Impedance Tube Measurements

Vibro-Acoustics Consortium Web Meeting
University of Kentucky
For a pipe, the cutoff frequency is defined as:

\[ f_{\text{cutoff}} = \frac{c}{1.71d} \]
Plane Waves in a Pipe

Daniel Russell, Penn State University
https://www.acs.psu.edu/drussell/Demos/superposition/superposition.html
Part 1 Measurement of Sound Impedance

ASTM E1050-95 (ISO 10534-2) Test Method

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Coordinate System and Microphone Locations

Sound source

Material sample

\( x_1 \)

\( x_2 \)
Plane Wave Theory

Total sound pressure at any point in the tube:

\[ P(x) = p_+ e^{-jkx} + p_- e^{jkx} \]

\( +x \) traveling wave \( -x \) traveling wave

The transfer function between points 1 and 2:

\[ H_{12} = \frac{P(x_2)}{P(x_1)} = \frac{p_+ e^{-jkx_2} + p_- e^{jkx_2}}{p_+ e^{-jkx_1} + p_- e^{jkx_1}} = \frac{e^{-jkx_2} + Re^{jkx_2}}{e^{-jkx_1} + Re^{jkx_1}} \]

\[ R = \frac{p_-}{p_+} \text{ is the pressure reflection coefficient of the material} \]
Solving for $R$:

$$R = \frac{e^{-jkx_2} - H_{12}e^{-jkx_1}}{H_{12}e^{jkx_1} - e^{jkx_2}}$$

Normalized specific boundary impedance:

$$\frac{z}{\rho c} = \frac{1 + R}{1 - R}$$

Normal incident sound absorption

$$\alpha = 1 - |R|^2$$
Two-Microphone Standards

1. ISO 10534-2, *Acoustics-Determination of sound absorption coefficient and impedance in impedance tubes - Part 2: Transfer-function method*

2. ASTM E1050-10, *Standard Test Method for Impedance and Absorption of Acoustical Material Using a Tube, Two Microphones and a Digital Frequency Analysis System*
Sound Absorption Measurement

Sample holder with rigid piston
Cutting with Rotating Blade

Inexpensive and accurate if kept sharpened

Stanley, Internoise 2012
Stamping Press System

Used for low-density fibrous materials

Stanley, Internoise 2012

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Water Jet Cutting

Expensive but accurate

Stanley, Internoise 2012
“Not so Good” and “Good” Specimens

Stanley, Internoise 2012
Stanley, 2012 Specimen Preparation

- Face uniformly flush with cell lip
- Front surface even across lip of sample holder
- Extremely small (at most) and consistent gap between specimen and sample holder
- No specimen compression in the holder
Variability of Melamine

6 Samples of 1.91 cm Melamine

Absorption Coefficient

Frequency (Hz)
Variability Glass Fiber

8 Samples of 5 cm Glass Fiber

Sound Absorption

Frequency (Hz)

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Effect of Cutter Size

8 Samples of 1 inch thick 0.6 lbs/ft³ Melamine

Absorption Coefficient vs. Frequency (Hz)

- 1.375 inch Diameter cutter
- 1.360 inch Diameter Cutter
- 1.375 inch Diameter Cutter (with needles)
Effect of Adding Needles

1.25 cm thick Melamine

Absorption Coefficient vs. Frequency (Hz)

- Original
- 3 needles
- 6 needles
- 12 needles
- 20 needles
- 25 needles
Summary

“While the use of an impedance tube system to measure acoustic absorption is not an extremely precise and repeatable process due to unavoidable variations of specimen cutting and cell fit, the disciplined use of the guidelines stated in this paper will help to insure that test results maintain a consistent level of accuracy and validity. The experience gained with repeated preparation and testing will also contribute to a better feel for more subtle aspects of preparation and specimen fitting for testing.”

Stanley, Internoise 2012
Part 2 Determination of Bulk Properties

- Determining the bulk properties
  - Complex wave number and characteristic impedance
  - Complex speed of sound and density
- Bulk properties are used
  - For designing layered absorbers
  - In FEM and BEM models
Porous Absorbers Property Determination

\[ k_c = \frac{\omega}{c'} \quad z_c = \rho'c' \]

Determination of Sound Absorption

\[
\begin{pmatrix}
    p_1 \\
    u_1
\end{pmatrix} =
\begin{bmatrix}
    \cos(k_c L) & jz_c \sin(k_c L) \\
    j/z_c \sin(k_c L) & \cos(k_c L)
\end{bmatrix}
\begin{pmatrix}
    p_2 \\
    u_2
\end{pmatrix}
\]

\[
z_s = \frac{p_1}{u_1} = -jz_c \cot(k_c L)
\]

\[
R = \frac{z_s - \rho c}{z_s + \rho c}
\]

\[
\alpha = 1 - |R|^2
\]
Porous Absorbers Layered

\[
[T] = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} = [T_1][T_2][T_3] \ldots [T_n]
\]

\[
z_s = \frac{T_{11}}{T_{21}}
\]

\[
R = \frac{z_1 - \rho c}{z_1 + \rho c}
\]

\[
\alpha = 1 - |R|^2
\]
Overview of Approaches

Direct Measurement
- Two load method  ASTM E 1050
- Two cavity method  Utsuno, 1989
- Three microphone method  Iwase et al., 1998

Measure Flow Resistance
- Curve fit to find flow resistivity based on empirical equations  Simón et al., 2006
- Find bulk properties from measured flow resistivity  Wu, 1988; Mechel et al., 2002

Measure Sound Absorption
- Curve fit to find Biot parameters based on theory
- Find bulk properties from fitted Biot parameters  Allard, J. F., 1999
Why Two-Loads?

\[
\begin{pmatrix}
    p_1 \\
    u_1
\end{pmatrix} =
\begin{bmatrix}
    T_{11} & T_{12} \\
    T_{21} & T_{22}
\end{bmatrix}
\begin{pmatrix}
    p_2 \\
    u_2
\end{pmatrix}
\]
Two-Load Method

For each load:

\[
A = j \frac{H_{1,ref} e^{-jkl_1} - H_{2,ref} e^{-jk(l_1+s_1)}}{2 \sin k s_1}
\]

\[
B = j \frac{H_{2,ref} e^{jk(l_1+s_1)} - H_{1,ref} e^{jkl_1}}{2 \sin k s_1}
\]

\[
C = j \frac{H_{3,ref} e^{jk(l_2+s_2)} - H_{4,ref} e^{jkl_2}}{2 \sin k s_2}
\]

\[
D = j \frac{H_{4,ref} e^{-jkl_2} - H_{3,ref} e^{-jk(l_2+s_2)}}{2 \sin k s_2}
\]
Transmission Loss Measurement

Pressures and particle velocities at two ends of the sample:

\[ p_0 = A + B \]
\[ u_0 = \frac{(A - B)}{\rho c} \]
\[ p_d = Ce^{-jkd} + De^{jdk} \]
\[ u_d = \frac{(Ce^{-jdk} - De^{jdk})}{\rho c} \]

Four-pole matrix (subscripts \(a\) and \(b\) indicate acoustic loads)

\[
T = \begin{bmatrix}
    p_{0a}u_{db} - p_{0b}u_{da} & p_{0b}p_{da} - p_{0a}p_{db} \\
    p_{da}u_{db} - p_{db}u_{da} & p_{da}u_{0b} - p_{db}u_{0a} \\
    u_{0a}u_{db} - u_{0b}u_{da} & p_{da}u_{db} - p_{db}u_{da}
\end{bmatrix}
\]

\[
TL = 20 \log_{10} \left| \frac{1}{2} \left( T_{11} + \frac{T_{12}}{\rho c} + \rho cT_{21} + T_{22} \right) \right|
\]
Two Loads

Load A: Open tube. Load B: 10 cm sound absorbing material.

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Two Load Method

\[
\begin{bmatrix}
T_{11} & T_{12} \\
T_{21} & T_{22}
\end{bmatrix} =
\begin{bmatrix}
\cos(k_c d) & jz_c \sin(k_c d) \\
\frac{j \sin(k_c d)}{z_c} & \cos(k_c d)
\end{bmatrix}
\]

\[ z_c = \frac{T_{12}}{\sqrt{T_{21}}} \]

\[ k_c d = \arctan \left( \frac{T_{12}}{j T_{11} z_c} \right) \]
Two Cavity Method

\[ z_c = \pm \sqrt{\frac{z_1'z_1(z_2 - z_2') - z_2z_2'(z_1 - z_1')}{(z_2 - z_2') - (z_1 - z_1')}} \]

\[ k_c = \left( \frac{1}{2jd} \right) \ln \left( \frac{(z_1 + z_c)(z_2 - z_c)}{(z_1 - z_c)(z_2 + z_c)} \right) \]
Three Microphone Method

Iwase et al., 1998

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Reflection coefficient:

\[ R = \frac{e^{jks} - H_{12}}{H_{12} - e^{-jks}} e^{2jkl} \]
Three Microphone Method

The complex wave number of the sample is:

\[ k_c = \frac{1}{d} \arccos \left( \frac{1 + R}{e^{jkL} + Re^{-jkL}} \right) H_{23} \]

The characteristic impedance of the sample is:

\[ z_c = jz_0 \frac{1 + R}{1 - R} \tan(k_c d) \]
2.5 cm Melamine Bulk Properties

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Melamine Sound Absorption Coefficient

2.5 cm Melamine Foam

![Graph showing sound absorption coefficients for different setups.](image)

- Blue line: Two Load
- Red line: Two Cavity
- Green line: Three Microphone

Frequency (Hz) vs. Absorption Coefficient

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Melamine Transmission Loss

2.5 cm Melamine Foam

Transmission Loss (dB)

<table>
<thead>
<tr>
<th>Transmission Loss (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two Load</td>
</tr>
<tr>
<td>Two Cavity</td>
</tr>
<tr>
<td>Three Microphone</td>
</tr>
</tbody>
</table>

Frequency (Hz)

0 1000 2000 3000 4000 5000

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Find bulk properties from measured flow resistivity Wu, 1988; Mechel et al., 2002
Measure Flow Resistance

Sample (thickness $t$)

Flow resistance:
$$r_s = \frac{\Delta P}{u}$$

Flow resistivity:
$$\sigma = \frac{r_s}{t}$$

Measure Flow Resistivity Using ASTM C522

Plug into Empirical Models
See Sound Absorptive Material Webinar

Wu, 1988; Mechel et al., 2002
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Allard, J. F., 1999

Find bulk properties from measured flow resistivity
Wu, 1988; Mechel et al., 2002
Curve Fitting Methods

- Measure sound absorption coefficient
  - Minimize least squares error with respect to empirical equations
    - Flow Resistivity
      - Plug into empirical models
        - Wu (1988) or Mechel et al. (2002)
  - Measure sound absorption coefficient
    - Minimize least squares error with respect to analytical equations
      - Biot Parameters
        - Plug into theoretical model
          - Johnson-Champoux-Allard model

Simon et al., 2006; Allard, J. F., 1999
2.5 cm Melamine Bulk Properties

- Measured Flow Resistivity (Re): 12,100 Rayls/m
- Simón Curve Fitting Flow Resistivity (Re): 11,400 Rayls/m

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Summary

- Compared the various methods for determining the bulk properties of sound absorbers.
- Curve fitting approaches are adequate for common sound absorbing materials.
Part 3 Covers and Adhesives

• How do adhesives and covers between sound absorbing layers affect the overall performance?
• How can we determine the acoustic properties of an adhesive or a cover?
Melamine Effect of Adhesives

1.8 cm thick 9.6 kg/m³ Melamine Foam

Absorption Coefficient vs. Frequency (Hz)

- 0.0 g
- 0.2 g
- 0.3 g
- 0.4 g
- 0.5 g
- 0.6 g
- 0.7 g
- 0.8 g
- 0.9 g
- 1.0 g
Fiber Effect of Adhesives

2.5 cm thick 19.2 kg/m³ Fiber

Absorption Coefficient vs. Frequency (Hz)

- 0.0 g
- 0.2 g
- 0.3 g
- 0.4 g
- 0.5 g
- 0.6 g
- 0.7 g
- 0.8 g
- 0.9 g
- 1.0 g
Transfer Impedance

A transfer impedance is commonly used to model perforates, covers and source impedance. Particle velocity is assumed to be continuous across the layer.

\[ z_{tr} = \frac{p_1 - p_2}{u} \]

\[ u_1 = u_2 = u \]

\[ p_1 \quad p_2 \]
Transfer Impedance Measurement

Measurement of perforates or fabrics

\[ z_1 = \frac{p_1}{u_1} \]

\[ z_2 = \frac{p_2}{u_2} \]

\[ Z_{tr} = z_1 - z_2 \]
Transfer Impedance Measurement

Measurement of adhesive layer or bonded cover

\[ z_1 = \frac{p_1}{u_1} \]

\[ z_2 = \frac{p_2}{u_2} \]

\[ Z_{tr} = z_1 - z_2 \]
Transfer Impedance Glue

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0.6 g of Glue Effect of Different Substrate

Transfer Impedance (Re)

Frequency (Hz)

Melamine
Polyester
Fiber

Transfer Impedance (Im)

Frequency (Hz)

Fiber
Melamine
Polyester
Effect of Covers

![Absorption Coefficient vs. Frequency](image)

- Fiber with Cover
- Fiber Only

Absorption Coefficient

Frequency (Hz)
Measurement Transfer Impedance

\[ z_{tr} = z_1 - z_2 \]

\[ z_1 = \frac{p_1}{u_1} \]

\[ z_2 = \frac{p_2}{u_2} \]

Transfer Impedance

Frequency (Hz)

0.0 1.0 2.0 3.0 4.0 5.0

0 1000 2000 3000 4000 5000

Resistance

Reactance
Test Case 2

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Test Case 3

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Test Case 3

Top Cover Transfer Impedance

Top Cover + Foam Layer

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Test Case 3

Glue Transfer Impedance

- Transfer Impedance (Re)
- Transfer Impedance (Im)

Glue + Fiber Layer

- Transfer Matrix Method
- Measured

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Test Case 3

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Summary

- Transfer impedance is determined using an impedance difference approach.
- Demonstrated capability to predict properties of layered absorbers.
Transmission Loss Standards


This standard
• Is designed to measure TL of ‘soft’ barrier materials
• Is not specifically aimed at mufflers
• Only mentions the two-load method*

* Also one-load method for uniform materials

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Two-Load Method

To and Doige, 1979

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Two-Source Method

Configuration A

Loudspeaker 1 2 3 4 arbitrary $Z_r$

Muffler Impedance tube

Configuration B

$Z_r$

1 2 3 4 Loudspeaker

Munjal and Doige, 1990
Transmission Loss Calculation

$$TL = 20 \log_{10} \left( \frac{1}{2} \left| T_{11} + \frac{T_{12}}{\rho c} + \rho c T_{21} + T_{22} \right| \right)$$

![Graph showing transmission loss in dB vs frequency (Hz)](image)

- Two-source method
- Two-load method
Future Directions

• Develop a better method (or refine the approach) for determining the transmission loss of hard samples in an impedance tube.

• Explore best practices for cutting samples using the new water jet cutter at the University of Kentucky.
References