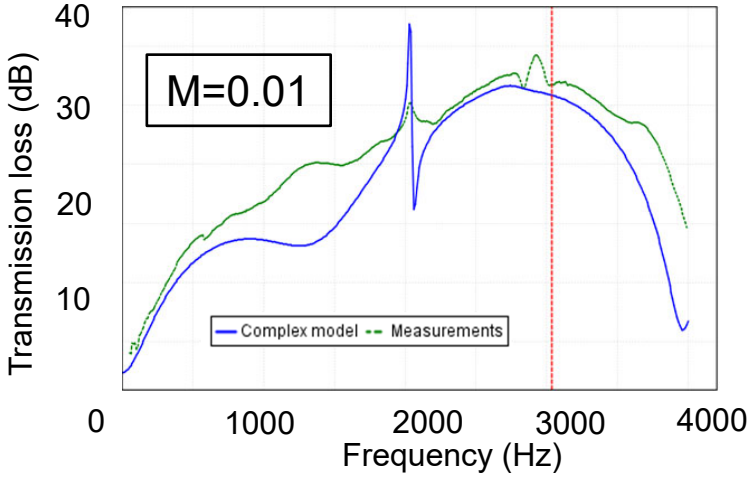


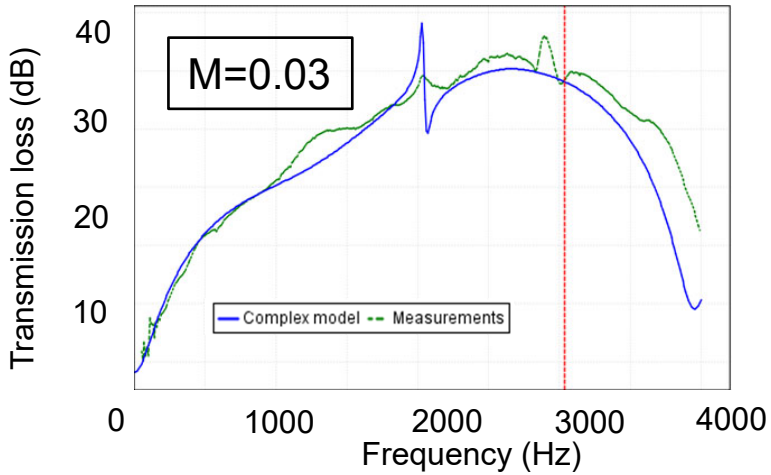
Example Small Engine Muffler

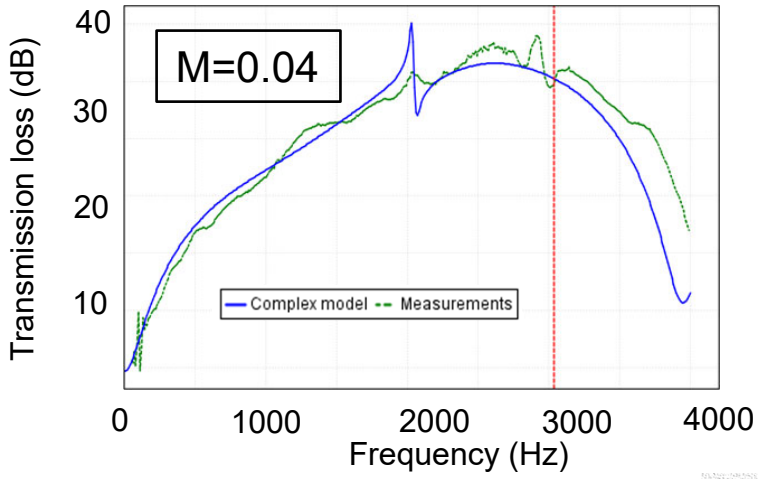


A Note on Perforates and Flow

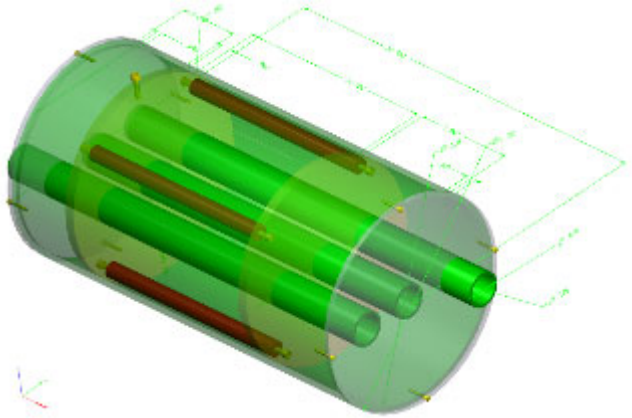


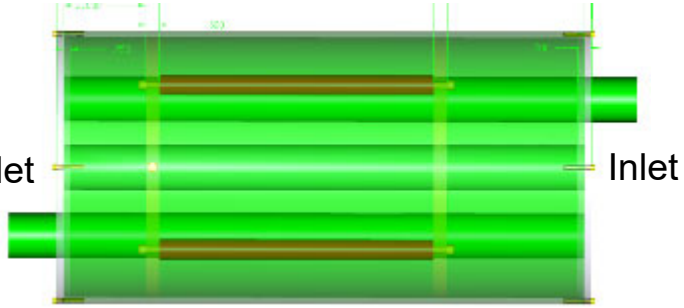


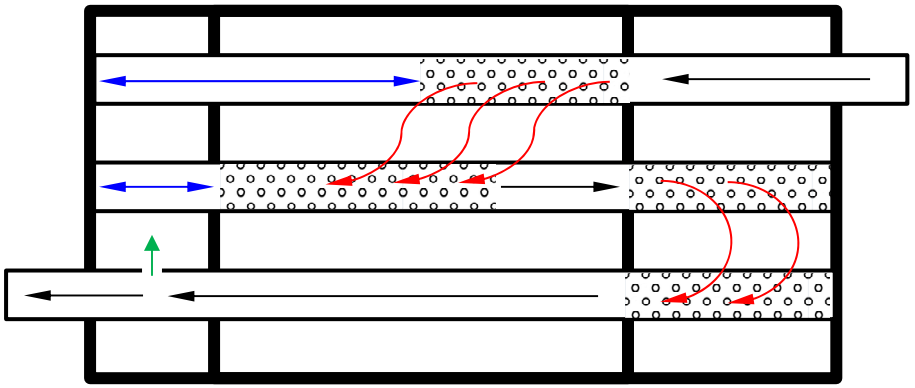




Example Cross Flow Muffler







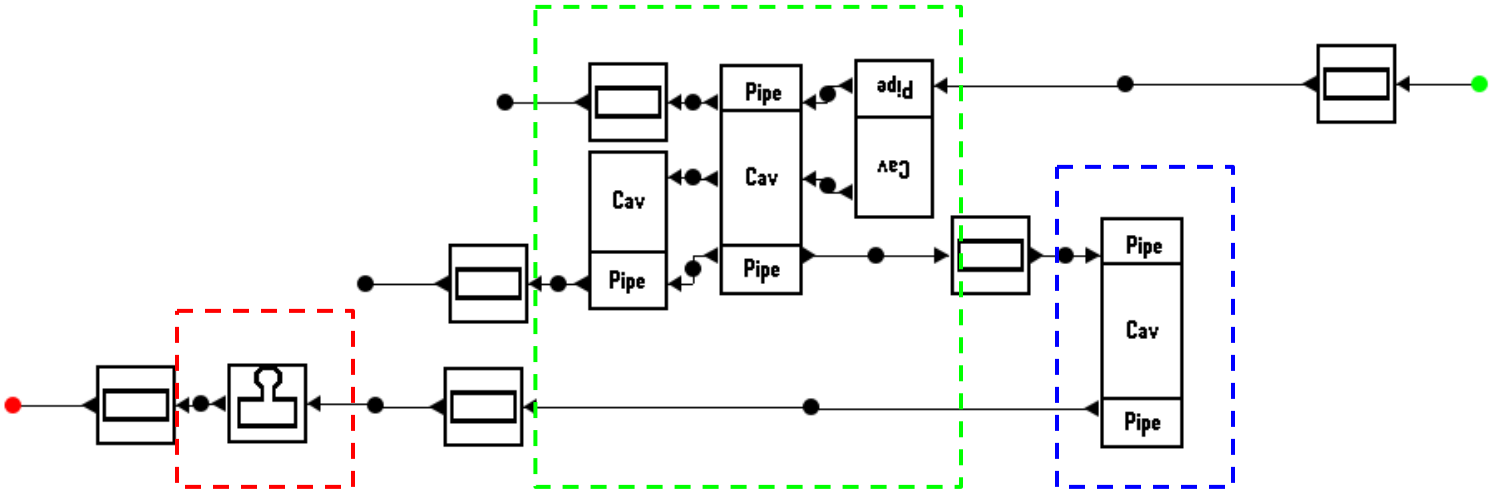
: Propagation

: Quarter-wavelength tube

: Cross-flow

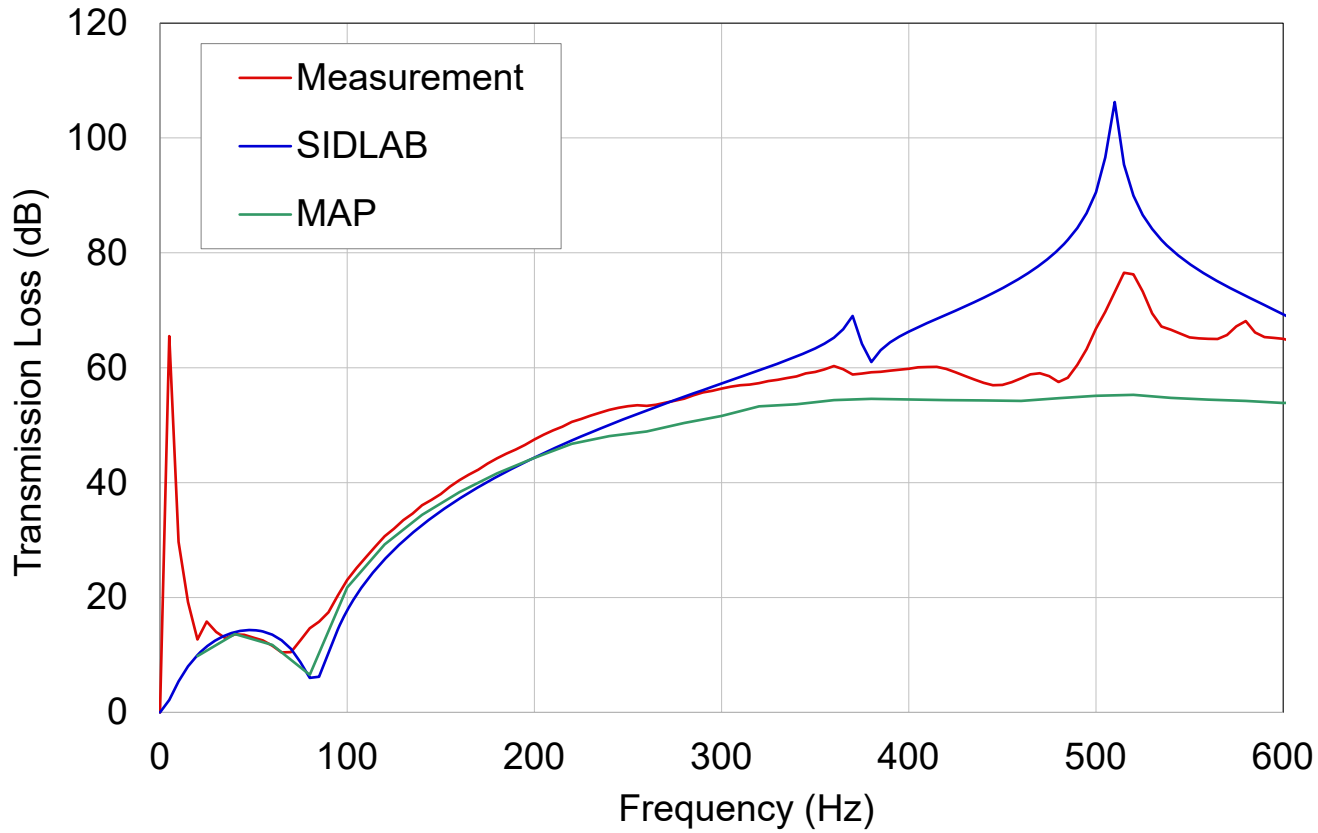
: Helmholtz resonator

Example Cross Flow Muffler



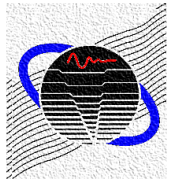
3rd chamber 2nd chamber 1st chamber

Example Cross Flow Muffler

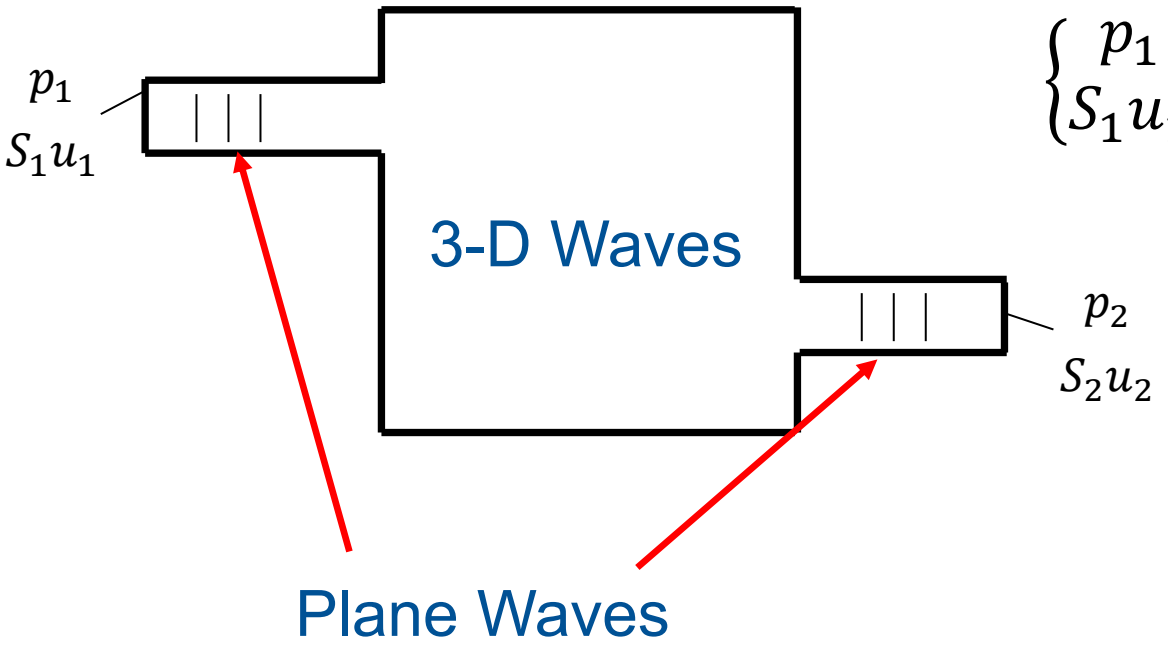


Overview

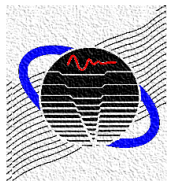
- Fundamental Concepts
- Illustrative Examples
- Conceptual Design
- Detailed Design
- Prototyping / Testing



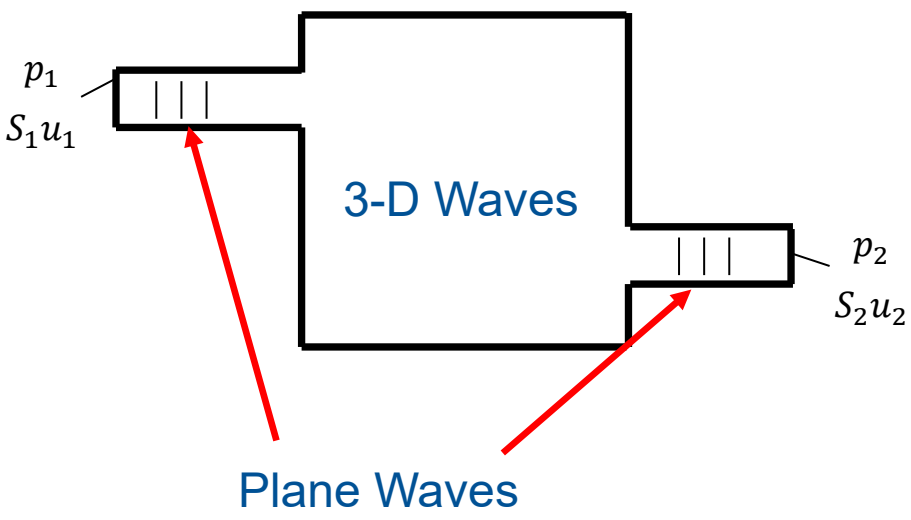
Determination of Transfer Matrix



$$\begin{Bmatrix} p_1 \\ S_1u_1 \end{Bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{Bmatrix} p_2 \\ S_2u_2 \end{Bmatrix}$$



Determination of Transfer Matrix



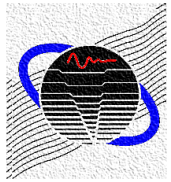
$$\begin{Bmatrix} p_1 \\ S_1u_1 \end{Bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{Bmatrix} p_2 \\ S_2u_2 \end{Bmatrix}$$

$$A_{11} = p_1\Big|_{u_1=1,u_2=0} \qquad A_{12} = p_1\Big|_{u_1=0,u_2=-1}$$

$$A_{21} = p_2\Big|_{u_1=1,u_2=0} \qquad A_{22} = p_2\Big|_{u_1=0,u_2=-1}$$

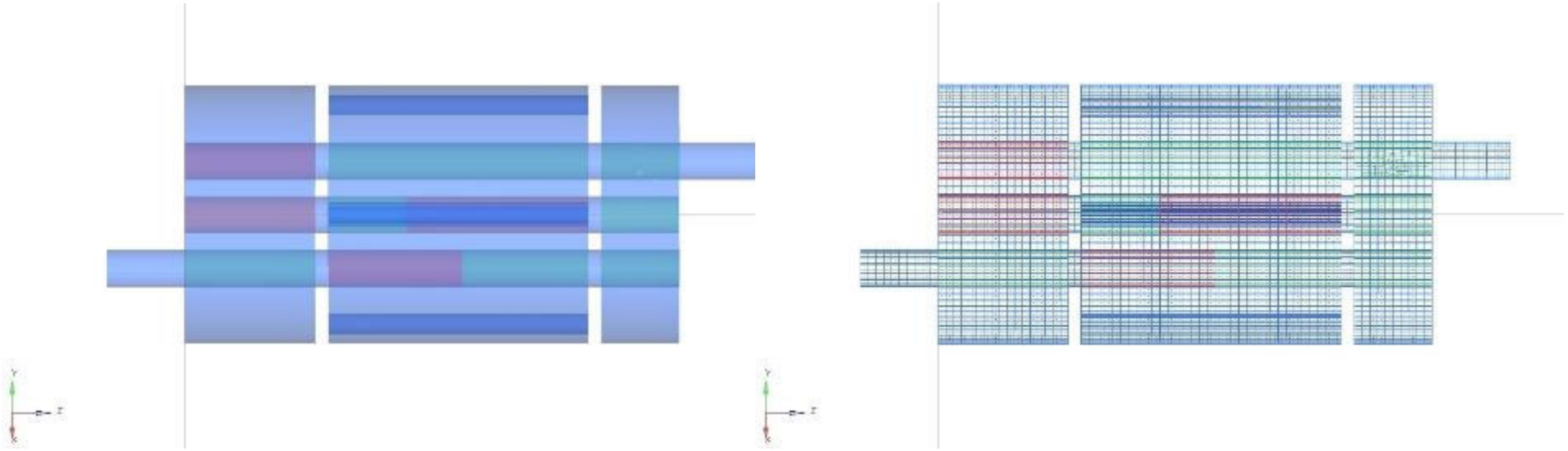
$$T_{11} = A_{11}/A_{21} \qquad T_{12} = \frac{1}{S_2}\left(A_{12} - \frac{A_{11}A_{22}}{A_{21}}\right)$$

$$T_{21} = S_1\frac{1}{A_{21}} \qquad T_{22} = \frac{S_1}{S_2}\left(-\frac{A_{22}}{A_{21}}\right)$$



Example Cross Flow Muffler

Virtual prototyping using acoustic BEM (MAP software)



Overview

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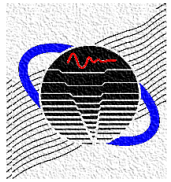
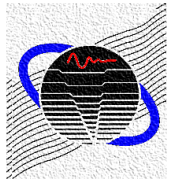
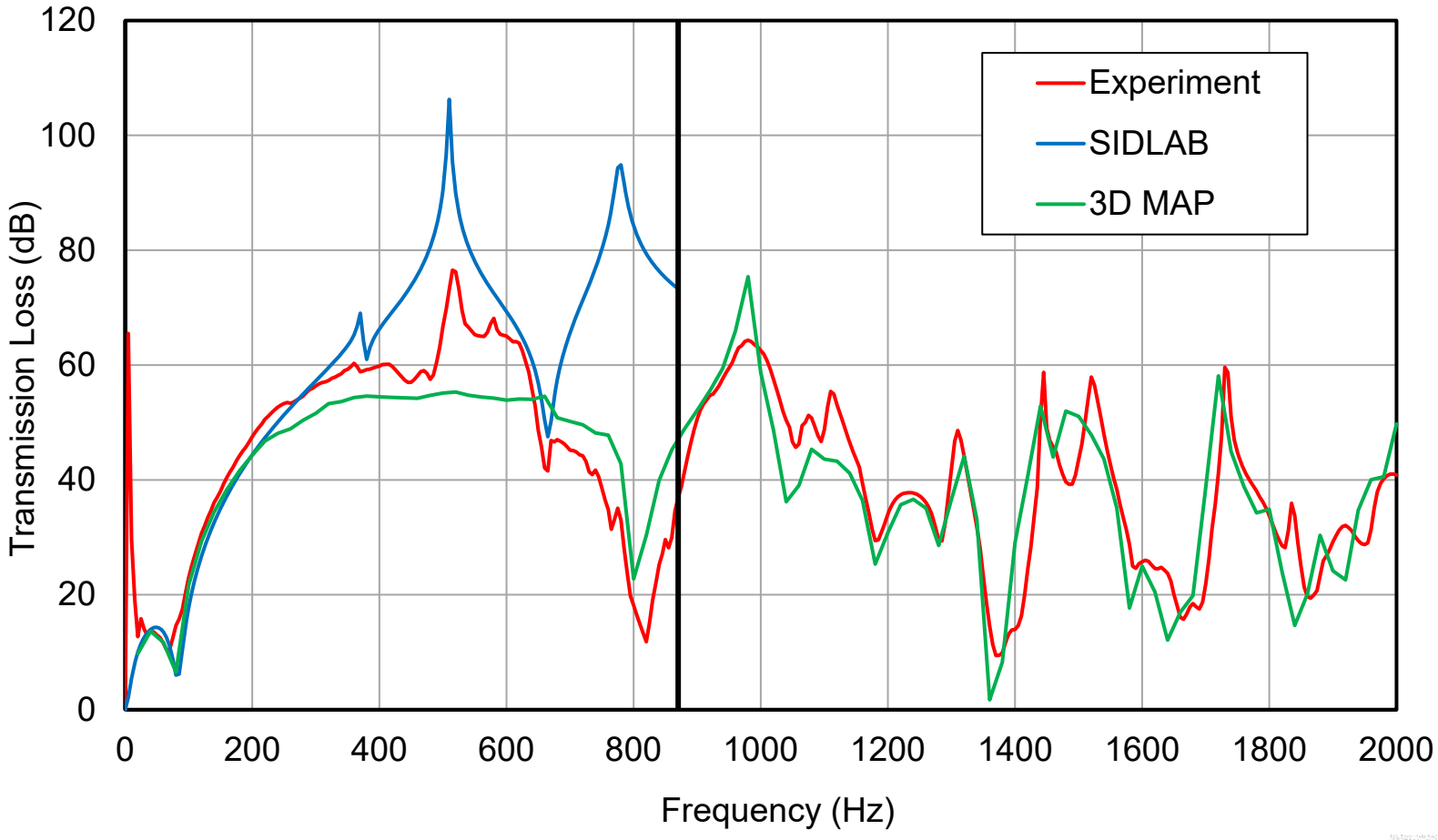


Transmission Loss Standards

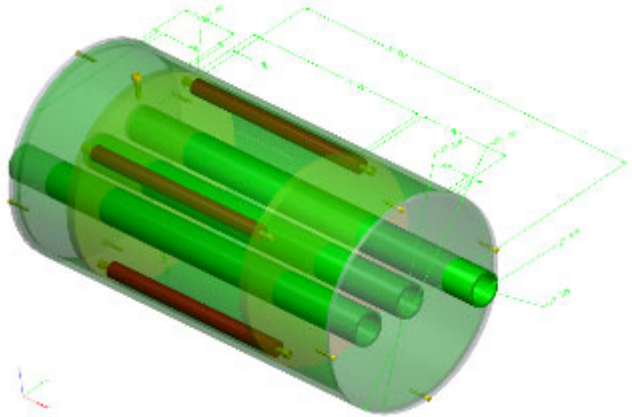
ASTM E2611-09, “Standard Test Method for Measurement of Normal Incidence Sound Transmission of Acoustical Materials Based on the Transfer Matrix Method,” American Society of Testing and Materials, Philadelphia, 2009.

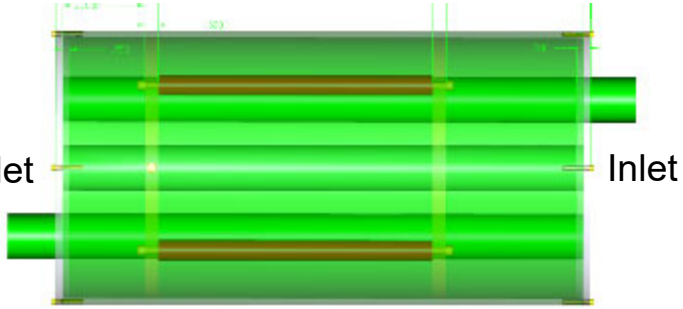


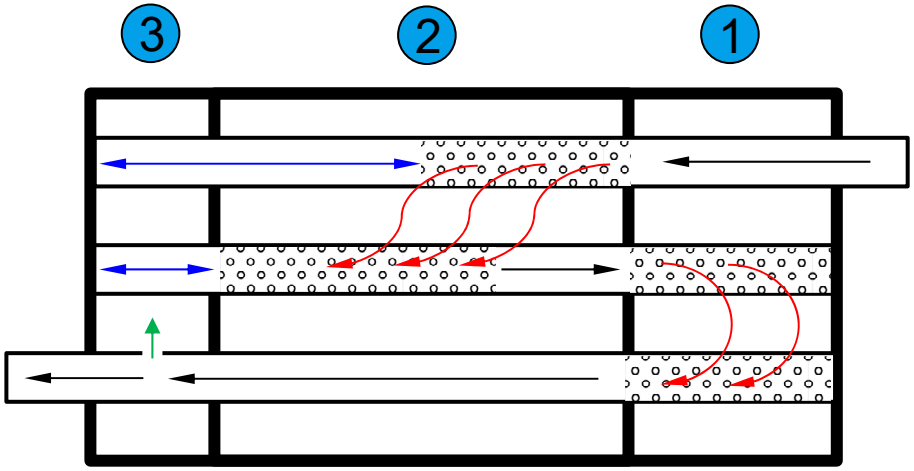
Example Cross Flow Muffler



Example Cross Flow Muffler







: Propagation

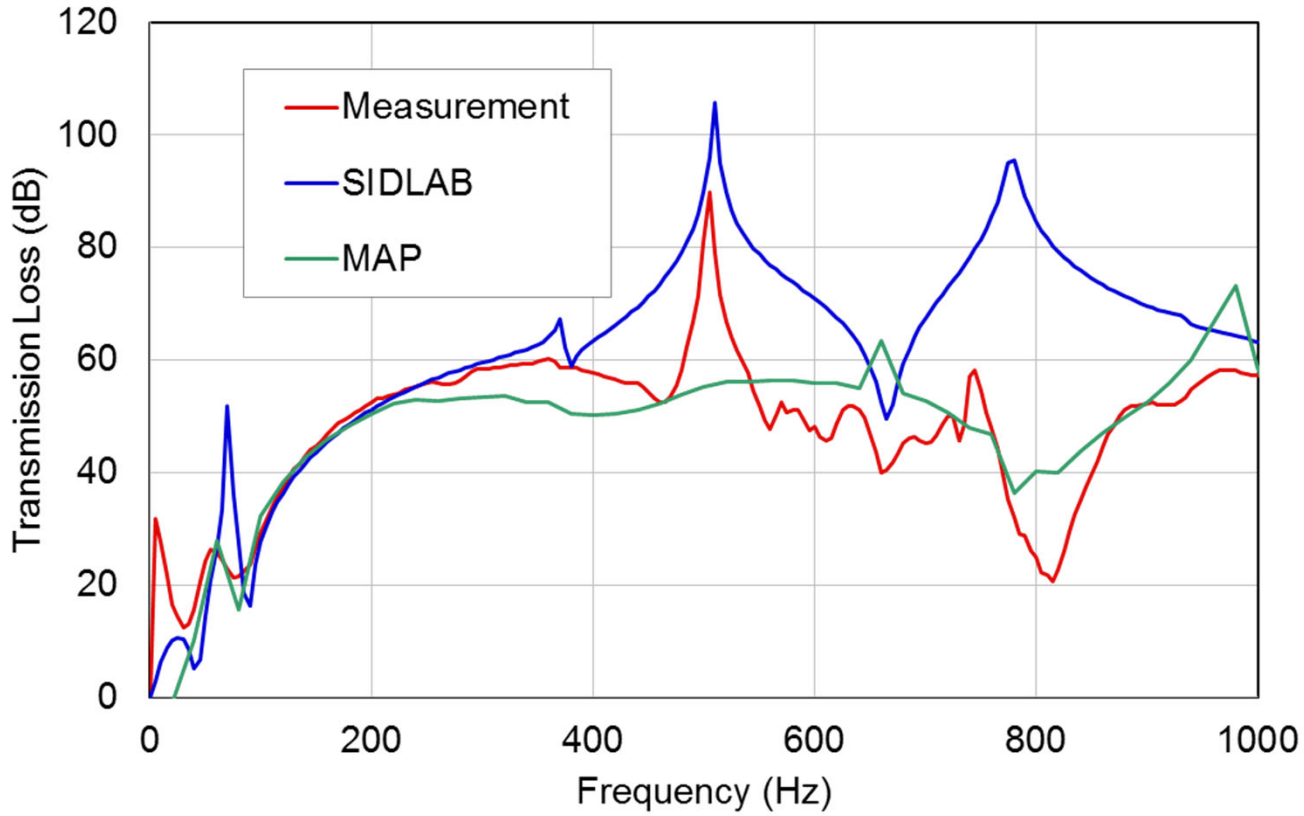
: Quarter-wavelength tube

: Cross-flow

: Helmholtz resonator

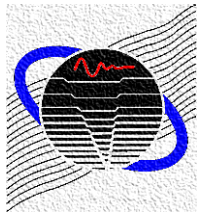
Example Helmholtz Resonator Included

Muffler Design



Summary

- Muffler fundamentals introduced.
- Recommend the use of plane wave simulation in the early design stages.
- Numerical simulation is useful for finalizing the geometry and details.
- Prototyping using 3D printing is recommended for an initial test.



References

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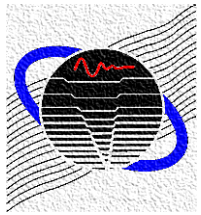
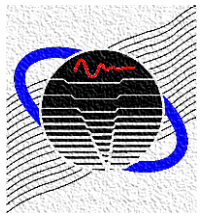
- D. W. Herrin, X. Hua, Y. Zhang, and T. Elnady, “The Proper Use of Plane Wave Models for Muffler Design,” SAE International Journal of Passenger Cars – Mechanical Systems, Vol. 7, No. 3 (2014).
- M. L. Munjal, Acoustics of Ducts and Mufflers, 2nd Edition, Wiley-Interscience, United Kingdom, 2014.



List of Variables

ρ	density of air
c	speed of sound in air
f	frequency
ω	angular frequency
k	wave number
σ	porosity
t	panel thickness
d_h	hole diameter
υ	air kinematic viscosity
M	flow Mach number
C_1	end correction coefficient
J_1	Bessel function of the first kind of order 1
b	the distance between two neighboring holes in axial directions
h	the distance between two neighboring holes in circumferential directions
ε_{mn}	If either m or n is equal to zero, $\varepsilon_{mn} = 1$; otherwise, $\varepsilon_{mn} = 0.5$;



Empirical Equations Perforated Panel

Eq. 4 (Rao and Munjal, 1986)

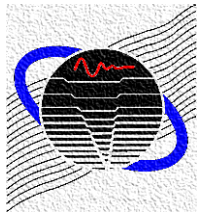
$$Z_{tr} = (1/\sigma)\left(7.337 \times 10^{-3}(1 + 72.23M) + j2.2245 \times 10^{-5} f(1 + 51t)(1 + 204d_h)\right)$$

Eq. 5 (Ji, 2010)

$$Z_{tr} = (1/\sigma)\left(\left(1 + t/d_h\sqrt{8kv/\rho c}\right) + jk(t + C_1 d_h)\right)$$

$$C_1 = \frac{4}{\pi^2}\frac{1}{(\zeta\eta)^{1/2}}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\varepsilon_{mn}\frac{J_1^2\left(\pi\sqrt{(m\zeta)^2 + (n\eta)^2}\right)}{\left(m^2(h/b) + n^2(b/h)\right)^{3/2}}$$

$$\zeta = d_h/b\,, \qquad \eta = d_h/h$$



Empirical Equations Perforated Panel

Eq. 6 (Allam and Åbom, 2011)

$$Z_{tr} = r_c + jx_c$$

β is equal to 2 for holes with rounded edges and 4 for holes with sharp edges

Surface Resistance $R_s = \frac{1}{2}\sqrt{2\eta\rho\omega}$

$$r_c = \mathrm{Re}\left(\frac{j\omega t}{\sigma c}\left(1 - \frac{2}{\kappa\sqrt{-j}}\frac{J_1\left(\kappa\sqrt{-j}\right)}{J_0\left(\kappa\sqrt{-j}\right)}\right)^{-1}\right) + \frac{2\beta R_s}{\sigma\rho c} + \frac{|\hat{u}_h|}{\sigma c}$$

the absolute value or the peak particle velocity inside the holes

$$x_c = \mathrm{Im}\left(\frac{j\omega t}{\sigma c}\left(1 - \frac{2}{\kappa\sqrt{-j}}\frac{J_1\left(\kappa\sqrt{-j}\right)}{J_0\left(\kappa\sqrt{-j}\right)}\right)^{-1}\right) + \frac{\delta_c\omega\left(1 + \frac{|\hat{u}_h|}{\sigma c}\right)^{-1}}{\sigma c}$$

Dimensionless shear wave-number

$$\kappa = d_h\sqrt{\omega/4\upsilon}$$

End correction

$$\delta_c = 0.85 d_h$$